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STRUCTURAL MODEL AND MATERIALS EVALUATION PROCEDURES

BALLAST AND FOUNDATION MATERIALS RESEARCH PROGRAM

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16. Abstract

An adequate engineering analysis of conventional railway track support system (CRTSS) requires the consideration of all the major components of the track support system. Past efforts in this area have not been satisfactory because of lack of proper material characterization and very simplified modelling of the CRTSS. An attempt has been made to model the CRTSS using the finite element method that would allow a determination of the transient response of the CRTSS by incorporating proper material characterization. Because of the complex three-dimensional geometry, the analytical modelling was divided into two stages; namely, a longitudinal analysis stage and a transverse analysis stage. Stress dependent material properties of the ballast, the subballast, and the subgrade can be used with the finite element method.

The finite element model has been validated using the measured response at Section 9 of the Kansas Test Track. Good agreement was obtained between the measured response and that calculated using the finite element model.

The report also describes ballast and subgrade materials evaluation procedures. The repeated load triaxial testing procedure that has been selected for evaluating the resilient and permanent deformation characteristics of ballast and subgrade soil materials is described in detail.

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PREFACE

This report has been generated as part of a sub-contract between the Association of American Railroads Research and Test Department, and the University of Illinois.

This sub-contract is part of a larger contract which is a cooperative effort between the Federal Railroad Administration and the Association of American Railroads on improved track structures. The entire program is in response to recognition of the desire for a more durable track structure. To this end, the program is a multi-task effort involving 1) the development of empirical and analytical tools for the description of the track structure so that the economic trade-offs among track construction parameters such as tie size, rail size, ballast depth and cross section, type, subgrade type, stiffness, may be determined. 2) methodologies to upgrade the existing track structures to withstand new demands in loading, 3) development of performance specifications for track components, and 4) investigating the effects of various levels of maintenance.

This particular report describes an analytical tool for investigating track behavior.

A special note of thanks is given to Mr. William S. Autrey, Chief Engineer of Santa Fe, Mr. R. M. Brown, Chief Engineer of Union Pacific, Mr. F. L. Peckover, Engineer of Geotechnical Services, Canadian National Railway, Mr. C. E. Webb, Asst. Vice President, Southern Railway System, as they have served in the capacity of members of the Technical Review Committee for this Ballast and Foundation Materials Program, and Dr. R. M. McCafferty as the Contracting Officer's Technical Representative of the FRA on the entire research program.

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Association of American Railroads

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LIST OF ABBREVIATIONS AND SYMBOLS

C = cohesion

E = Young's modulus

$|E|$ = complex modulus

E_R = resilient modulus

e_N = permanent axial strain after N loading cycles

I = moment of inertia of beam (rail) section

$\{K\}$ = overall structural stiffness matrix

$\{k\}$ = element structural stiffness matrix

N = number of repeated loading cycles

n = initial porosity

$\{P\}$ = applied nodal forces for the whole system

$\{p\}$ = element nodal force vector

R_f = a ratio relating the stress difference at failure to the asymptotic value of stress difference

$\{U\}$ = nodal displacements for the whole system

$\{u\}$ = element nodal displacement vector

u_i = displacement at node i in the x direction

v_i = displacement at node i in the y direction

x,y = coordinates

α_{xy} = shear strain in the plane of x, y

ε_p = permanent axial strain

E_x, E_y = normal strains in the x and y directions respectively

u = Poisson's ratio

σ_1 = major principal stress

σ_2 = intermediate principal stress

σ_3 = minor principal stress

σ_d = $\sigma_1-\sigma_3$ = deviator stress

σ_x,σ_y = normal stresses in the x and y directions respectively

τ_{xy} = shear stress in the plane of x, y

θ = sum of principal stresses = $\sigma_1+\sigma_2+\sigma_3$

= $\sigma_1+2\sigma_3$ in a triaxial test

ϕ = angle of internal friction

CHAPTER 1

INTRODUCTION

Two integral parts of the "Ballast and Foundation Materials Research Program are the development of a "mechanistic structural analysis model" and the simultaneous development of testing procedures for evaluating material properties needed as inputs to the structural model.

The analysis model which is described herein provides for adequate engineering analysis of the track system response. It considers the rail-tie assemblage to be composed of an elastic beam (rail) on springs (ties). This assemblage is founded on the ballast-subgrade system.

Previous track support system analysis models have not used adequate material models for representing the behavior and properties of ballast and subgrade materials. It has been conclusively shown that the structural response characteristics of ballast and subgrade soil materials subjected to repeated loads of short duration are dependent on the state of stress. The structural analysis model which has been developed considers the "stress-dependent" nature of these materials.

In order to provide the required input to this model, it is necessary to evaluate the stress-dependent nature of the ballast and subgrade soil materials. Appropriate laboratory testing procedures have been established for evaluating the repeated, dynamic loading response properties of ballast and subgrade soil. These procedures are described herein.

REPORT OBJECTIVES

The general objective of this report is to summarize the activities accomplished during the project phase entitled: "Development of a Structural Model and Materials Evaluation Procedures." Table 1 summarizes the

Table 1. Summary of Project Work Phases

Phase I - Technical Data Bases
Phase II - Development of Structural Model and Materials Evaluation Procedures
Phase III - Parameter Studies and Sensitivity Analyses
Phase IV - Materials Evaluation Study
Phase V - Economic Evaluation
Phase VI - Preparation of Conclusions Summary, and Recommendations

various phases of endeavor of this project. An earlier report (1) summarized Phase 1 activities. Specific objectives of this report are:

1. Describe the background, development, and validation of the mechanistic structural analysis model and

2. Describe the testing procedures that will be used for evaluating needed material properties.

REPORT ORGANIZATION

Chapters 2 and 3 deal with background, development and validation of the structural analysis model. Chapters 4 and 5 contain a description of the materials testing procedures.

CHAPTER 2

EXISTING RAILWAY SUPPORT SYSTEM ANALYSIS METHODS

GENERAL

Over the years, sporadic efforts have been made to develop analytical methods for evaluating the structural response of railway support systems. Early attempts to closely model the support system were hampered by the lack of satisfactory theories for representing the behavior of the various components of the system. In the last quarter century, extensive developments in track and pavement system modelling have coincided with developments in electronic computer technology.

One of the major limitations with "track system behavior models" that existed at the time of initiation of this project was the inadequate representation of the structural behavior of ballast and subgrade soil materials. In most track support system models, the ballast-subgrade system has been represented by an "elastic" half space or by a "spring constant".

Recent developments in highway and airfield pavements technology have demonstrated that the repeated load response of ballast-like granular materials and fine-grained soils is very much dependent on the state of stress. For example, Figures 1 and 2 illustrate typical "elastic" or "resilient" response of granular and fine-grained soil materials as a function of stress level. As shown in Figure 1, the resilient modulus* of a granular material increases as either $(\sigma_1+2\sigma_3)$ or σ_3 (where σ_1 is the major principal stress and σ_3 is the minor principal stress) increases. Figure 2 shows that the resilient modulus of a fine-grained soil decreases as the magnitude of

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*Resilient modulus = $\dfrac{\text{repeated deviator stress}}{\text{elastic or recoverable axial strain}}$

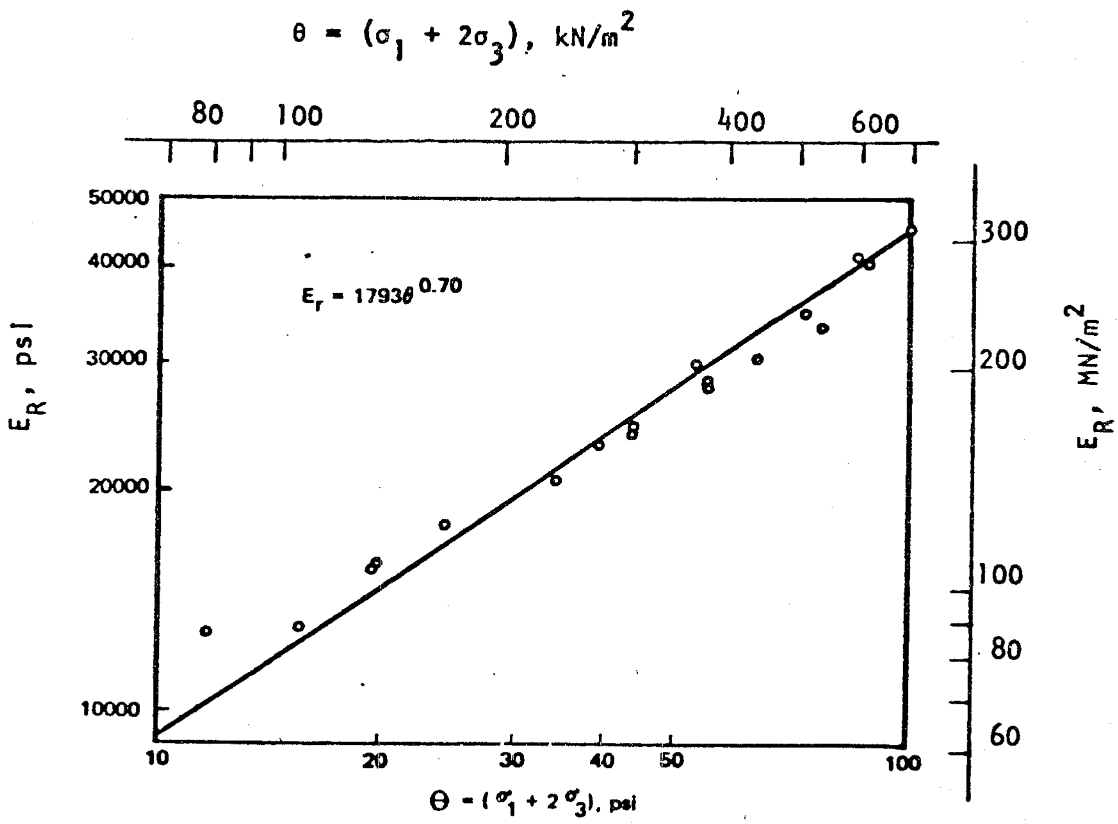


Figure 1. Typical Resilient Modulus vs Sum of Principal Stresses for Granular Material (From Ref. 35).

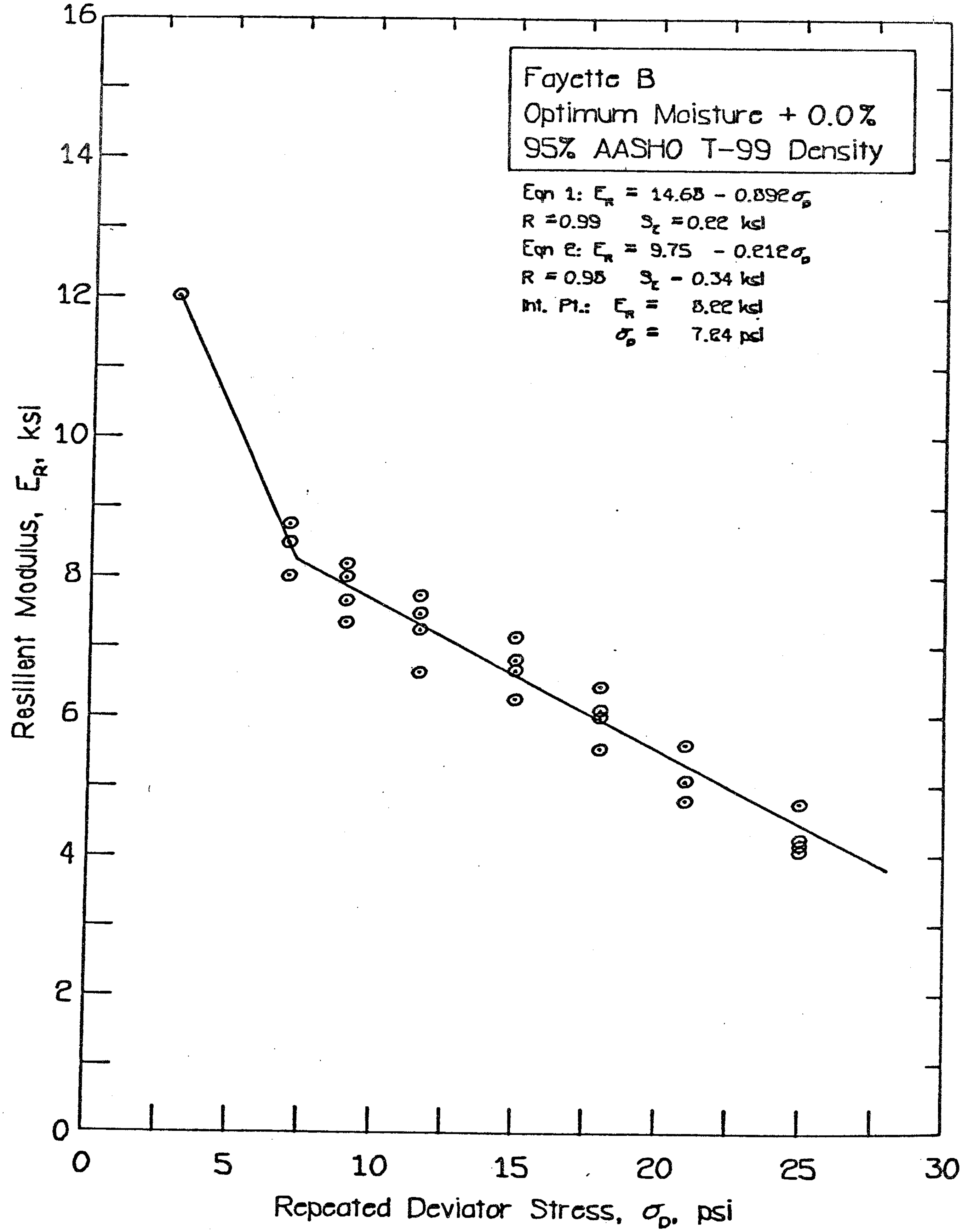


Figure 2. Typical Resilient Response Data for a Fine-Grained Soil (From Ref. 35).

repeated deviator stress, σ_d ($\sigma_d = \sigma_1 - \sigma_3$), increases.

The significance of these behavior patterns is that in a track support system, the "stiffness" properties of the ballast and subgrade soil materials are not constant since the stress state varies as a function of position in the system.

It must be emphasized that one of the primary objectives of the structural model development as undertaken in this project is to develop a procedure which will calculate with a reasonable degree of accuracy the stress and strain conditions in the ballast and subgrade.

EXISTING ANALYTICAL METHODS

In an earlier project report (1) a detailed discussion was presented concerning the aspects of the various existing track system analytical methods. These various methods are listed below and pertinent references given.

1. Beam on Elastic Foundation (4, 5)
2. Meacham's Modification of Beam on Elastic Foundation (6)
3. Finite Beam on an Elastic Foundation (7, 8)
4. Analytical Method for Track Structure Subjected to Moving Loads (9)
5. General Boussinesq Method (as described by Ireland) (10)
6. Finite Element Methods (11, 12, 13)

GENERAL CONCLUSIONS CONCERNING APPLICABILITY AND VALIDITY OF EXISTING METHODS

Numerous theories, techniques and/or procedures have been developed for calculating stress-strain and deflection or deformation conditions in railway support systems. Even the most recent developments in track structure analysis methodology have concentrated on "realistic" representation of the rail-fastener-tie components while representing the ballast and subgrade as either springs or as linear-elastic-homogeneous materials. Most of the attention has been directed to rail behavior rather than ballast and subgrade soil behavior.

Recent developments in highway and airfield pavement technology have demonstrated that the repeated load response of granular materials and subgrade soils is very much dependent on the stress conditions existing in the materials. Thus, realistic representation of these materials in a track system analysis model requires that their stress-dependent nature be adequately considered.

For this project it was concluded that in order to properly evaluate the influence of ballast and subgrade materials on the behavior of the track structure, it would be necessary to develop a model which would consider actual repeated load properties of ballast and subgrade materials.

Chapter 3 describes in detail the model that has been developed.

CHAPTER 3

DEVELOPMENT OF THE MODEL FOR ANALYSIS OF CONVENTIONAL TRACK SUPPORT SYSTEMS

GENERAL

Based on current technology, it appears that the ballast-subgrade components of the track support system can be more realistically represented than has been the case with existing "track support system analysis models". Technology and structural models in the field of highway and airfield pavements have advanced to the point where the "stress dependent" nature of granular and subgrade soil materials can be considered. Basically three modeling approaches have been recently developed for the highway and airfield pavements, these are:

1. Finite element procedures
2. Iterative closed form elastic procedures
3. Closed-form visco-elastic procedures

Of these three, the first and second have been found to be the most applicable.

THEORETICAL DEVELOPMENT

General

For many field problems it has been virtually impossible to obtain analytical solutions because of the complexity of geometry, boundary conditions, and material properties unless certain simplifying assumptions are made which result in a change or modification of the characteristics of the problem. With the advent of high speed digital computer methods, solution of complex field problems has been facilitated. One of the most powerful methods that has evolved is the "finite element" method. This method is applicable to a wide spectrum of complex, boundary value problems in engineering.

Realistic analysis of the conventional track support system requires that primary response parameters, i.e., deflections, stresses, and strains be evaluated for each component of the system.

An effective analytical model for the track system should be capable of:

1. Evaluation of the system comprised of the basic structural components of the system, viz, rails, tie-plates, ties, ballast, sub-ballast, and subgrade.
2. Consideration of realistic material characterization for each system component.
3. Consideration of the finite horizontal extent of the ballast and sub-ballast in a typical cross section of a track support system.
4. Consideration of arbitrary loading conditions including non-uniform, non-symmetrical and dynamic loading conditions.
5. Evaluation of the response of the system to changes in material properties caused by changing environmental conditions (viz, the effect of temperature on the response of the rails, the effect of various degress of saturation of the subgrade, etc.).

With proper application, the finite element method is capable of the foregoing.

The Finite Element Method

Basically, in the "finite element" method the system to be analyzed is represented by an assemblage of subdivisions or discrete bodies called "finite elements." These elements are considered interconnected at joints which are called nodes or nodal points. Simple functions are chosen to approximate the distribution or variation of the actual displacements over each finite element. Such assumed functions are called displacement functions or shape functions. Relationships are then established between these generalized displacements (usually denoted as $\{u\}$) and generalized forces (usually denoted as $\{p\}$) applied at the nodes using the principal of virtual work or some other variational principle. This element force-displacement relationship is expressed in the form of element stiffness matrix (usually denoted as $[k]$) which incorporates the material and geometrical properties of the element, viz.

$$[k]\ \{u\} = \{p\} \tag{1}$$

The overall structural stiffness matrix, $[K]$, is then formulated by superimposing the effects of the individual element stiffness using the topological or the element connectivity properties of the structure. The overall stiffness matrix is used to solve the set of simultaneous equations of the form:

$$[K]\ \{U\} = \{P\} \tag{2}$$

where:

$\{P\}$ = applied nodal forces for the whole system

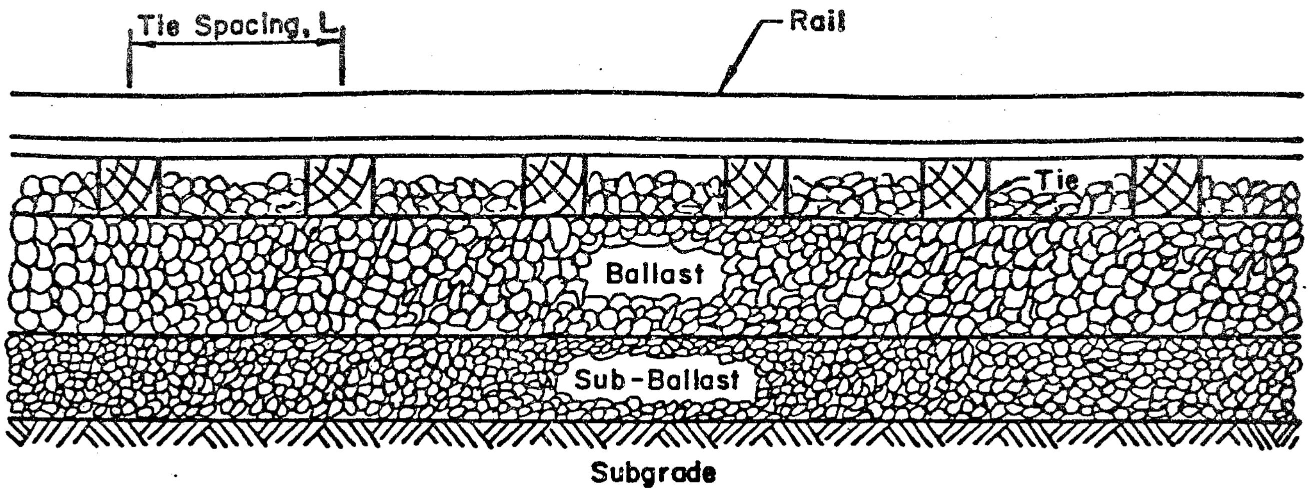
$\{U\}$ = resulting nodal displacements for the whole system

Extensive discussion of the principles and concepts of finite element methods can be found in many references including 14 and 15.

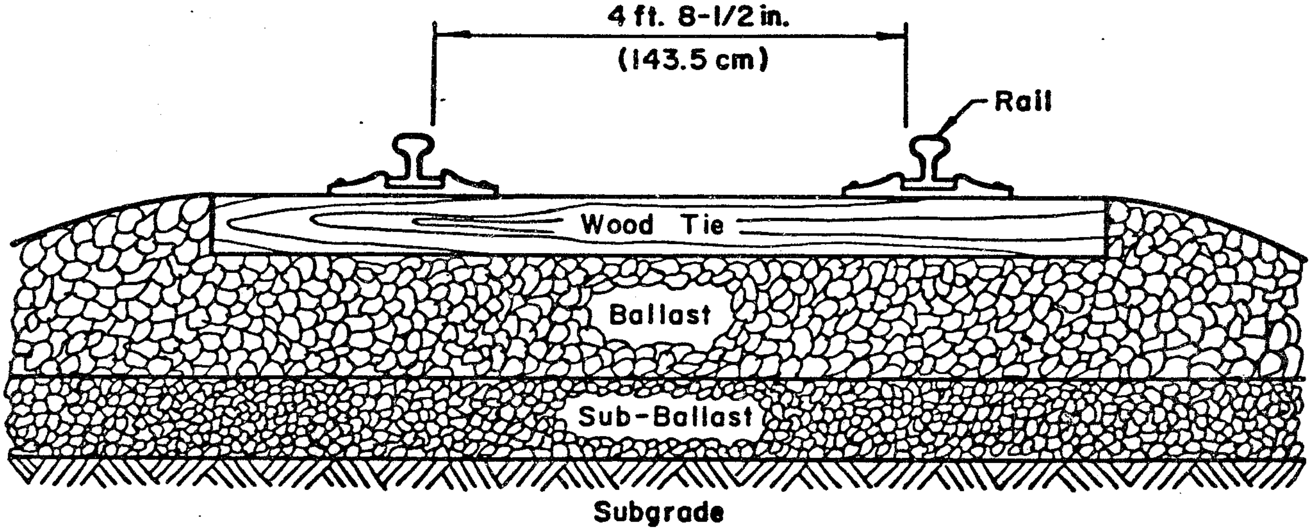
Representation of the Conventional Railway Track as an Assemblage of Discrete Bodies (Finite Elements)

A typical longitudinal section and a typical transverse section of a conventional railway track are shown in Figure 3. It can be seen that because of the three-dimensional geometry and non-uniform loading conditions, analysis of the conventional railway track structure should consider a three-dimensional approach. While it is possible to formulate a three-dimensional finite element model that would represent the system, the amount of discretization and the computer costs required for solution of the problem would be high and probably impractical.

If the symmetrical nature of the loading in the transverse direction is examined, Figure 3, it is apparent that a two stage analysis might provide a



(a) Longitudinal Section



(b) Transverse Section

Figure 3. A Typical Longitudinal and a Typical Transverse Section of a Conventional Railway Track.

reasonable engineering approach. In this two stage analysis, a longitudinal analysis is performed followed by a transverse analysis.

The longitudinal analysis considers point loads (corresponding to wheel loads), acting on a single rail sitting on the tie-ballast-subgrade system. Figure 4 shows a typical finite element mesh used for the longitudinal analysis. The rail-tie subsystem is represented as a continuous beam supported on tie springs. Rectangular planar elements are used to represent the ballast, subballast and the subgrade. The thickness of the elements is varied with depth using a "pseudo" plane strain technique (discussed later) to account for the spread of loading in the direction perpendicular to the plane. This allows a three-dimensional load spread which is known to exist in practice, to be simulated with a two-dimensional model. The displacement components are assumed to vary linearly over each element.

The transverse analysis uses the output from the longitudinal analysis for input. Either the maximum reaction or the maximum deflection at a tie obtained from the longitudinal analysis is used as input as a tie which rests on the ballast-subgrade system. Experience with the use of the model indicates that the maximum deflection at a tie would be more appropriate for use as input for the transverse analysis. Again the "pseudo" plane strain technique is used. Rectangular element representation is used for the tie, ballast, sub-ballast, and subgrade materials, and the displacement components are assumed to vary linearly over each element. Figure 5 shows the finite element mesh used for the transverse analysis.

Incorporation of the Pseudo-Plane Strain State

As a first approximation a longitudinal analysis can be performed using plane strain considerations with the thickness of the plane strain section being equal to half the tie length. However, experience (Ref. 4, 16, 17, 18)

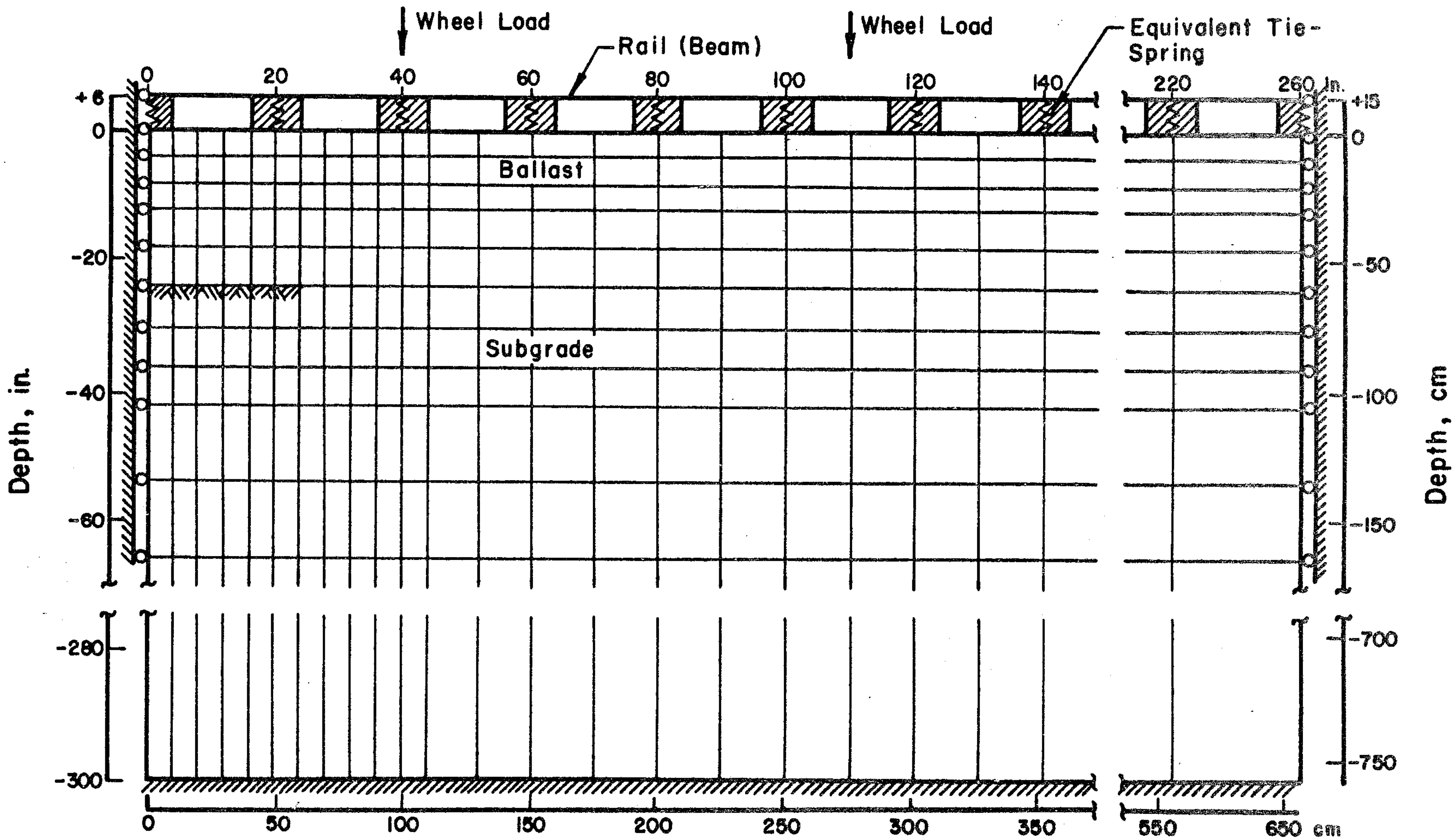


Figure 4. A Typical Finite Element Mesh Used for Longitudinal Analysis.

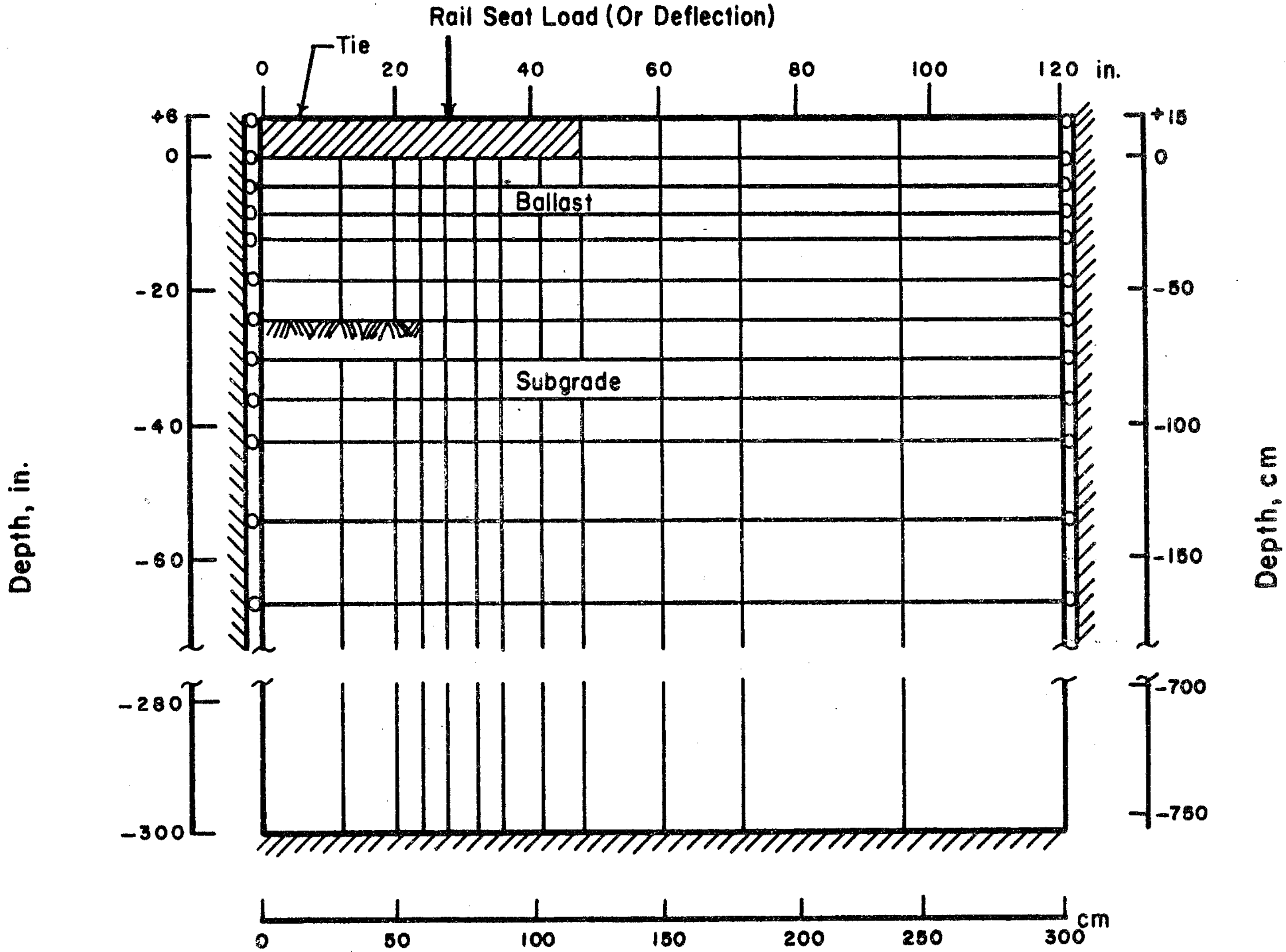


Figure 5. A Typical Finite Element Mesh Used for Transverse Analysis.

and preliminary calculations have shown that most of the load transfer between the tie and the ballast takes place within a smaller tie bearing area as shown schematically in Figure 6. As a result, the extent of the tie length under each rail that is effectively utilized in the load transfer to the ballast was then taken as 18 in. (45.7 cm).

The plane strain approximation for the longitudinal analysis is shown in Figure 7. Initial computer runs using the above discussed plane strain approximation showed that the normally expected stress dissipation (distribution) with depth was not obtained. The plane strain state generally distributes the load in two directions only and thus severely restricts the diminishing of stress with depth as would be expected under the actual track system.

After much consideration it was decided to use a "psdueo" plane strain state for the longitudinal model. In a pseudo-plane strain analysis, the finite element thickness is allowed to increase with depth to simulate 3 dimensional stress distribution with depth. The pseudo-plane strain state is shown in Figure 8.

An important factor in using the pseudo-plane strain state is the determination of the rate of increase of element thickness with depth. Presently, it is assumed that the rate of increase of element thicknesses with depth is constant. This rate of increase is denoted by a parameter called the "angle of distribution", ϕ, as shown in Figure 8.

A similar approach, using the pseudo-plane strain analysis, is used for the transverse analysis. The complete tie is assumed as being effective in transferring the load from the tie to the ballast surface and the thickness of the finite elements at the surface of the ballast is made equal to the width of the tie. The thickness of the finite elements is increased with

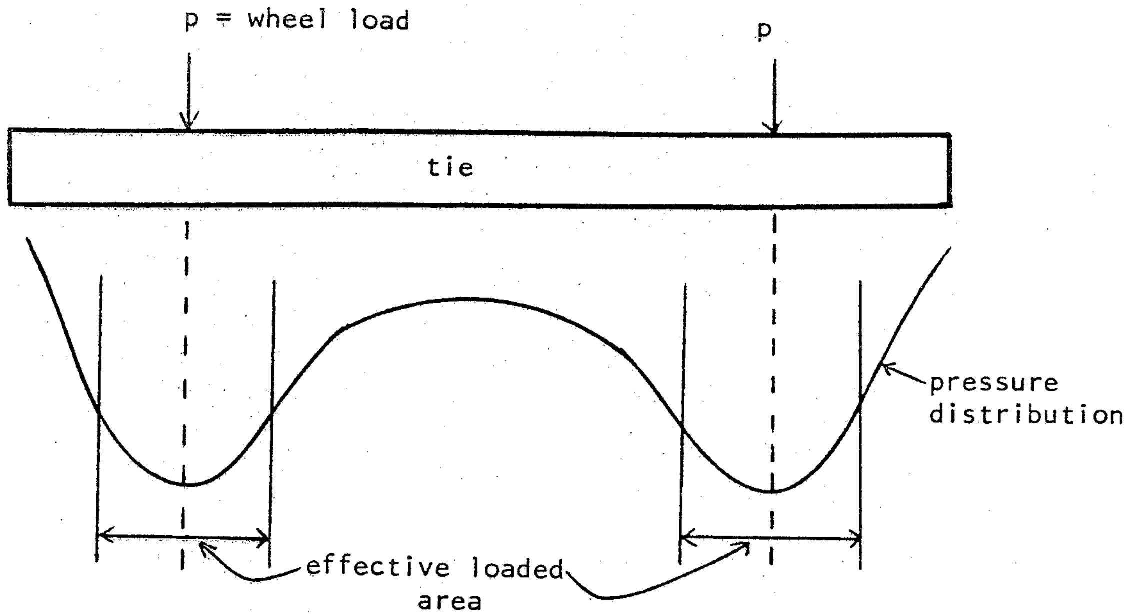


Figure 6. Vertical Pressure Distribution Under a Tie (schematic).

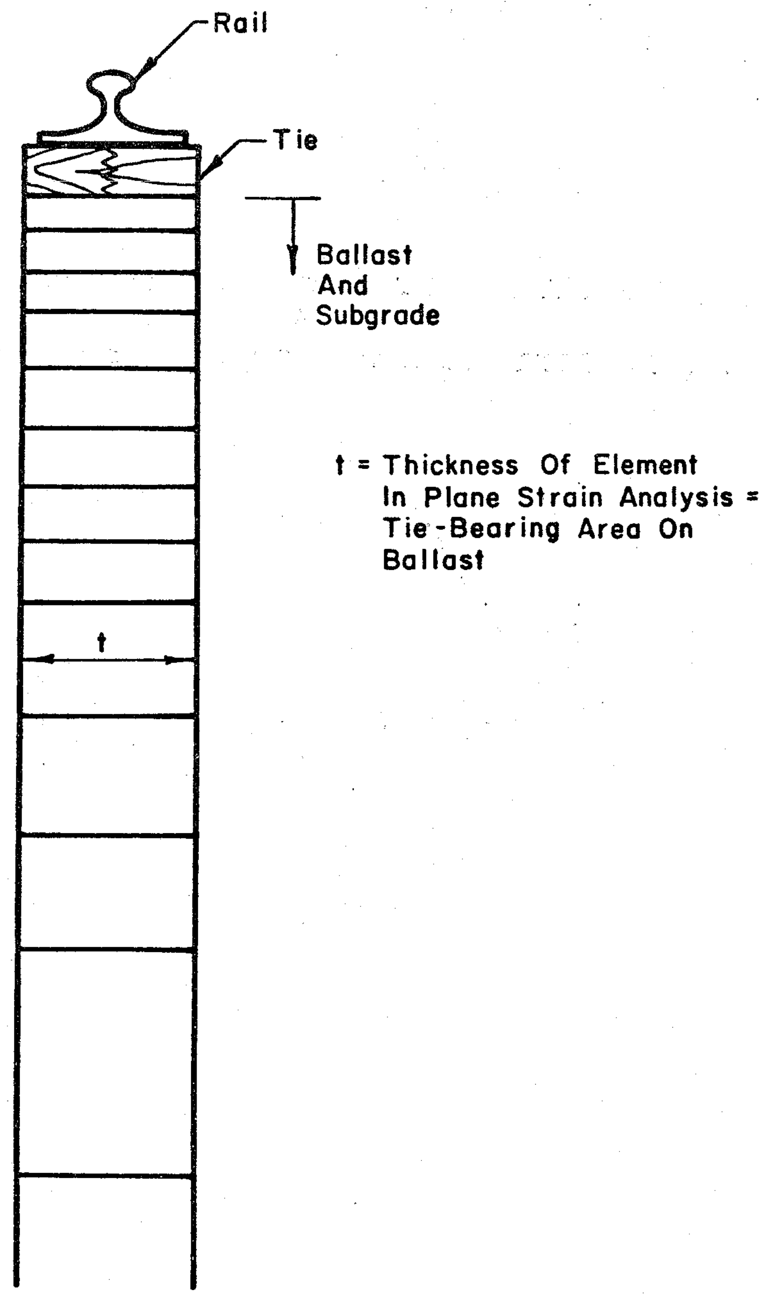


Figure 7. Plane Strain Approximation For Longitudinal Analysis.

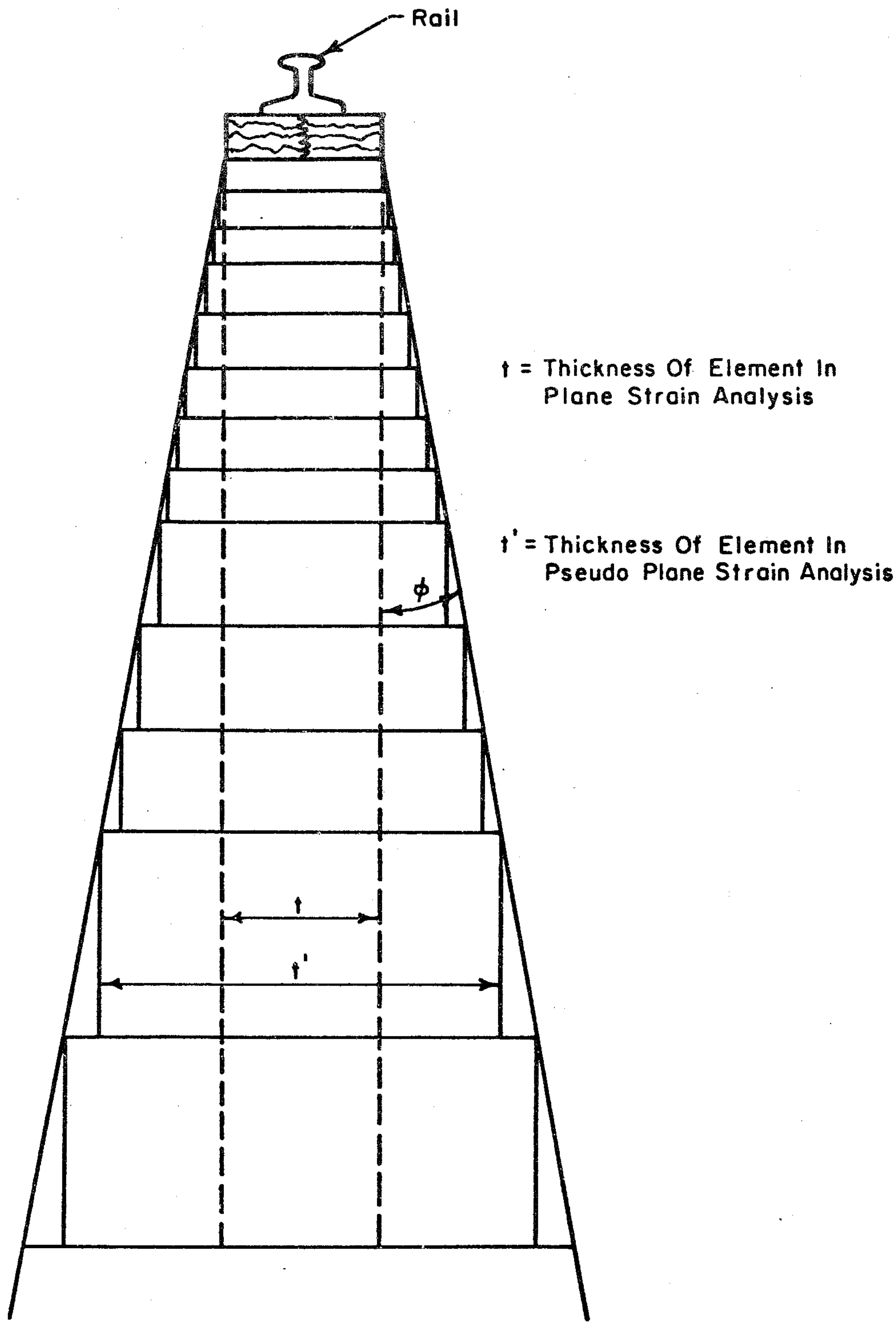


Figure 8. Pseudo Plane Strain Approximation for Longitudinal Analysis.

depth using the angle of distribution, ϕ, concept as shown in Figure 9.

Development of Stiffness Matrix for Each Element Type

a. Rectangular Plane Strain Element for Ballast, Sub-ballast and Subgrade

The displacements are assumed to vary bi-linearly over the element. If u and v represent the displacements in the x and y direction, Figure 10, the displacement functions can be written in terms of the nodal point displacements using the shape function approach, i.e.

$$u = N_1 u_1 + N_2 u_2 + N_3 u_3 + N_4 u_4 \tag{3}$$

and

$$v = N_1 v_1 + N_2 v_2 + N_3 v_3 + N_4 v_4 \tag{4}$$

where: u_n, v_n = displacements at node n, and N_1, N_2, N_3, and N_4 are shape functions which give a unit nodal value for the particular nodal displacement under consideration and zero nodal values for all others, and are given by:

$$N_1 = \eta\ (1-\zeta) \tag{5}$$

$$N_2 = (1-\zeta)\ (1-\eta) \tag{6}$$

$$N_3 = \zeta(1-\eta) \tag{7}$$

$$N_4 = \zeta\eta \tag{8}$$

where:

$$\zeta = \frac{x}{a} \tag{9}$$

$$\eta = \frac{y}{b} \tag{10}$$

and a, b = element dimensions.

The strains are then given by:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \alpha_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \tag{11}$$

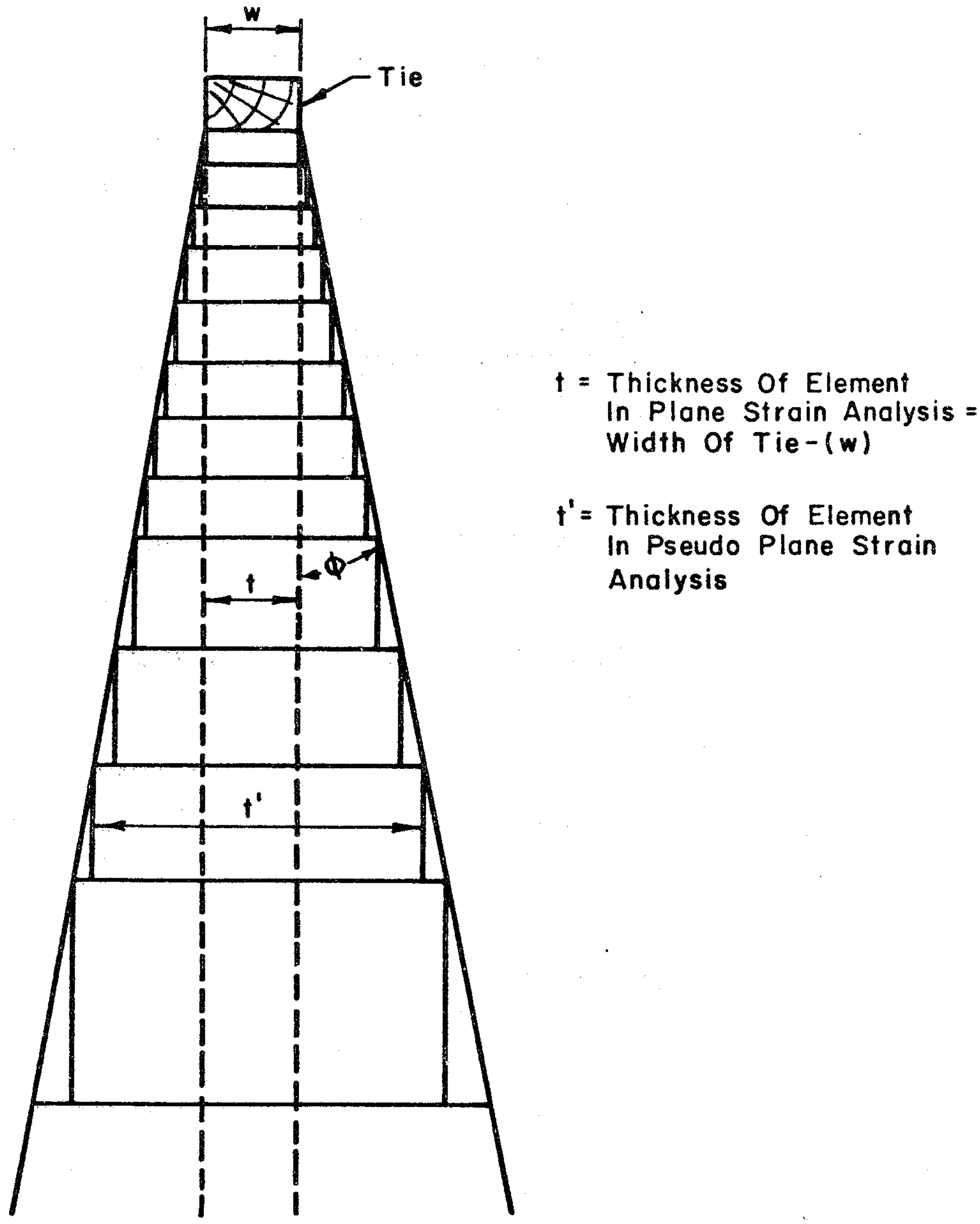


Figure 9. Pseudo Plane Strain Approximation for Transverse Analysis

$$u = N_1u_1 + N_2u_2 + N_3u_3 + N_4u_4$$

$$v = N_1v_1 + N_2v_2 + N_3v_3 + N_4v_4$$

$$N_1 = \eta\,(1 - \zeta) \quad N_2 = (1 - \zeta)\,(1 - \eta) \quad N_3 = \zeta\,(1 - \eta) \quad N_4 = \zeta\eta$$

$$\zeta = \frac{x}{a} \qquad \eta = \frac{y}{b}$$

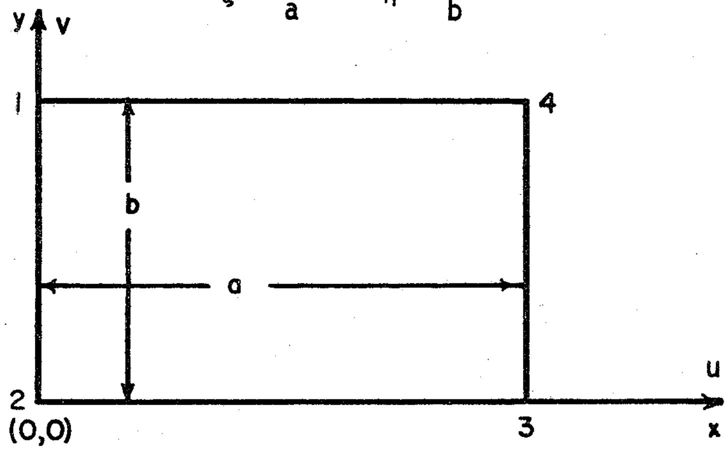


Figure 10. Rectangular Plane Strain Element.

$$\text{i.e.,}\quad \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \alpha_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} \qquad (12)$$

$$\text{i.e.,}\quad \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \alpha_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{-\eta}{a} & 0 & -\frac{(1-\eta)}{a} & 0 & \frac{(1-\eta)}{a} & 0 & \frac{\eta}{a} & 0 \\ 0 & \frac{(1-\zeta)}{b} & 0 & -\frac{(1-\zeta)}{b} & 0 & -\frac{\zeta}{b} & 0 & \frac{\zeta}{b} \\ \frac{(1-\zeta)}{b} & -\frac{\eta}{a} & -\frac{(1-\zeta)}{b} & -\frac{(1-\eta)}{a} & -\frac{\zeta}{b} & \frac{(1-\eta)}{a} & \frac{\zeta}{b} & \frac{\eta}{a} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix} \qquad (13)$$

$$\text{i.e.,}\quad \{\varepsilon\} = [B]\ \{u\} \qquad (14)$$

$$\{\delta\varepsilon\} = [B]\ \{\delta u\} \qquad (15)$$

The stresses are then given by (for plane strain):

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \alpha_{xy} \end{Bmatrix} \qquad (16)$$

$$\text{i.e.,}\quad \{\sigma\} = [D]\ \{\varepsilon\} \qquad (17)$$

$$\text{i.e.,}\quad \{\sigma\} = [D]\ [B]\ \{u\} \qquad (18)$$

Using the principal of virtual work,

$$\delta W_{ext} = \{\delta u\}^T \{p\} \qquad (19)$$

$$\delta W_{int} = \int_{vol} \{\delta\varepsilon\}^T \{\sigma\}\ dV = t \int_{area} \{\delta u\}^T [B]^T [D]\ [B]\ \{u\}\ dydx \qquad (20)$$

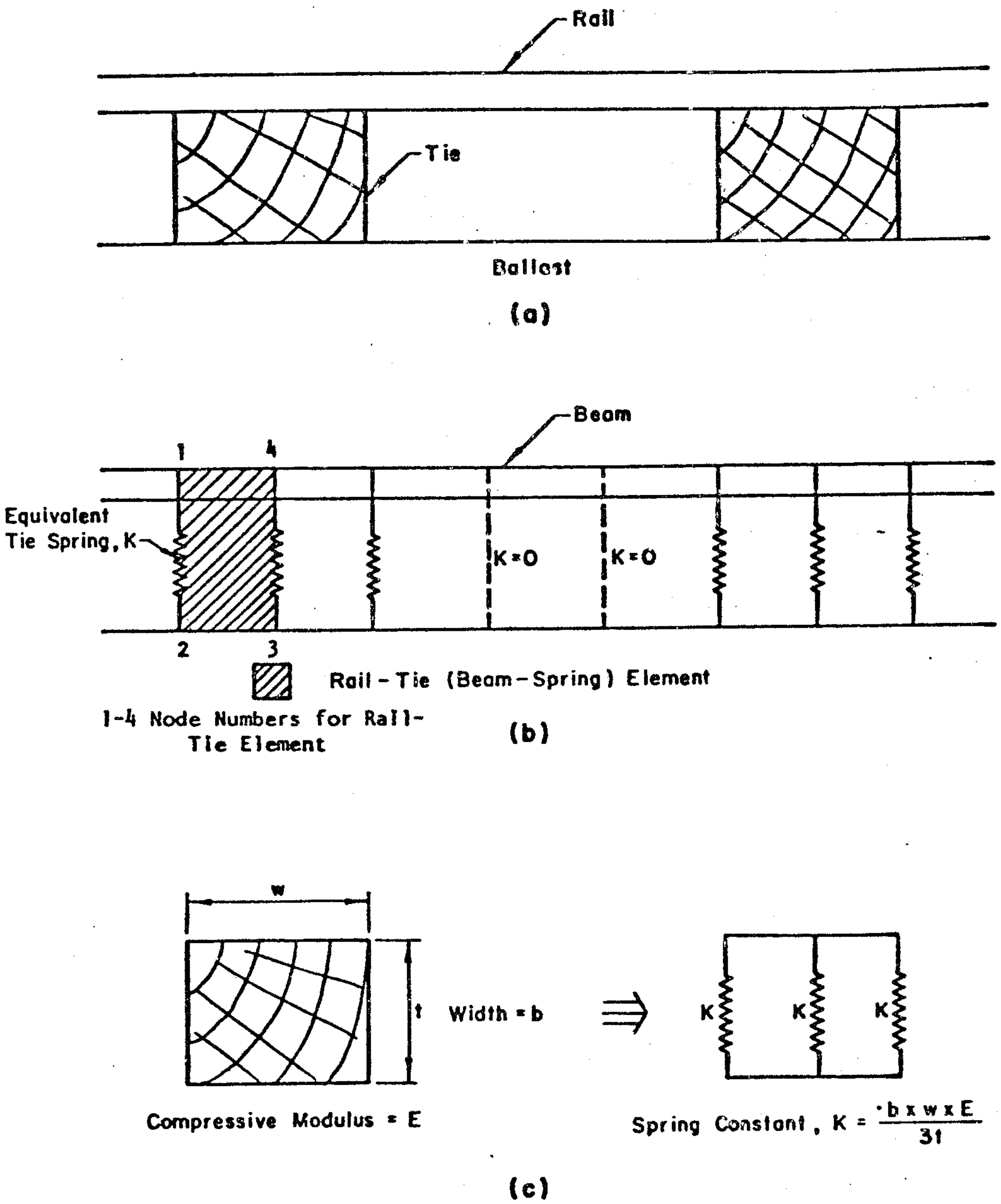


Figure 11. Rail-Tie Representation for the Longitudinal Analysis.

To include the springs in terms of equivalent tie spring constants, K_1 and K_2, and to confirm the number of degrees of freedom of the beam-spring element to the number of degrees of freedom to the rectangular elements, the modified stiffness matrix for the beam-spring element is written as:

$$[k_m] = \begin{bmatrix} 4\ \ell^2 c & 6\ \ell c & 0 & 0 & 0 & 0 & 2\ \ell^2 c & -6\ \ell c \\ 6\ \ell c & K_1+12c & 0 & -K_1 & 0 & 0 & 6\ \ell c & -12\ c \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -K_1 & 0 & K_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_2 & 0 & -K_2 \\ 2\ \ell^2 c & 6\ \ell c & 0 & 0 & 0 & 0 & 4\ \ell^2 c & -6\ \ell c \\ -6\ \ell c & -12\ c & 0 & 0 & 0 & -K_2 & -6\ \ell c & K_2+12c \end{bmatrix} \rightarrow \begin{Bmatrix} \theta_1 \\ w_1 \\ \theta_2 \\ w_2 \\ \theta_3 \\ w_3 \\ \theta_4 \\ w_4 \end{Bmatrix} \qquad (26)$$

(Note: θ_2, θ_3 are non-existent).

The curvature in the beam element is given by (using the notations of Figure 11):

$$Kc = \frac{-\partial^2 w}{\partial x^2} \qquad (27)$$

$$\text{i.e., } Kc = \left[\frac{4}{\ell} - \frac{6x}{\ell^2}, \frac{6}{\ell^2} - \frac{12x}{\ell^3}, \frac{2}{\ell} - \frac{6x}{\ell^3}, \frac{-6}{\ell^2} + \frac{12x}{\ell^3}\right] \begin{Bmatrix} \theta_1 \\ w_1 \\ \theta_4 \\ w_4 \end{Bmatrix} \qquad (28)$$

and the moment is given by:

$$M = EIKc \qquad (29)$$

Incorporation of Material Non-Linearity in the Model

Finite element problems with materials that exhibit, under repeated loading, stress-dependent behavior can be solved using two analytical procedures:

1. Iterative technique
2. Incremental technique

In the iterative technique the entire load is applied in one step and the problem is solved by iteration until some specific convergence is obtained. In the incremental load technique, the applied load is usually subdivided into equal increments. The problem is solved for each increment, choosing for the successive solutions a value of the modulus for a given element corresponding to the state of stress calculated for that element in the preceeding cycle. These solution techniques are referred to as pseudo elastic technique as in each solution cycle for both methods the problem is solved as an elastic problem.

The incremental load technique, using equal increments of load, has been used here in developing the analytical solution of the finite element model representation of the conventional railway track system. Initial moduli values are assumed for the stress dependent materials and are used to solve the problem with the first load increment. After the total load is applied, a single iterative analysis is carried out so that the moduli values used in the final load increment are more compatible with the state-of-stress existing at the end of the final load increment. Presently, only the load induced stresses are considered in the analysis and in calculating the bulk stress, θ, ($\theta = \sigma_1 + \sigma_2 + \sigma_3$) for the granular materials, the the deviatoric stress, σ_d, ($\sigma_d = \sigma_1 - \sigma_3$), for the subgrade soils.

Failure criteria are incorporated in terms of:

1. minimum value of the minor principal stress, σ_3, for the granular materials
2. maximum principal stress ratio, σ_1/σ_3, for the granular materials, and
3. maximum shear stress, $(\frac{\sigma_1 - \sigma_3}{2})$, for the subgrade soils.

Boundary Conditions Used in the Longitudinal Analysis

As the longitudinal analysis now exists, a symmetrical loading is considered and only half of the system is considered in the modelling and analysis. Loads are input as point loads acting on the rail.

Due to loading symmetry, the nodes along the vertical boundary representing the centerline of the system are restrained from horizontal movement. Also, since horizontal deformation dissipates rapidly with distance from the loaded area, a horizontal restraint on the other vertical boundary has been placed at a distance of 260 in. (6.6 m) from the center of the loaded area. The nodes along the bottom boundary are restrained both from horizontal as well as vertical movement to simulate a rigid boundary.

Boundary Conditions Used in the Transverse Analysis

Due to loading symmetry in the transverse direction (assuming the axle load is equally distributed on both wheels), only half the transverse section is considered in the modelling and analysis. Loads are input as point loads and deflections are input as point deflections.

Again, due to loading symmetry, the nodes along the vertical boundary representing the centerline of the system are restrained from horizontal movement and as explained previously the other vertical boundary at a distance of 120 in. (3.1 m) is also restrained from horizontal movement. The nodes along the bottom boundary are restrained both from horizontal as well as vertical movement to simulate a rigid boundary.

Output

The output given by the program is:

1. Rail moment
2. Rail deflection
3. Tie reaction

4. Stress pattern in the ballast and the subgrade

5. Deformation pattern in the ballast and the subgrade

JUSTIFICATION AND VALIDATION OF THE FINITE ELEMENT MODEL

As already mentioned, accurate modelling of a conventional track structure is impractical for most cases due to the requirement of a very large computer and the subsequent cost. The two-stage finite element model which has been described models the conventional track structure at a much lower cost than would be required for a single three dimensional model. Clearly, a certain degree of accuracy as well as "rationality" had to be sacrificed. Initial results obtained from the two-stage finite element model indicate the model adequately characterizes the response of the track system subjected to repeated traffic loading. A brief discussion on the justification and the validation of the model follows.

Complex Geometry

The conventional railway track system has a complex three dimensional geometry as far as structural modelling is concerned, and therefore the two-stage two-dimensional modelling seems appropriate.

Pseudo-Plane Strain Considerations

When a load is applied to the surface of a foundation, the stress intensity at any point in the foundation decreases with depth because of a three dimensional stress distribution. The plane strain state analysis generally allows a two-dimensional stress distribution with the result that the stress dissipation with depth is neither representative nor realistic. The use of pseudo-plane strain state allows an approximate three dimensional analysis using a two dimensional model and gives a better distribution of stress with depth as would normally be expected. This is shown in Figure 12 which gives

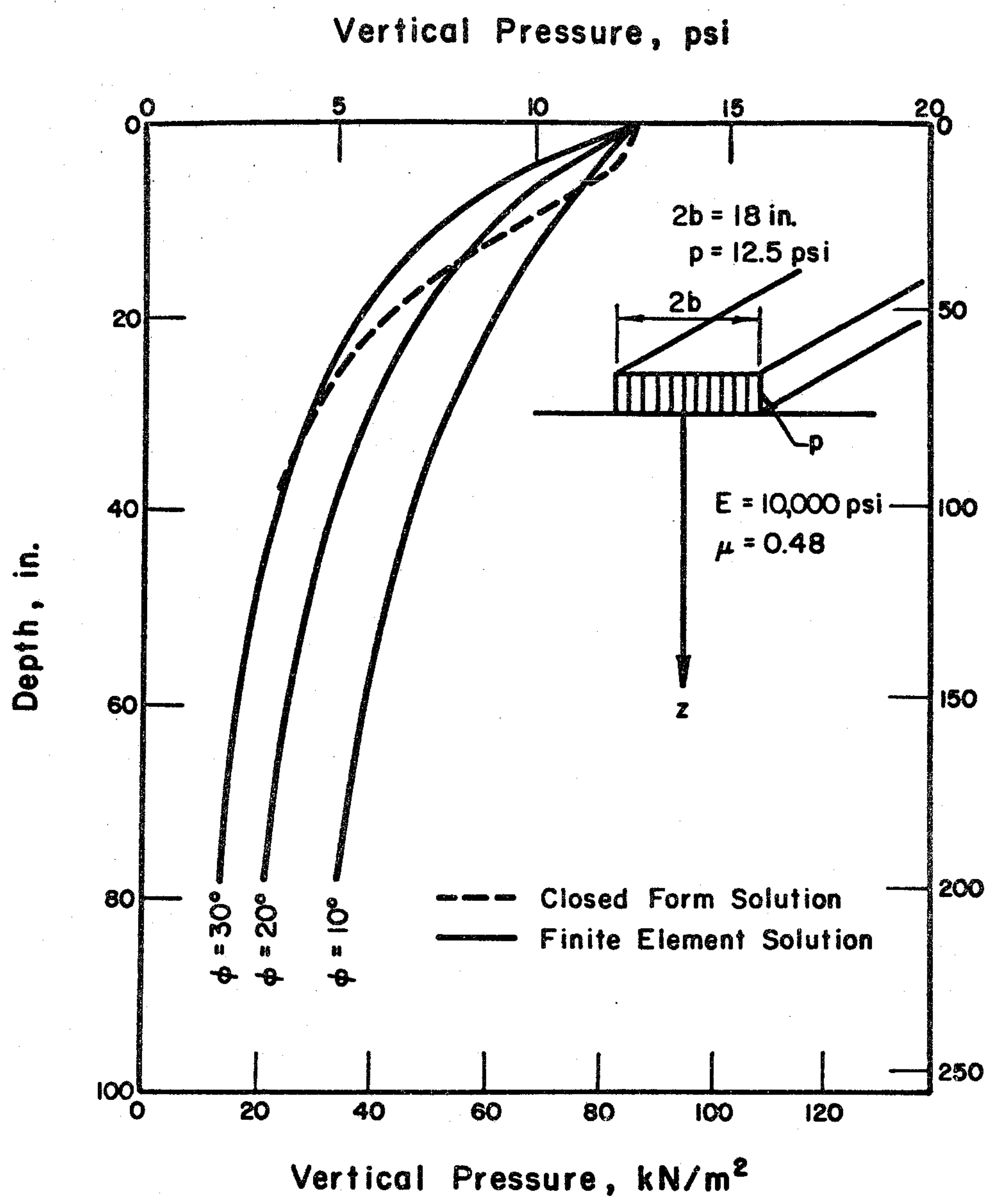


Figure 12. Vertical Stress Distribution Obtained by a Closed Form Analytical Method and by Using Pseudo-Plane Strain Finite Element Approximation for a Uniform Strip Load.

the vertical stress distribution in an isotropic elastic foundation subjected to a uniform strip load of width 8 in. (20.3 cm) and pressure intensity of 12.5 psi (86.2 kN/m^2). Analytical results (56) as well as the results using the finite element model (transverse section model) using different angle of distributions for pseudo-plane strain considerations are shown in Figure 12. It is seen that the analytical results agree well with that of the finite element model using an angle of distribution of 30 degrees, thus demonstrating the validity of the pseudo-plane strain analysis approach.

Non-Linear Material Properties

It is known that granular and fine-grained materials exhibit, under repeated loading, strength characteristics that depend on the state-of-stress existing in the material. This response, termed the resilient response, can be evaluated in the laboratory and is discussed later. This response has been modelled from the results of the laboratory tests as follows.

a. Ballast Material

$$E_R = K_3 \, \theta^{K_4} \tag{30}$$

where:

E_R = resilient modulus = $\dfrac{\text{repeated deviator stress}}{\text{elastic or recoverable strain}}$, psi

θ = sum of the principal stresses = $\sigma_1 + \sigma_2 + \sigma_3$ ($\sigma_1 + 2\sigma_3$ in a triaxial test), psi

K_3, K_4 = constants determined from laboratory tests

A typical resilient response curve for ballast material is shown in Figure 1.

b. Fine-Grained Soils

Generally, the resilient modulus of fine-grained soils decreases with an increase in deviatoric stress. At higher values of deviatoric stress, the resilient modulus is almost constant. A typical resilient response curve for fine-grained soils is shown in Figure 2.

Experience with Models Incorporating Non-Linear Material Characterization

The finite element technique has been applied with success in the analysis of layered pavement systems. The two finite element models that are widely used are the ones developed by Wilson (Ref. 19, 20, 21) and by Barksdale (Ref. 22, 23). Both the models are axi-symmetric and allow only a single circular load to be considered for non-linear analysis. The material can be characterized as temperature dependent for bituminous mixtures and stress dependent for granular material and the fine-grained soils. The stress dependent characterization of the granular material and fine-grained soils is done as mentioned previously. Several investigations (Ref. 19, 20, 24) have shown that when the conditions of the materials used in laboratory strength evaluation tests correspond to field conditions the calculated results using the non-linear finite element models are in good agreement with the measured field response.

Validation Using the Kansas Test Track Data

The Kansas Test Track is located between Aikman and Chelsea, Kansas and is connected into the Atchinson, Topeka and Santa Fe Railway Company (ATSF) line (Ref. 25, 26). The project is sponsored jointly by the Federal Railroad Administration and ATSF. There are nine test sections, each incorporating a different design concept. Section 9 was designed as a conventional track to be used primarily as a control section. Subsequent discussion will be limited to Section 9 only.

a. Section 9

Section 9 situated between Stations 8589+17 and 8597+17. It is 800 ft (244 m) in length. The embankment is about 8 ft (2.4 m) thick and consists of gray to brown silty clay (CL) and reddish brown clay (CH). The bedrock under the embankment consists of limestone often interbedded with thin clay shale and shale layers.

The track consists of 136 lb/yd (68 kg/m) rail with timber ties (7 in. by 8 in. by 9 ft (17.8 cm by 20.3 cm by 2.74 cm) ties spaced at 19.5 in. (49.4 cm) center to center. The slag ballast is 12 in. (30.5 cm) thick and is placed over 6 in. (15.2 cm) of lime-modified soil.

b. Material Details

1) ballast - Figure 13 shows the gradation of the slag ballast used and Table 2 details other properties of the slag ballast.

2) lime-modified soil - Details of material properties are given in Table 3.

3) embankment material - The material properties are detailed in Table 4.

c. Field Investigations

The details of a field investigation made during November, 1974 are given below.

1) loading data (Ref. 27) - The details of the work train used during the field investigation are given in Table 5. The work train consisted of a locomotive, 2 loaded cars, and a caboose. The wheel loads of the loaded cars were about 30,000 lb. (13,600 kg).

2) measured results (Ref. 27, 28) - Figures 14 and 15 show the profiles of deflection of the rail, and the tie reaction (rail seat load) at a north rail-tie interface and the tie reaction at a south rail-tie interface obtained when the work train passed at 30 mph (48 km/h). The measured values for deflection obtained at different depths in the embankment are given in Table 6.

d. Finite Element Modelling

A structural analysis was conducted on test Section 9 using the two-stage finite element model and is detailed below.

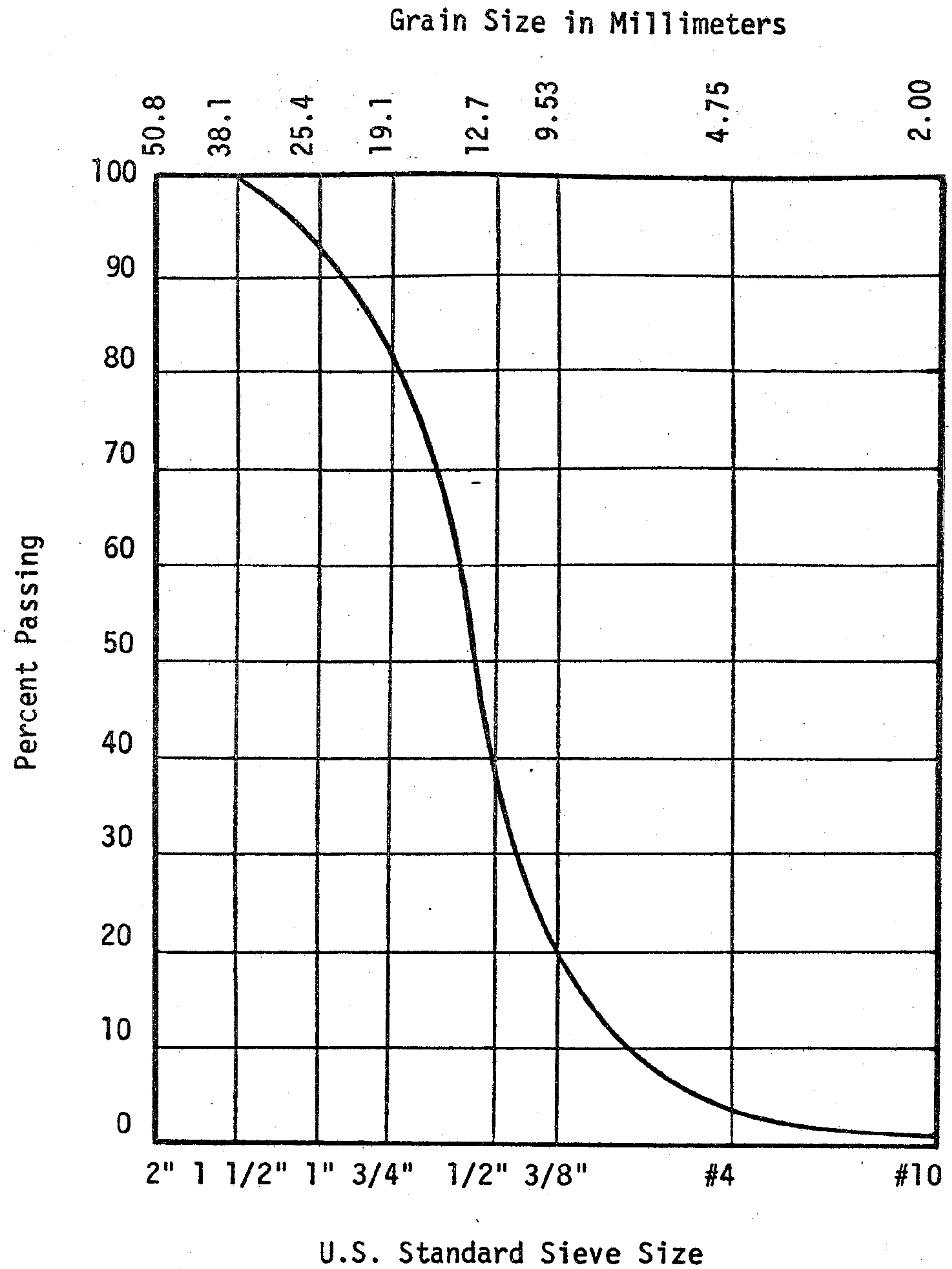


Figure 13. Gradation of the Slag Ballast Used at the Kansas Test Track.

Table 2. Properties of the Slag Ballast Used at the Kansas Test Track

Property	Value
Particle Index	14.1
Flakiness Index, %	5.5
Soundness (Na), %	0.9
L. A. Abrasion Resistance, %	26.7
Bulk Specific Gravities	
Air Dry Surface	2.52
Absorbtion Capacity	1.68
Crushing Value, %	25.2

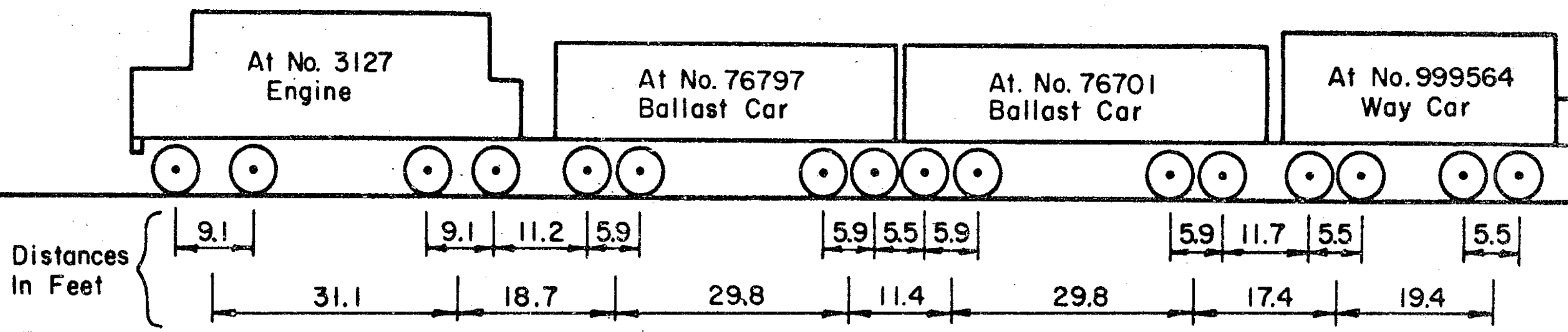
Table 3. Properties of the Lime-Modified Soil Used at the Kansas Test Track (Ref. 55)

Property	Range
Dry Density - pcf	85.7 to 89.6
- kg/m^3	1376 to 1440
Water Content, %	26.6 to 29.4
Unconfined Compressive Strength - psi	114 to 126
- kN/m^2	785 to 870

Table 4. Anticipated Properties of the Embankment Material (Ref. 26) (at 95% Relative Compaction Modified AASHO and w = optimum + 2)

Soil Property	Range	Average
Dry Density - pcf	92 to 106	100
- kg/m^3	1470 - 1690	1600
Water Content, %	18 - 24	22
Liquid Limit, %	33 - 89	67
Plastic Limit, %	15 - 25	18
Plasticity Index, %	15 - 63	49
Shrinkage Limit, %	10.5 - 12.1	11.4
Specific Gravity	2.54 - 2.65	2.59
Unconfined Compressive Strength - psi	50 - 75	60
- kN/m^2	350 - 520	410
Initial Tangent Modulus from Unconfined Tests,		
- psi	1800 - 6000	3700
- kN/m^2	12500 - 41000	25500

Table 5. Details of the Work Train Used at the Kansas Test Track (Ref. 27).



Axle Loads								
Kips	70.35	69.19	62.35	59.85	61.30	59.70	16.55	13.99
kg.	31910	31384	28282	27148	27806	27080	7507	6346
Wheel Loads								
Kips	35.17	34.60	31.17	29.92	30.65	29.85	8.27	6.99
kg.	15955	15692	14141	13574	13903	13540	3754	3173

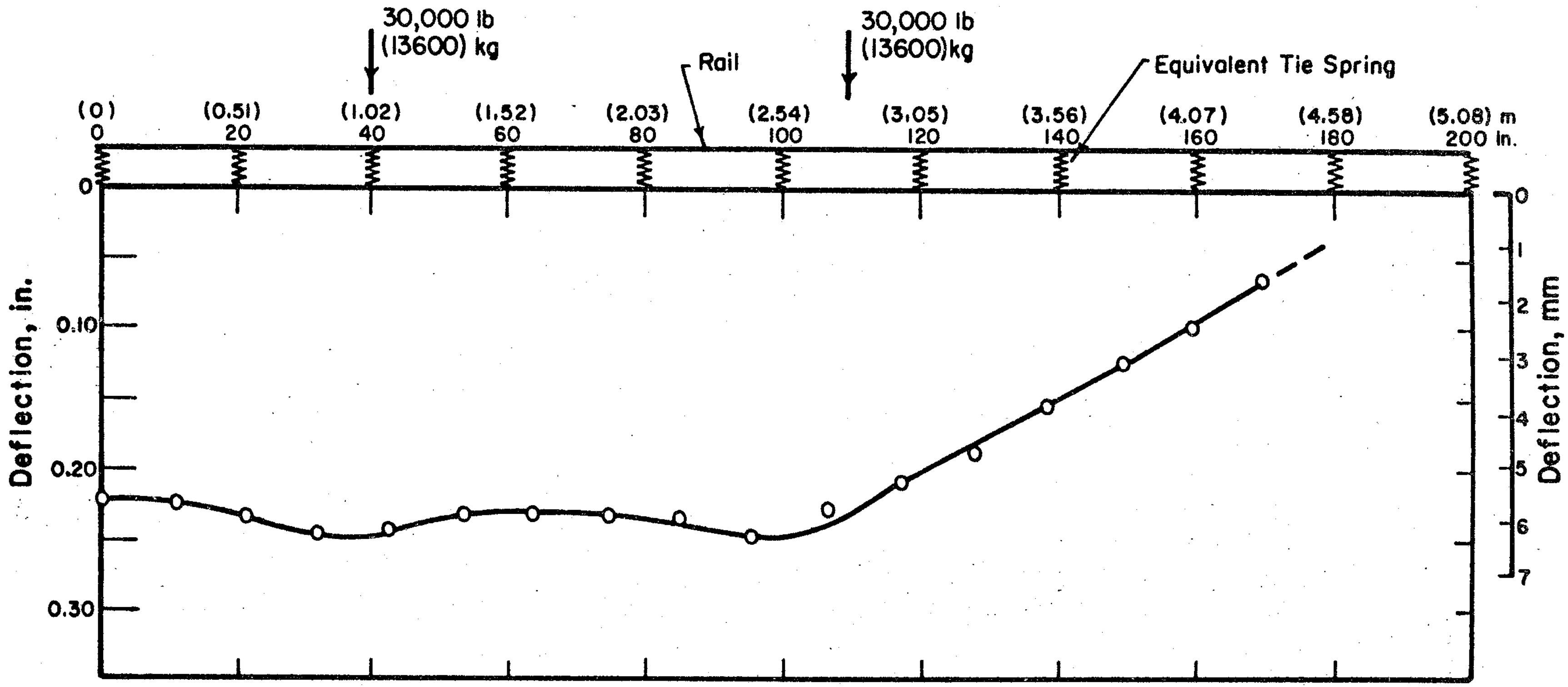


Figure 14. Measured Rail Deflection at Section 9 (From Ref. 27).

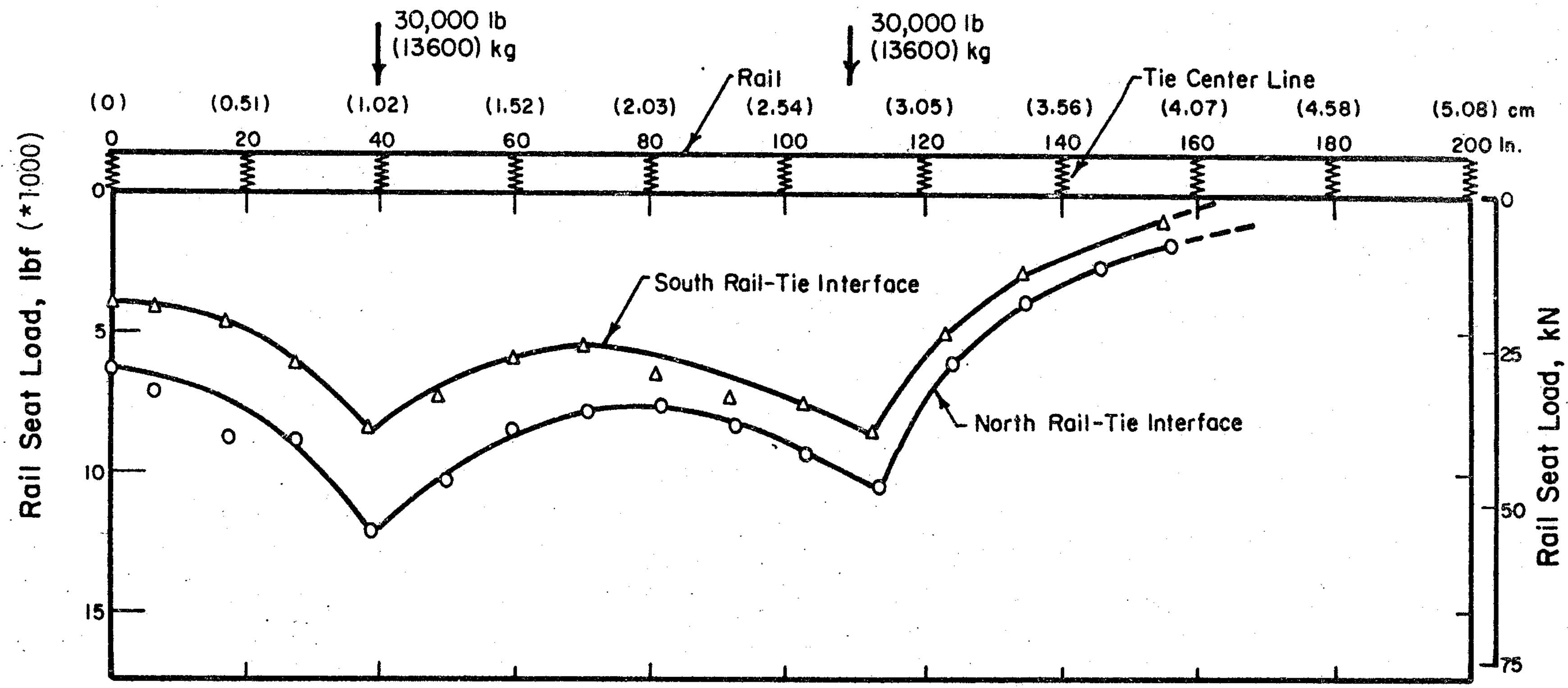


Figure 15 Measured Rail Seat Load (Tie-Reaction) at a North Rail Tie Interface and at a South Rail-Tie Interface(From Ref. 27).

Table 6. Embankment Deflection Data for Section 9

LVDT Position	LVDT Location	Measured Deflection[1] at 30 mph in. (mm)	Calculated Deflection in. (mm)
1	Top Embankment - 18"[2]	0.037 (0.94)	0.027 (0.69)
2	Top Embankment - 36"	0.054 (1.37)	0.038 (0.97)
3	Top Embankment - 66"	0.058 (1.47)	0.050 (1.27)
4	Top Embankment - Rock	0.082 (2.08)	0.090 (2.29)
	(Max. Rail Deflection)	0.245 (6.22)[3]	0.134 (3.40)

1 Reference 28

2 Top Embankment - n" means that the deflection was measured between the top of the embankment and n inches below the top of the embankment.

3 Reference 27

Careful attention was paid to close simulation of the material properties of the track. The field results given in Table 6 show that there was an abnormally high deformation in the upper 18 in. (45.7 cm) of the track support system. This indicated that the upper level of the embankment (including the lime-modified soil) had softened and/or was in a loose state. Samples taken during a field investigation (University of Illinois) in December, 1974 showed that the moisture content in the lime-modified soil layer ranged from 30 to 40 percent and below the lime modified soil layer the moisture content ranged from about 25 to 40 percent, much about their optimum moisture contents. Therefore it was decided to assume a lower modulus value for the lime-modified soil layer and to consider the upper 24 in. (61.0 cm) layer of the embankment material separately from the rest of the embankment material. The resilient response data for the ballast and the embankment materials were obtained from University of Illinois laboratory tests made on samples of these materials.

1) <u>input data</u>

<u>Rail</u>: 136 lb/yd (68 kg/m) rail, I = 94.90 in.^{4} (3954 cm^4)
E = 30,000 ksi (207,000 MN/m^2)

<u>Ties</u>: Timber ties - width = 8 in. (20.3 cm)
thickness = 7 in. (17.8 cm)
length = 9 ft (2.74 m)
Compressive modulus = 1,250 ksi (8,618 MN/m^2)
Effective bearing length under each rail = 18 in. (45.7 cm)
Tie Spacing = 20 in. (50.8 cm)

<u>Ballast</u>: Slag ballast - depth = 12 in. (30.5 cm)
Initial modulus used = 30,000 psi (207,000 kN/m^2)
μ = 0.35
Resilient Response Model: $E_R = 9600\ (\theta)^{0.55}$ psi

Lime-Modified Soil: Lime-modified soil depth = 6 in. (15.2 cm)
Initial modulus used = 6,000 psi (41,000 kN/m^2)
μ = 0.25
Resilient Response Model: E_R = 6000 psi

Embankment Material: Total depth used = 264 in. (670 cm)

Upper Layer: Depth = 24 in. (61 cm)
Initial modulus used = 3,000 psi (20,700 kN/m^2)
μ = 0.47
Resilient Response Curve Data:

σ_D, psi (kN/m^2)	E_R, psi (kN/m^2)
0.1 (0.7)	4000 (27,600)
3.0 (20.7)	4000 (27,600)
5.0 (34.5)	3000 (20,700)
10.0 (68.9)	2500 (17,200)
20.0 (137.9)	2000 (13,800)

Bottom Layer: Depth = 235 in. (594 cm)
Initial modulus used = 5,000 psi (34,500 kN/m^2)
μ = 0.47

Resilient Response Curve Data:

σ_D, psi (kN/m^2)	E_R, psi (kN/m^2)
0.1 (0.7)	9000 (62,100)
3.0 (20.7)	9000 (62,100)
5.0 (34.5)	6000 (41,400)
10.0 (68.9)	5000 (34,500)
20.0 (137.9)	3000 (20,700)

2) failure criteria used

The following failure criteria were used.

a) Tie springs cannot take tensile loads

b) Maximum stress ratio, σ_1/σ_3, for ballast = 10

c) Minimum compressive stress (σ_3) for ballast = 0 psi (0 kN/m^2)

d) Maximum shear stress $\left(\frac{\sigma_1-\sigma_3}{2}\right)$ of subgrade = 25 psi (172 kN/m^2)

e. Calculated Results

The length of track analyzed was 260 ft (79.3 m) and the angle of distribution used for the pseudo-plane strain consideration was 10^{0}. The angle of distribution of 10 degrees was selected because analysis using this angle gave better deflection results and also because it gave slightly conservative estimates for stress values in the ballast, the subballast, and the subgrade. The calculated results in terms of rail deflection and embankment surface deflection are shown in Figure 16. The rail deflection profile obtained from the field investigation is also shown in Figure 16. It is evident that the measured and the calculated rail deflection profile are not in good agreement. The maximum embankment deflection measured in the field and that calculated are comparable and are shown below:

Measured maximum embankment surface deflection = 0.082 in. (0.207 cm)
Calculated maximum embankment surface deflection = 0.090 in. (0.229 cm)

As can be seen from Table 6, a large part of the total deformation apparently takes place between the rail and the embankment surface. There are four factors that could be the cause of this anomaly. They are:

1) A gap between the rail base and the tie plate,
2) Improper seating of the tie on the ballast,
3) Deformation in the ballast layer,
4) Effects of dynamic loading.

It is the opinion of the project staff however, that the major causes of deformation between the rail and embankment were the first two factors mentioned above.

In a study conducted by the Batelle Memorial Institute (Ref. 6) it was found from field measurements of a track section that the track exhibited a non-linear response to traffic loading. A plot of measured rail seat load against rail deflection gave two distinct "slopes" as shown in Figure 17.

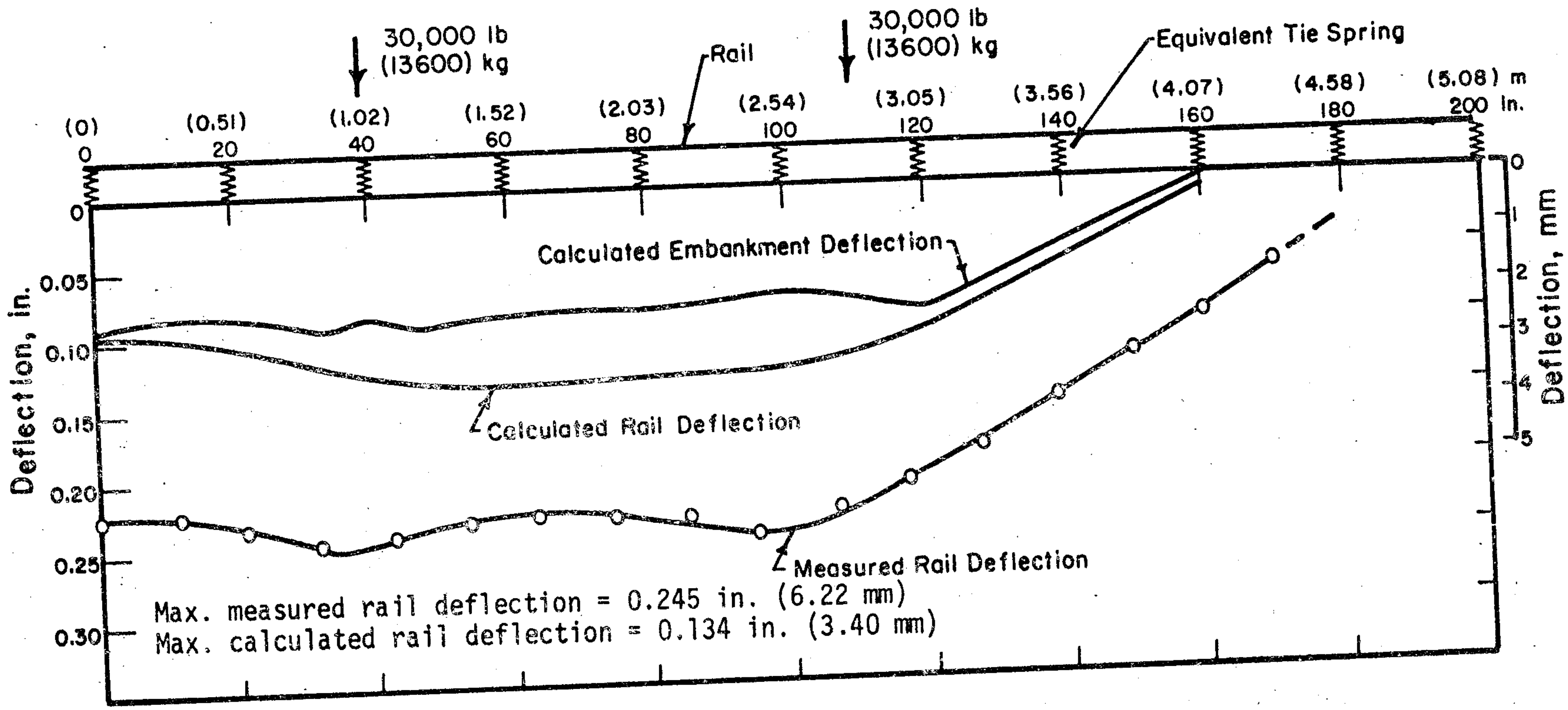


Figure 16. Calculated Rail and Embankment Deflections at Section 9.

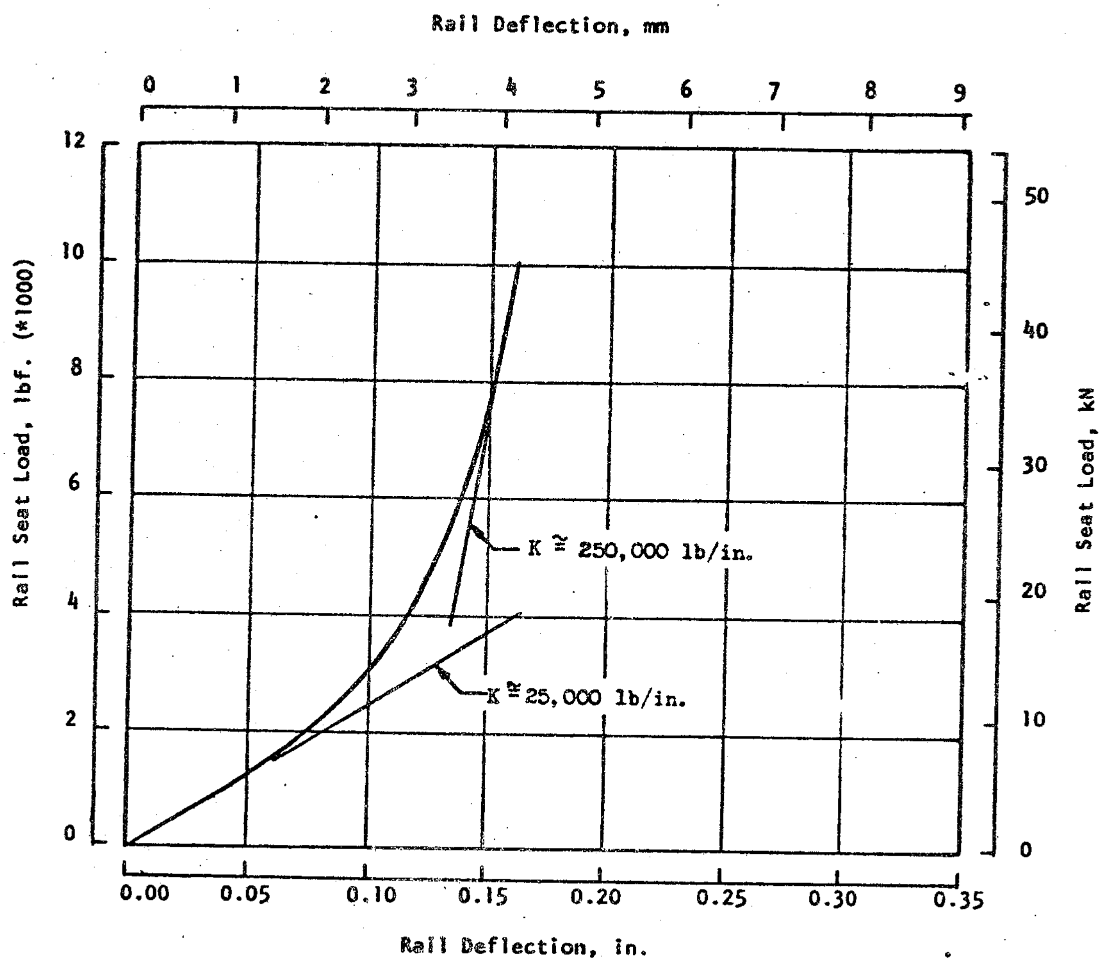


Figure 17. Plot of Measured Rail Seat Load Against Rail Deflection (Ref. 6).

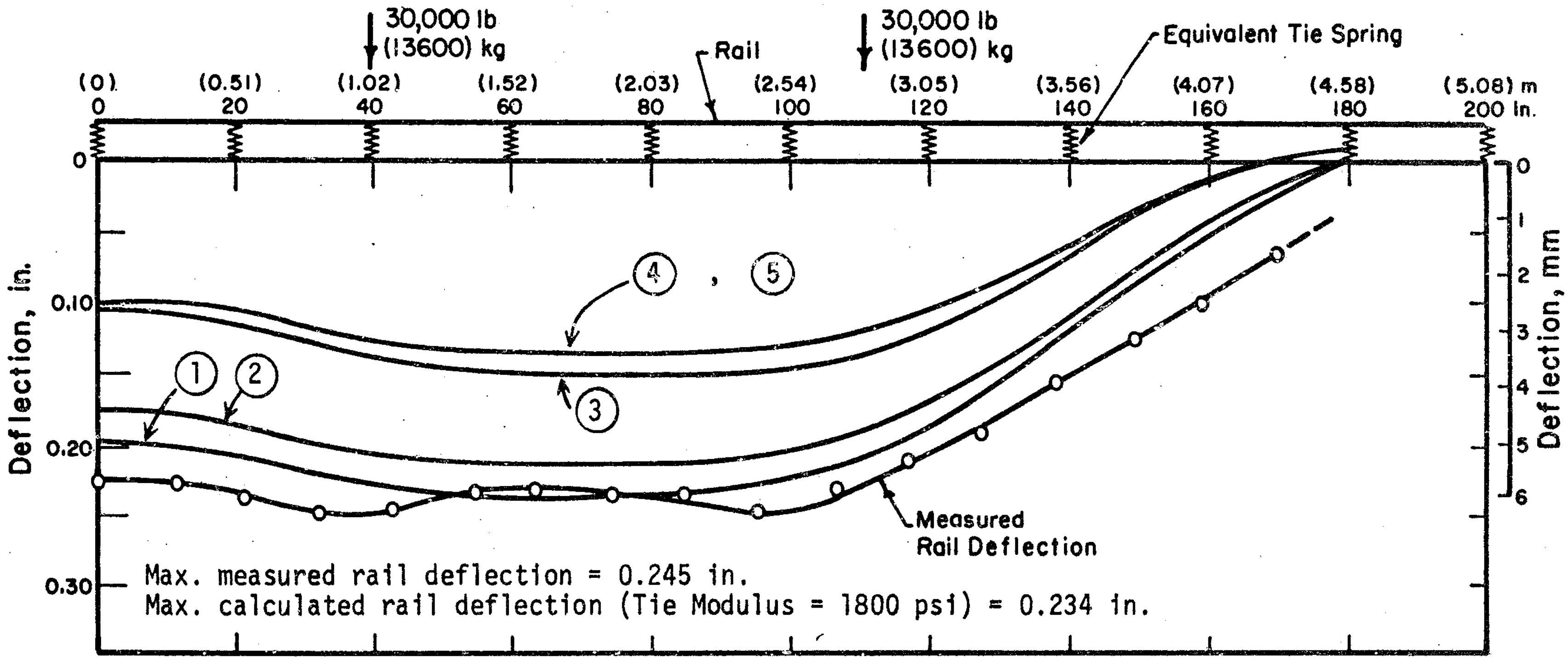
As shown in this figure, up to a certain tie-plate load, the rate of displacement with respect to the tie-plate load is greater than that for greater loads. A probable reason for this behavior is that an initial portion of the load is utilized in closing the gap between the rail base and tie-plate and for seating the tie firmly on the ballast.

This non-linear track behavior is taken into consideration by the Indian Railways when evaluating the track modulus of a track section. A "two-load" method of test is used to determine the track modulus (Ref. 29, 30). An initial load is applied to close gaps and compress the ballast and the subsequent load and the deflection are used in calculating the track modulus. Talbot has also reported this non-linear behavior of the track (Ref. 4).

In the study by the Batelle Memorial Institute (Ref. 6) it was concluded from the results of a lumped parameter model analysis of a railway track that the difference between the static and dynamic vertical deflection is negligible up to train speeds at which the frequency of wheel load application approaches the natural frequency of the track structure. For most cases, this train speed would be about 250 mph (400 km/h). Thus, the effects of dynamic loading on deflection for a train speed of 30 mph (48 km/h) would be insignificant.

These factors were taken into consideration and several computer runs were made with lower values for the modulus of compression of the ties. The computer program converts the modulus of compression of the tie value into equivalent spring constant value. Thus, lower modulus of compression values result in corresponding lower equivalent tie spring constant values which can simulate different levels of improper seating of tie on ballast or excessive play between rail and tie plates.

The rail deflection profiles obtained using lower tie moduli values are shown in Figure 18 together with the measured rail deflection profile. The



1) Tie Modulus = 1800 psi
2) Tie Modulus = 2500 psi
3) Tie Modulus = 25,000 psi
4) Tie Modulus = 250,000 psi
5) Tie Modulus = 1,250,000 psi

Figure 18. Calculated Rail Deflection Using Different Tie Compression Moduli Values.

results show that there is a close agreement between the measured profile and calculated profile using the compressive modulus of tie equal to 1800 psi (12,400 kN/m^2). Figures 19, 20, 21, and 22 show calculated rail moment, ballast surface pressure and embankment surface pressure, major and minor principal stress distributions with depth and tie reaction obtained using a compressive tie modulus of 1800 psi (12,400 kN/m^2).

A transverse analysis of the track support system for Section 9 was also conducted. The ballast surface deflection of 0.125 in. (2.8575 mm) obtained under the first wheel load from the longitudinal analysis was used as input deflection at the top of the tie for the transverse analysis. The deflection profile of the tie, the moment in the tie, and the vertical stress distribution under the tie and at the embankment surface are shown in Figure 23. The vertical stress distribution with depth at the intersection of the rail and tie under the front wheel load as obtained from the transverse analysis is compared with that obtained from the longitudinal analysis in Figure 24, and it can be noticed that the results of the two analyses compare very well.

Comparison With Other Analytical Methods

One of the analytical methods that has been widely used is the beam on an elastic foundation method. In this method, as already described in a previous section, a track modulus value represents the supporting characteristics of the ballast-subgrade support system. The rail moment and the rail deflection for a given track using different track moduli values are shown in Figure 25. The results of the finite element analysis of a similar track are also shown in Figure 25. It can be seen that the patterns of rail moment and rail deflection obtained by the analytical method and by the finite element method are very similar and are comparable for a track modulus of 1800 lb/in./in. (127 kgf/cm/cm). Also, the sensitivity of the maximum rail

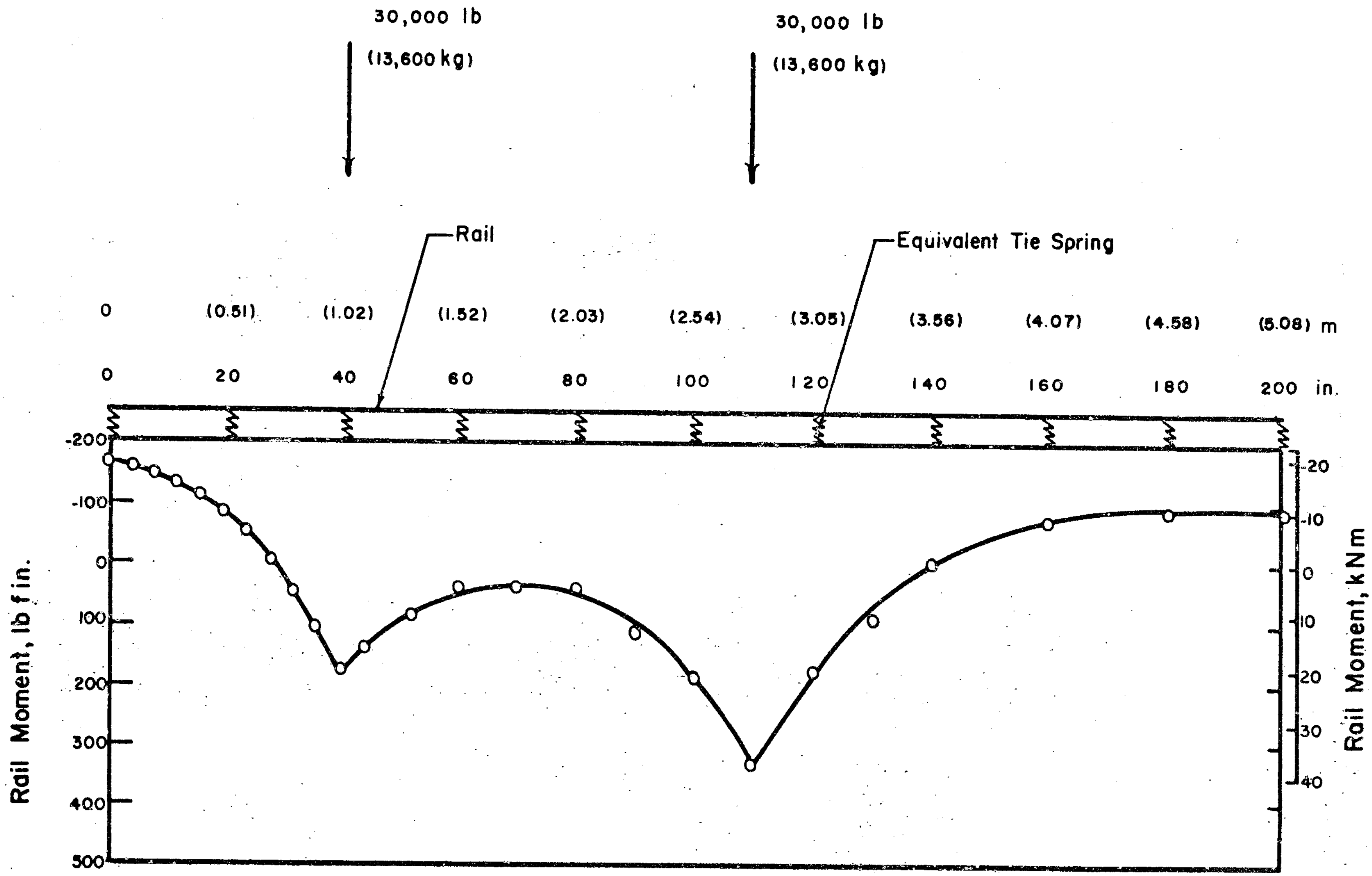


Figure 19. Calculated Rail Moment at Section 9.

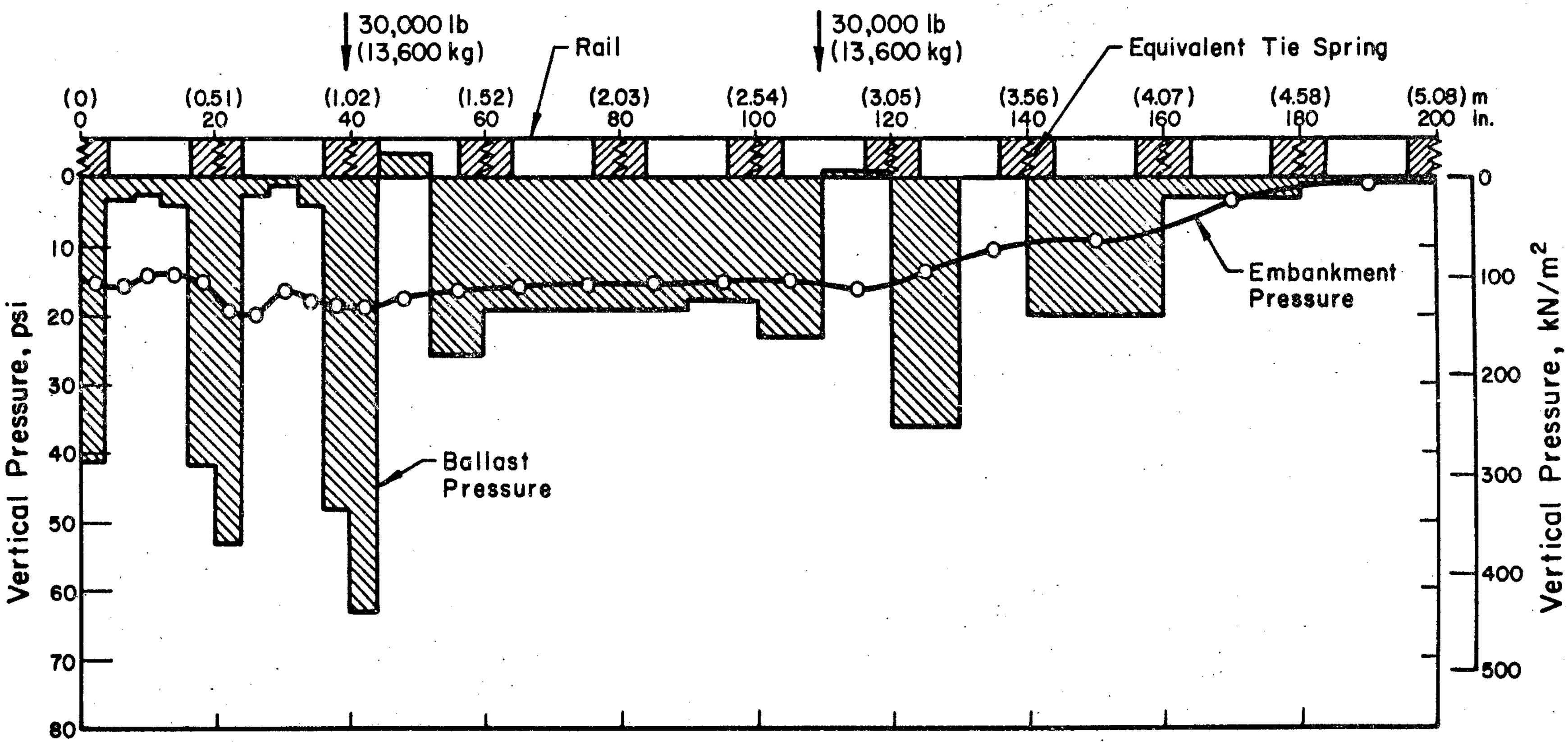


Figure 20. Calculated Ballast Surface Pressure and Embankment Surface Pressure at Section 9.

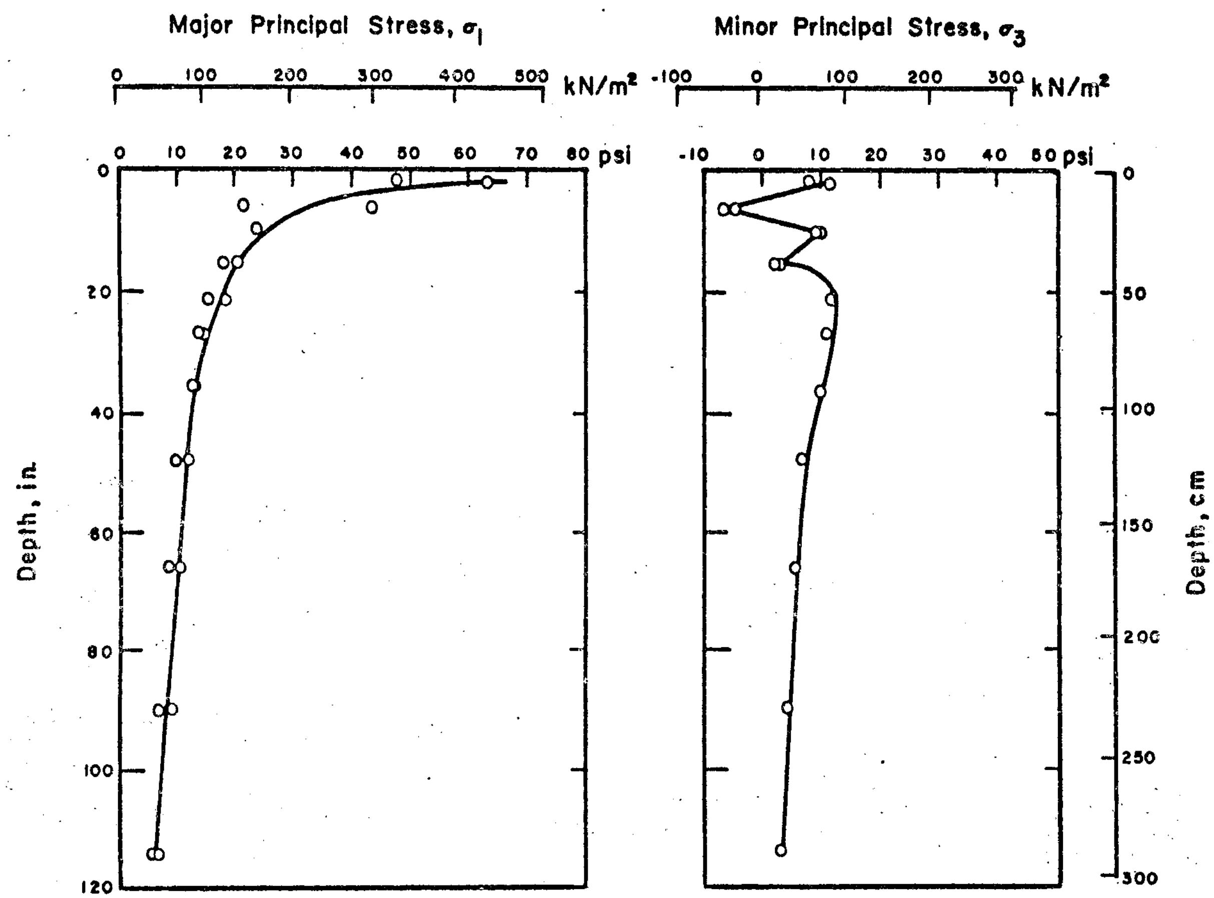


Figure 21. Calculated Major and Minor Principal Stress Distribution with Depth at Section 9.

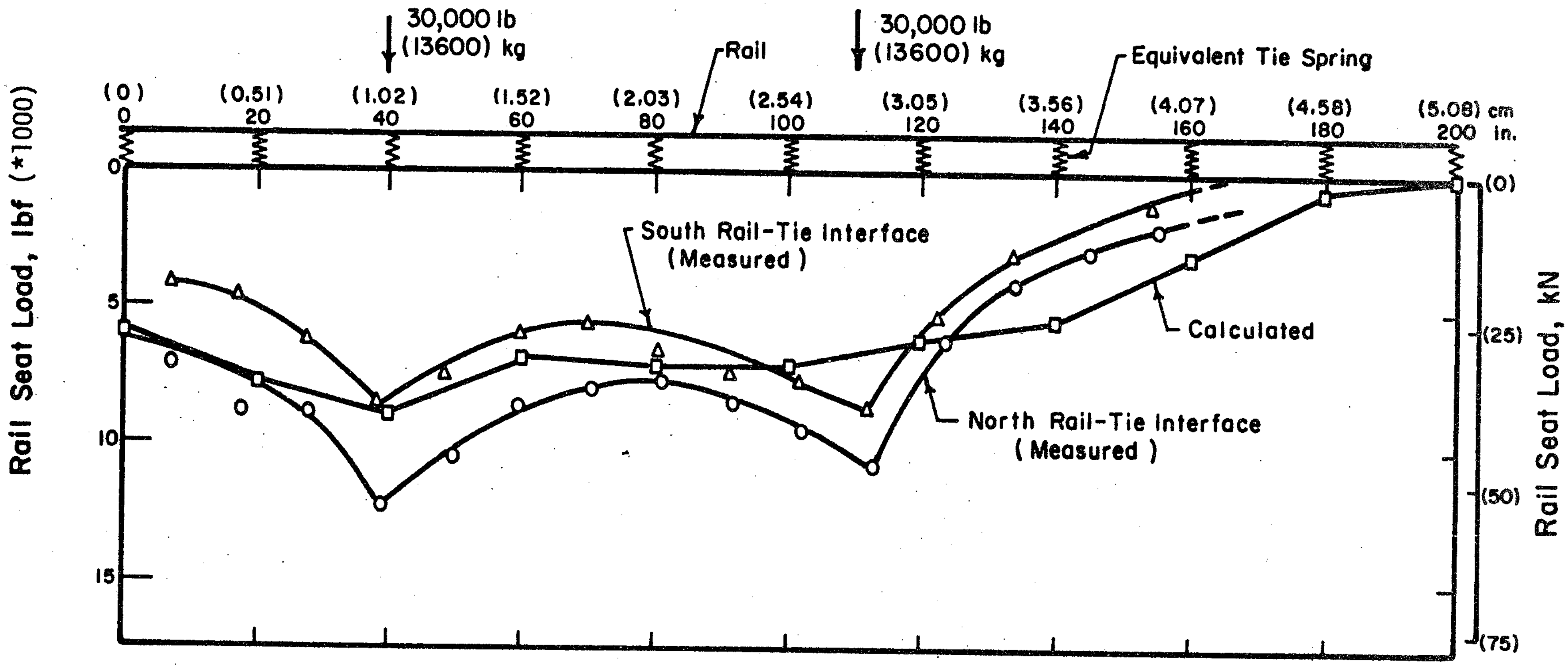


Figure 22. Calculated Rail Seat Load (Tie Reaction)

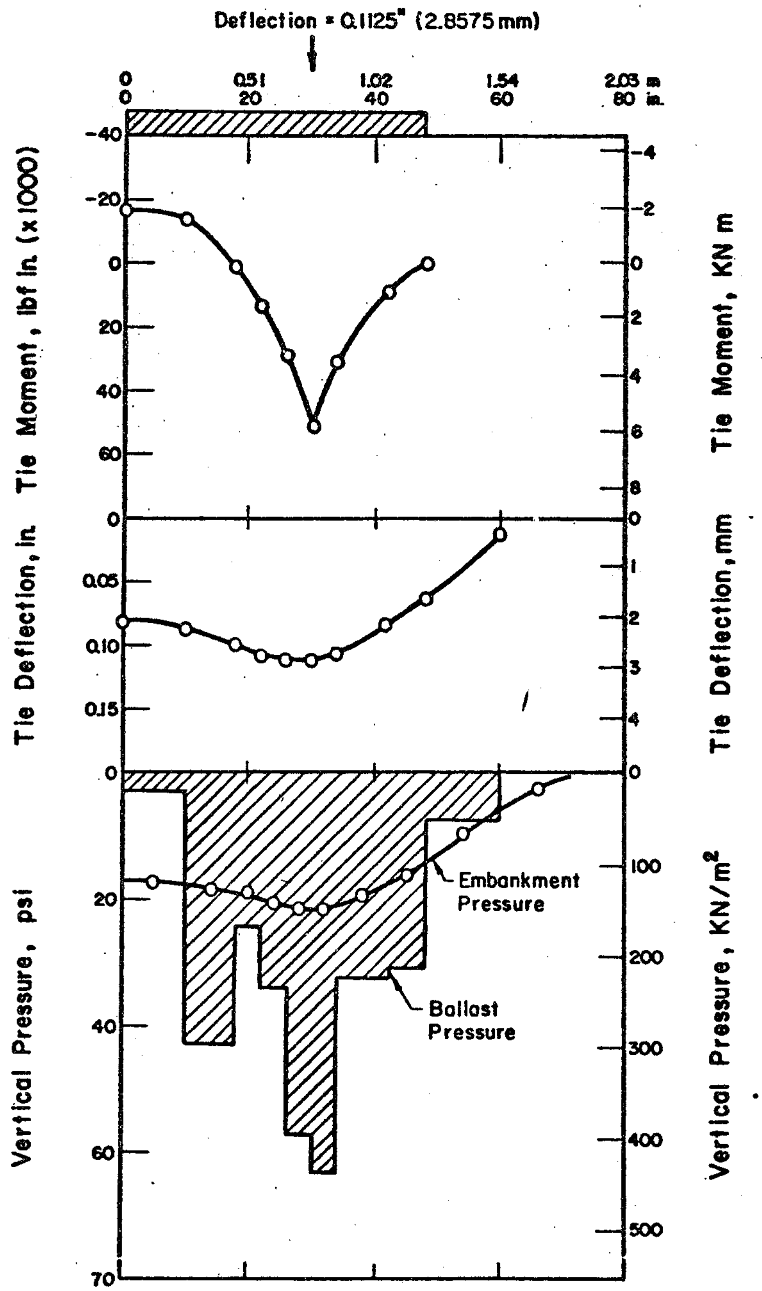


Figure 23. Transverse Analysis of a Tie at Section 9.

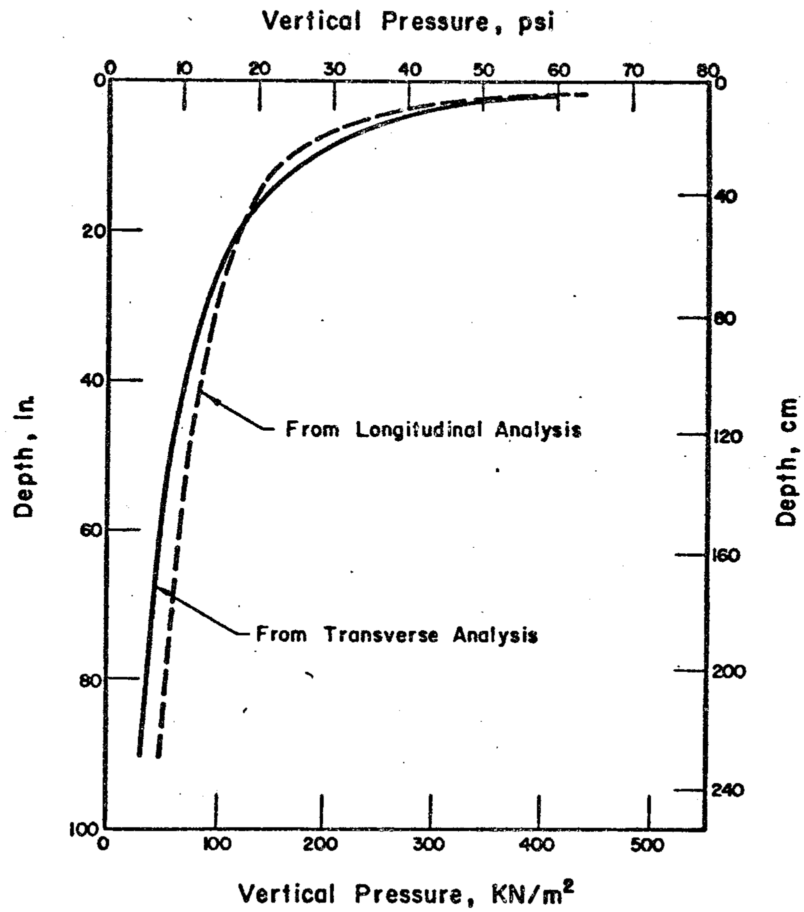


Figure 24. Stress Distribution with Depth as Given by Transverse Analysis and Longitudinal Analysis.

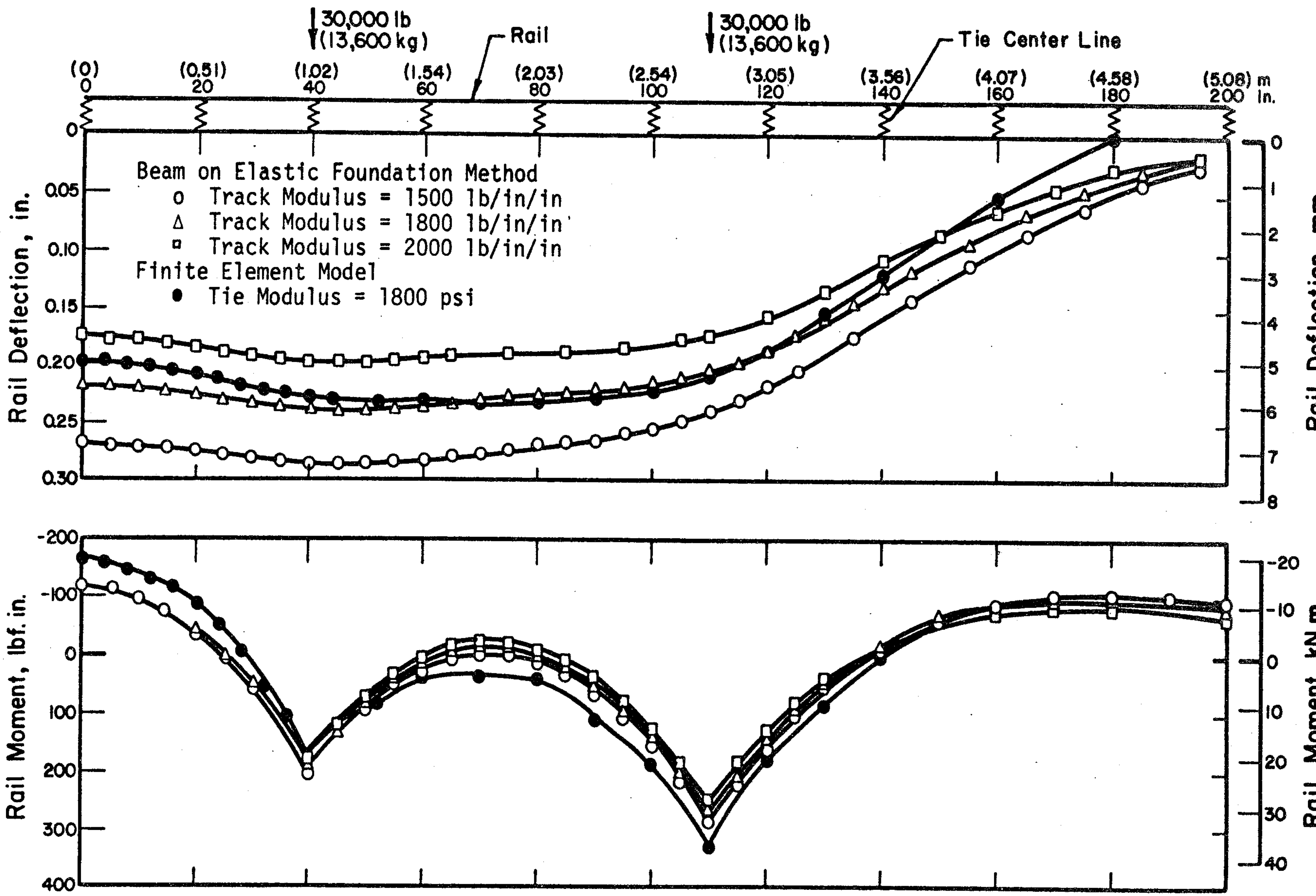


Figure 25. Comparison of Rail Moment and Rail Deflection as Calculated by the Finite Element Model and the Beam on Elastic Foundation Method.

deflection to the track modulus is evident. For the loading shown in Figure 25, a 25 percent decrease in the track modulus value from 2000 lb/in./in. (140 kgf/cm/cm) to 1500 lb/in./in. (105 kgf/cm/cm) increases the maximum rail deflection by about 33 percent from 0.216 in. (5.5 mm) to 0.287 in. (7.3 mm). Besides evaluating the rail moment and rail deflection, the finite element analysis also gives the tie reactions and the stress and the deformation pattern in the ballast and the subgrade systems. Also, the beam on elastic foundation method possesses the disadvantages already discussed in Chapter 2.

Outline of the Computer Program

The computer program for the analysis of conventional railway track support systems has been written using standard FORTRAN IV, and consists of 22 subroutines. A complete program listing and a User's Manual are given in Reference 57.

SUMMARY

A two-stage finite element model for the conventional railway track support system has been detailed. The use of the pseudo-plane strain technique in the model gives the stress distribution in the ballast, the subballast, and the subgrade. The model incorporates the basic components of the track support system and can accommodate stress dependent material properties.

Because of lack of field response data, validation of the model was attempted with a single set of data from the Kansas Test Track. As more appropriate field response data become available, it is expected that the model can be further validated.

CHAPTER 4

BALLAST AND SUBGRADE MATERIALS EVALUATION PROCEDURES

GENERAL

In order to utilize the "structural model" described in the foregoing chapter for the analysis of conventional track support structures, it is necessary to have component material behavior properties for input. The validity of the model is greatly influenced by how accurately the mechanistic response properties of the various materials can be determined.

In this research project, the emphasis is primarily concerned with ballast and foundation or subgrade materials. For this reason, materials evaluation procedures have been established for ballast and subgrade materials; the properties and behavior of other components of the track structure will be obtained from other sources.

The properties of materials should be evaluated under conditions that closely simulate actual in-service conditions. Thus, the in-service loading conditions must be considered in development of a realistic laboratory testing procedure. An examination of the vertical loading conditions that exist in the ballast and subgrade layers of a track structure indicate the following:

1. An extremely large number of rapidly applied loads may be imposed during the life cycle of a track structure. Figure 26 depicts average car movements per year as a function of gross track tonnage.

2. The duration of an individual stress pulse is typically very short, possibly on the order of 0.1 sec. Figure 27 depicts the frequency of load applications under a typical rail car as a function of speed.

3. The magnitude of stress varies as a function of vertical position in the support system, lower stresses generally coincide with greater depths.

Basically, there are four fundamental aspects of ballast and subgrade

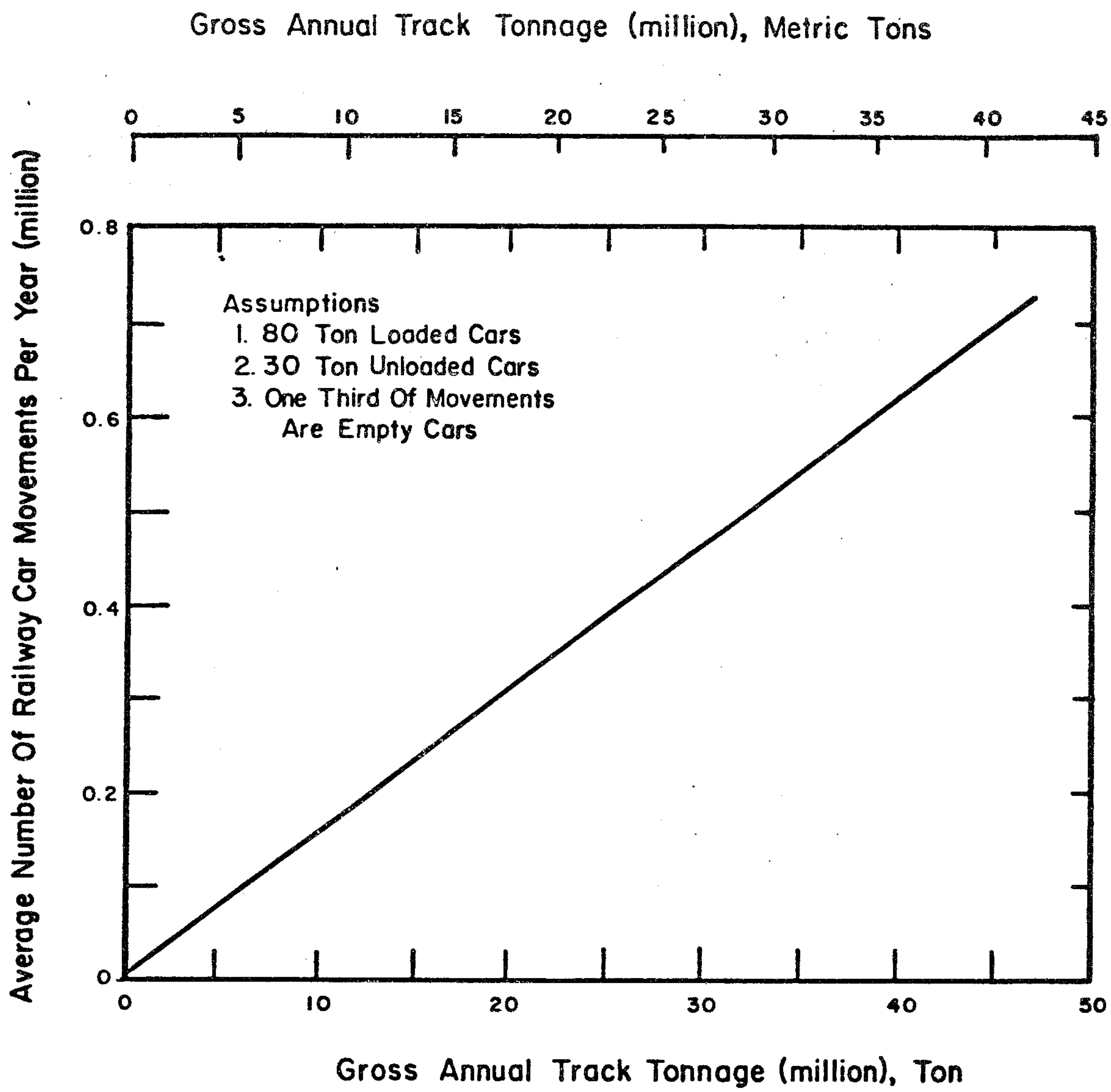


Figure 26. Average Number of Railway Car Movements Versus Gross Annual Track Tonnage.

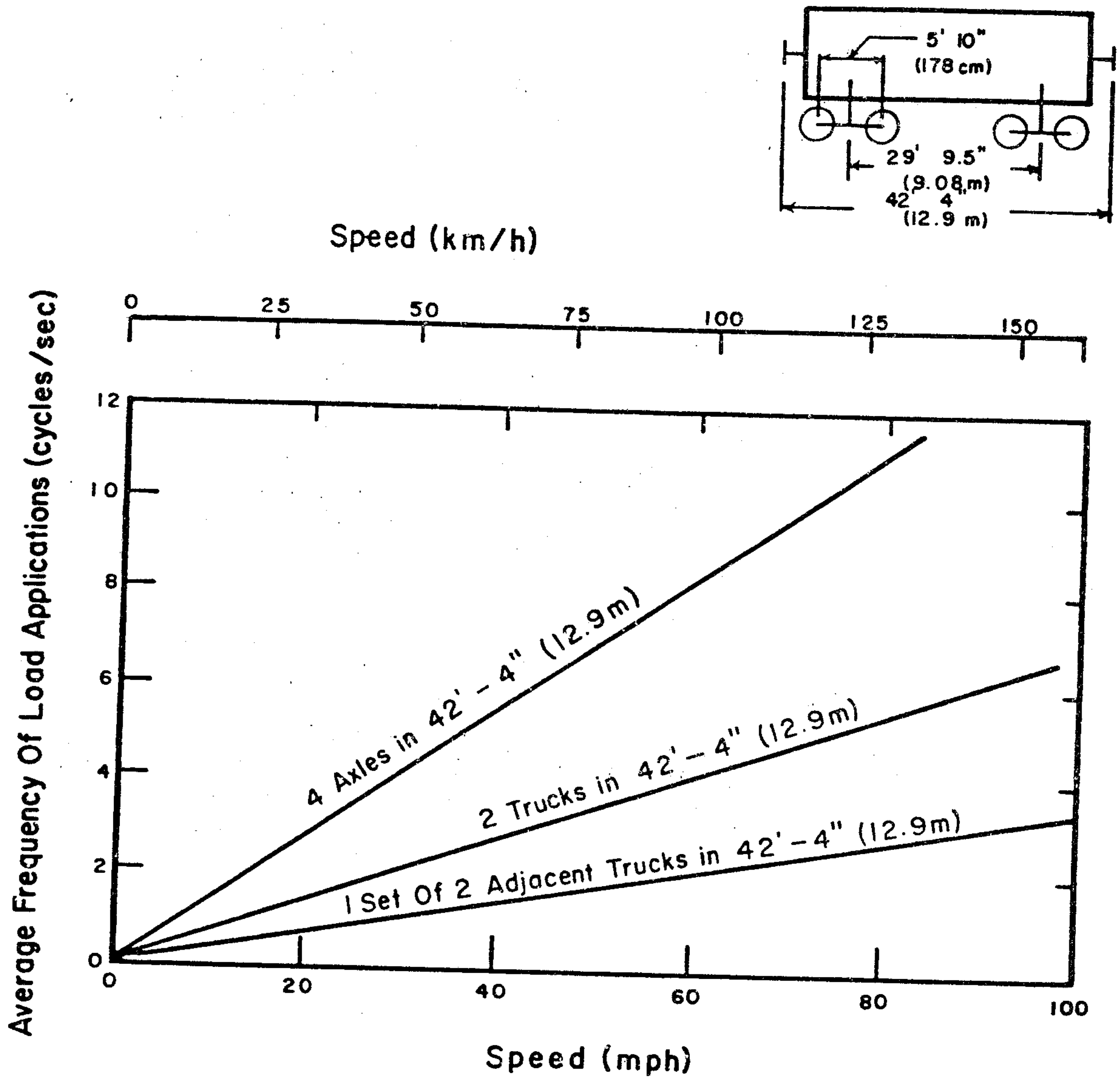


Figure 27. Average Frequency of Load Applications Versus Speed.

soil material behavior that are needed as input to the structural model; these are:

1. resilient or elastic behavior,
2. Poisson's ratio,
3. failure stress levels, and
4. stress-strain response after "failure"

The structural model utilizes these inputs and calculates the response parameters (stress, strain, and deflection or deformation) of the track system. These response parameters can then be used to evaluate the potential permanent deformation of the track system by using a permanent deformation "model" developed from laboratory testing. An earlier project report (1) discussed current permanent deformation models applicable to granular and fine-grained soil materials.

EVALUATION OF CHARACTERISTICS OF BALLAST MATERIALS

Based on current technology and broad experience with highway and airfield pavements and materials, realistic evaluation of the repeated, dynamic loading characteristics of granular materials can be obtained by use of a repeated load triaxial testing procedure.

Repeated Load Triaxial Testing

Several investigators (2, 31, 32, 33, 34, 35, 36) have used the conventional triaxial cell to evaluate the repeated load behavior of granular materials. Most often the confining pressure has been held constant and a repeated axial deviator stress has been applied, although Allen (35) and Brown and Hyde (34) have examined the effect of constant and pulsating confining pressure.

a. Resilient Behavior

One extremely successful method of determining the "elastic" or resilient behavior of granular materials is through the use of triaxial testing equipment, such as that shown in Figure 28. A conventional triaxial cell is instrumented and elastic vertical deflection measured at various magnitudes of confining pressure (typical σ_3 range = 3 to 25 psi (20-170 kN/m^2)) and various magnitudes of repeated axial stress (typical σ_1 range = 5-100 psi (34-690 kN/m^2). The resilient modulus for granular materials has been shown to increase as the cycled deviator stress increases and/or as the confining pressure increases. A general expression relating the resilient modulus to the stress state is (Equation 30):

$$E_R = K_3(\theta)^{K_4}$$

where E_R = resilient modulus, psi

K_3, K_4 = constants determined from laboratory tests

θ = sum of the principal stress = $\sigma_1 + \sigma_2 + \sigma_3$ ($\sigma_1 + 2\sigma_3$ in a triaxial test), psi

The resilient behavior of granular materials as evaluated in the repeated load triaxial procedure has also been related directly to the confining pressure by an expression similar to Equation 30:

$$E_R = K_5 \sigma_3^{K_6} \tag{31}$$

where K_5 = a constant

K_6 = a constant defining slope of line on log-log plot

b. Permanent Deformation

In addition to the determination of resilient properties of ballast materials subjected to repeated load, the permanent (also referred to as plastic or irrecoverable) deformation accumulated by the repeated application of a deviator stress is an important consideration. Barksdale (2) has shown that

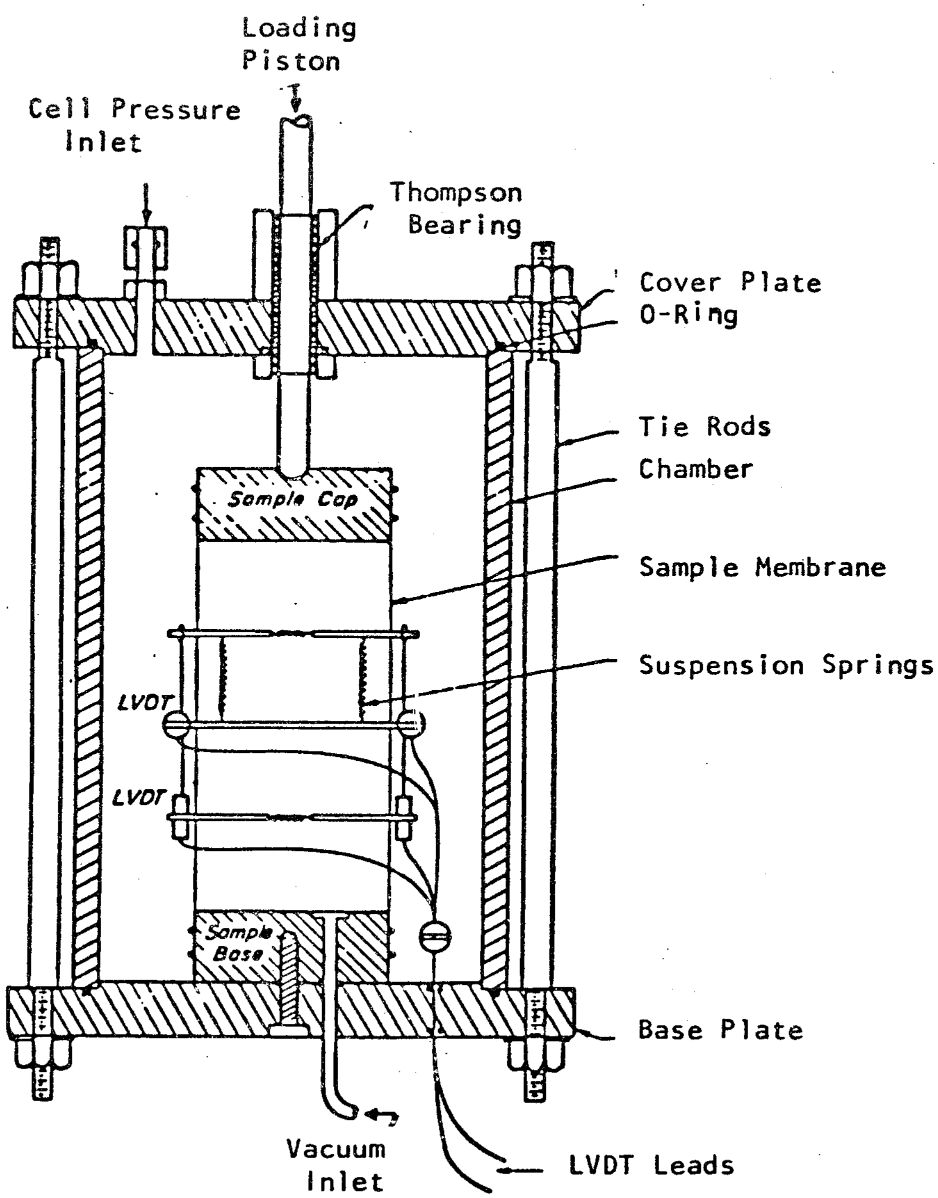


Figure 28. Typical Repeated Load Triaxial Testing Cell.

the plastic strain depends upon the number of load applications, confining pressure, the aggregate type and gradation, the density, and the degree of saturation. Typical results are shown in Figure 29. Haynes and Yoder (37) and Holubec (38) also have studied the permanent deformation behavior of granular materials subjected to repeated loading.

Although the plastic deformation attributable to one cycle of loading is small, the accumulated deformation after a number of cycles may be appreciable. The rate of accumulation of cyclic plastic deformation has been shown to approach a limiting value fairly rapidly.

Barksdale (2) modified the hyperbolic stress strain law originally developed by Konder (39) to predict the permanent strain from repeated load triaxial tests on well graded materials and developed the following expression:

$$\varepsilon_p = \frac{(\sigma_1 - \sigma_3)/(K_7 \sigma_3{}^{K_8})}{1 - \dfrac{(\sigma_1 - \sigma_3)(1 - \sin\phi)R_f}{2(C \cos\phi + \sigma_3 \sin\phi)}} \qquad (32)$$

where ε_p = permanent axial strain

$K_7 \sigma_3{}^{K_8}$ = a relationship defining the initial tangent modulus, psi

C = cohesion, psi

ϕ = angle of internal friction, degrees

R_f = a ratio relating the stress difference at failure to the asymptotic value of stress difference

The importance of the deviator stress and the confining pressure is obvious.

ORE (40) found the permanent deformation of ballast (triaxial test conditions) could be predicted by:

$$e_N = 0.082\ (100n - 38.2)\ (\sigma_1 - \sigma_3)^2\ (1 + 0.2 \log N) \qquad (33)$$

where:

e_N = permanent axial strain after N loading cycles

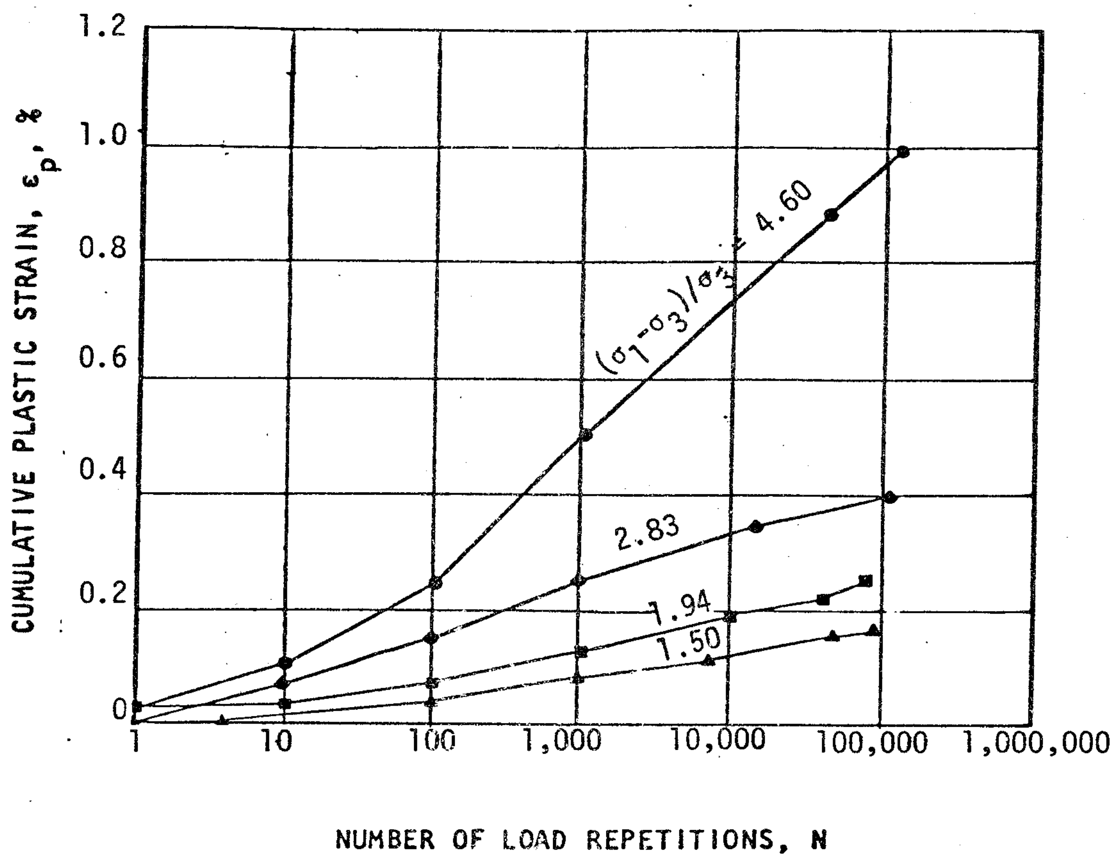


Figure 29. Influence of Number of Load Repetitions and Deviator Stress Ratio on Plastic Strain in a Porphyrite Granite Gneiss-Three Percent Fines, Confining Pressure Equal to 10 psi(From Ref. 2).

n = initial porosity

$\sigma_1 - \sigma_3$ = deviator stress, kgf/cm^2

N = number of repeated loading cycles

From the previous relationship the deviator stress and initial porosity are shown to be extremely important factors during the first few load cycles.

In an earlier project report (1) a methodology for estimating the permanent deformation in the ballast layer was presented.

EVALUATION OF CHARACTERISTICS OF FINE-GRAINED SOILS

Based on current technology and broad experience with highway and airfield pavements and materials, it can be concluded that realistic evaluation of the characteristics of fine-grained soils can be obtained with repeated load testing. Currently, repeated load triaxial and complex modulus testing (discussed later) are the most common procedures used to evaluate the repeated dynamic load response of fine-grained soils. The repeated load triaxial test has been the more widely used of the two.

General Description of Available Repeated Load Testing Procedures

The first extensive work done in repeated load triaxial testing of fine-grained soils dates back 15 to 20 years, to Seed and his associates (41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51). Recently, repeated load testing of a large number of fine-grained soils has been conducted at the University of Illinois by Robnett and Thompson (3). The equipment and testing procedures developed by Seed and Fead (52) have been modified slightly by other investigators such as Robnett and Thompson (53), but the basic procedure remains about the same. The basic repeated load triaxial testing equipment used with fine-grained soils is pictured in Figure 28.

Basically, the testing sequence consists of preparing cylindrical specimens from the fine-grained soil, placing a rubber membrane around the specimen, placing the specimen in the test chamber, applying either a constant or repeated confining pressure, subjecting the specimen to a repeated axial load of controlled stress pulse shape and duration, and monitoring the axial (and sometimes diametral) deformation.

The duration of repeated stress pulse can be closely controlled with typical repeated load triaxial testing equipment. Additionally, the repeated load triaxial testing procedure has the advantage that the confining pressure can be closely controlled (may be constant or repeated confining pressure) and a large number of repeated axial stress applications may be applied to allow for evaluation of long term loading response. Typically, 20-30 loads per minute are applied. Pulse duration times typically range from 0.06 to 0.25 seconds and pulse shapes vary depending on whether servo-hydraulic or pneumatic techniques are used for loading.

The types of data that can be obtained from this test include:

1. resilient modulus
2. permanent strain behavior, and
3. dynamic Poisson's ratio

Figures 2 and 30 illustrate typical resilient behavior and permanent strain data that can be obtained with the test.

Complex Modulus Testing

Papazian (54) has covered the fundamentals of the complex modulus concept. Basically, the complex modulus is determined by subjecting a sample to a sinusoidally applied steady state axial compressive stress while monitoring the strain which normally lags behind the stress by a phase angle.

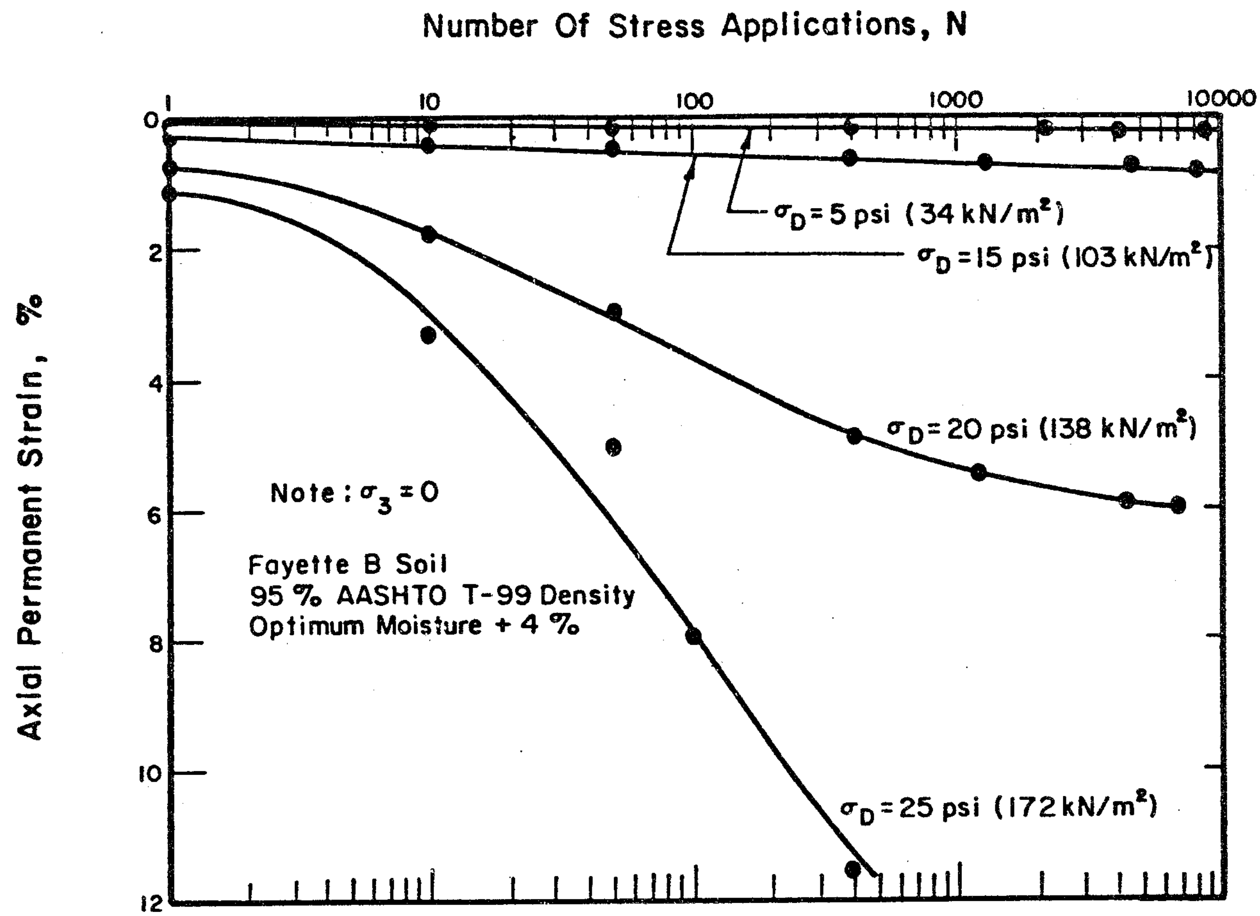


Figure 30. Typical Permanent Axial Strain Behavior of Subgrade Soil Subjected to Repeated Stress Applications.

The general equation for complex modulus is:

$$|E| = \frac{\sigma_o}{\varepsilon_o} \tag{34}$$

where:

$|E|$ = complex modulus

σ_o = amplitude of sinusoidal axial stress

ε_o = amplitude of steady state of recoverable axial strain

Normally the complex modulus is determined at a given frequency of dynamically applied load (usually in the range of 1-20 cycles per second). A more detailed discussion of the complex modulus test can be found in a previous project report (1).

To date complex modulus test results have not been widely used in pavement analysis work simply because of the lack of agreement between calculated and measured pavement response.

CHAPTER 5

DESCRIPTION OF TESTING PROCEDURES

GENERAL

The repeated load triaxial testing procedure has been selected for evaluating the resilient and permanent deformation characteristics of ballast and fine-grained soil materials for the following reasons:

1. It has been successfully used to evaluate fundamental properties of granular and fine-grained materials for use in analysis and design of highway and airfield pavements.
2. Actual loading conditions (stress state, frequency, duration, etc.) can be closely simulated witn the procedure.
3. The "structural model" previously described, requires as input the type of material property data that can be developed with this testing procedure.

BALLAST TESTING PROCEDURE

Sample Preparation

A sample of ballast is compacted in a split aluminum mold using a vibratory hammer or by a hand rodding technique. The mold is lined with a rubber triaxial membrane prior to placing and compacting the ballast. To promote uniform density and minimize segregation, the ballast is placed and compacted in three layers. By controlling the ballast weight and compacted volume, it is possible to prepare a specimen to a desired density. Depending on maximum ballast particle size considerations, the specimen size is either 6 inches dia. by 12 inches high or 8 inches dia. by 16 inches high (15.2 cm by 30.5 cm or 20.3 cm by 40.6 cm).

After compaction operations are completed, the specimen is removed from the mold and a second membrane placed on it. Optical tracking targets are positioned on the outside of the second membrane at upper and lower quarter points of the specimen height.

The specimen is then placed in the triaxial cell and the cell placed in a servo hydraulic loading machine.

Sample Testing

A constant confining pressure is applied to the specimen with compressed air. An axial repeated load is applied with the hydraulic ram. The loads are repeated at a frequency of 50 per minute with a haversine shaped stress pulse of 0.15 second duration.

The testing sequence is accomplished in two phases. The first phase consists of "conditioning" the sample for 5000 cycles at a constant confining pressure of 15 psi (103 kN/m^2) and a repeated deviator stress of 45 psi (310 kN/m^2). Measurements of permanent deformation are made at 10, 100, 1000, and 5000 load cycles.

The second phase of testing involves two types of measurements: resilient (elastic) deformations and permanent (irrecoverable) deformations. The resilient deformation measurements are taken with an optical scanner immediately after the 5000 load cycles of the conditioning phase and again later in the testing sequence. During the resilient deformation measurement sequences, constant confining pressures of 5, 10, and 15 psi (34, 69, and 103 kN/m^2) are used and the repeated deviator stress ranges from 10 to 60 psi (69 to 410 kN/m^2).

Permanent deformation measurements are taken at selected intervals during the second phase of testing. For a particular stress ratio, the pavement deformation is monitored over 5000 load cycles; the stress ratios used vary from 2 to 8. The permanent deformations of the total specimen height are measured by means of a linear variable differential transofmer located in the hydraulic actuator housing. A backup system for permanent deformation measurement is provided by the optical scanner system.

The entire testing sequence is shown in Figure 31.

Typical results of the resilient and permanent deformation portions of the testing program are shown in Figures 32 and 33.

SUBGRADE SOIL TESTING PROCEDURE

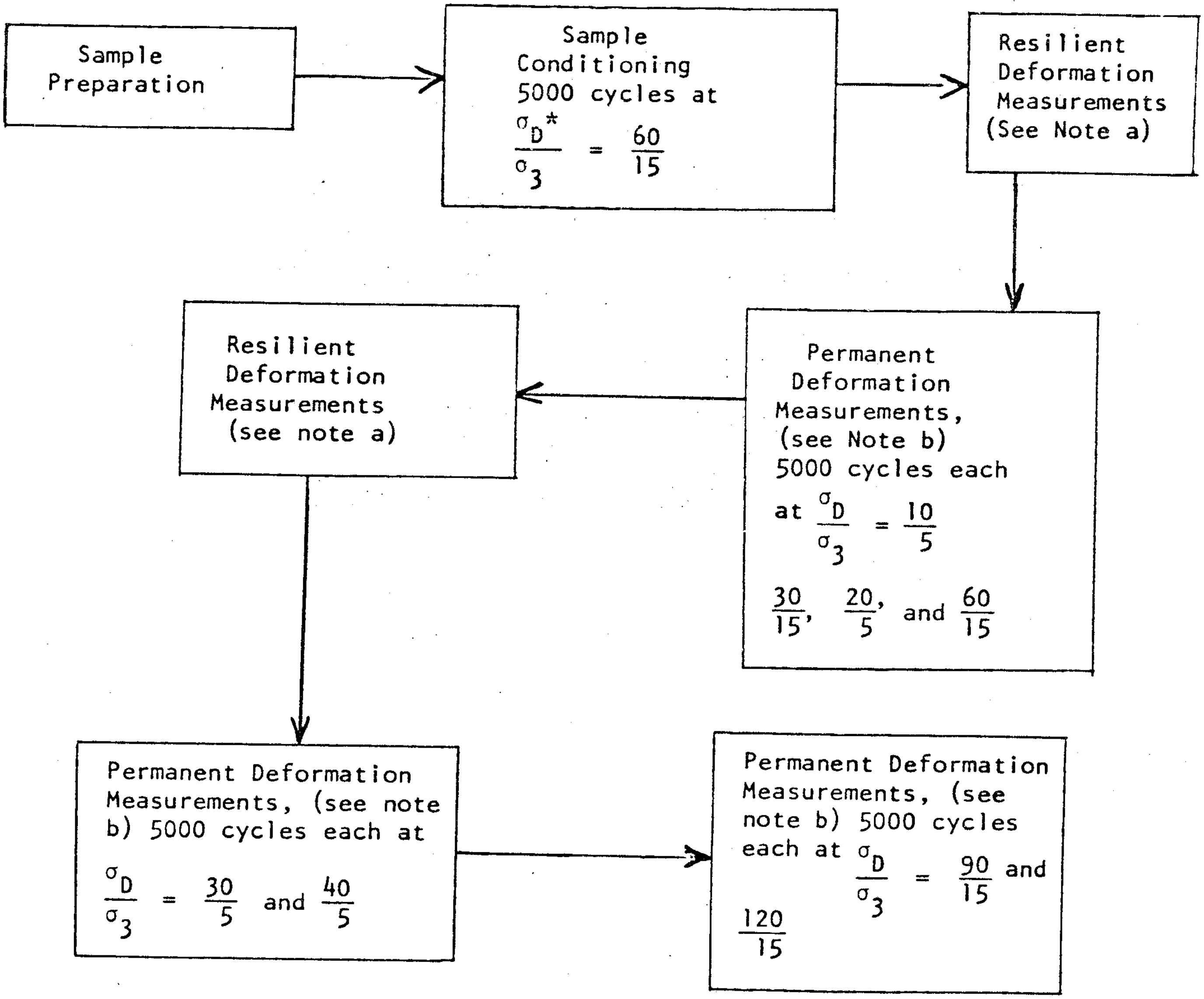
The repeated load triaxial testing procedure that has been selected for subgrade soil evaluation primarily consists of two parts, one in which the "resilient" behavior is evaluated and a second in which the "permanent deformation" behavior is evaluated.

Sample Preparation

The procedure for preparing samples is described in detail in Ref. 53. In general, it consists of preparing sets of at least three 2 inch diameter by 4 inch high (50.8 mm by 101.6 mm) cylindrical soil specimens using a miniature kneading type compactor, Figure 34. The specimen sets are prepared at moisture and density conditions representative of expected field conditions and then are tested using the repeated load equipment pictured in Figure 35.

Sample Testing

For the resilience test sequence and part of the permanent deformation test sequence, the confining pressure, σ_3, is set equal to zero. Justification for using no confining pressure during the testing of cohesive soils is a) the confining pressure normally encountered in a subgrade is very small, in the range of 1-5 psi (7-34 kN/m^2) and b) the effect of small magnitudes of confining pressure on the resilient response of fine-grained cohesive



(a) Readings at

$\sigma_D = 60$ psi, $\sigma_3 = 15$ psi
$\sigma_D = 30$ psi, $\sigma_3 = 15$ psi
$\sigma_D = 40$ psi, $\sigma_3 = 10$ psi
$\sigma_D = 20$ psi, $\sigma_3 = 10$ psi
$\sigma_D = 20$ psi, $\sigma_3 = 5$ psi
$\sigma_D = 15$ psi, $\sigma_3 = 5$ psi
$\sigma_D = 10$ psi, $\sigma_3 = 5$ psi

(b) Readings at

10, 100, 1000, and 5000 cycles

$^*\sigma_D = \sigma_1 - \sigma_3$ = repeated deviator stress

σ_3 = constant confining pressure

Figure 31. Testing Sequence For Ballast.

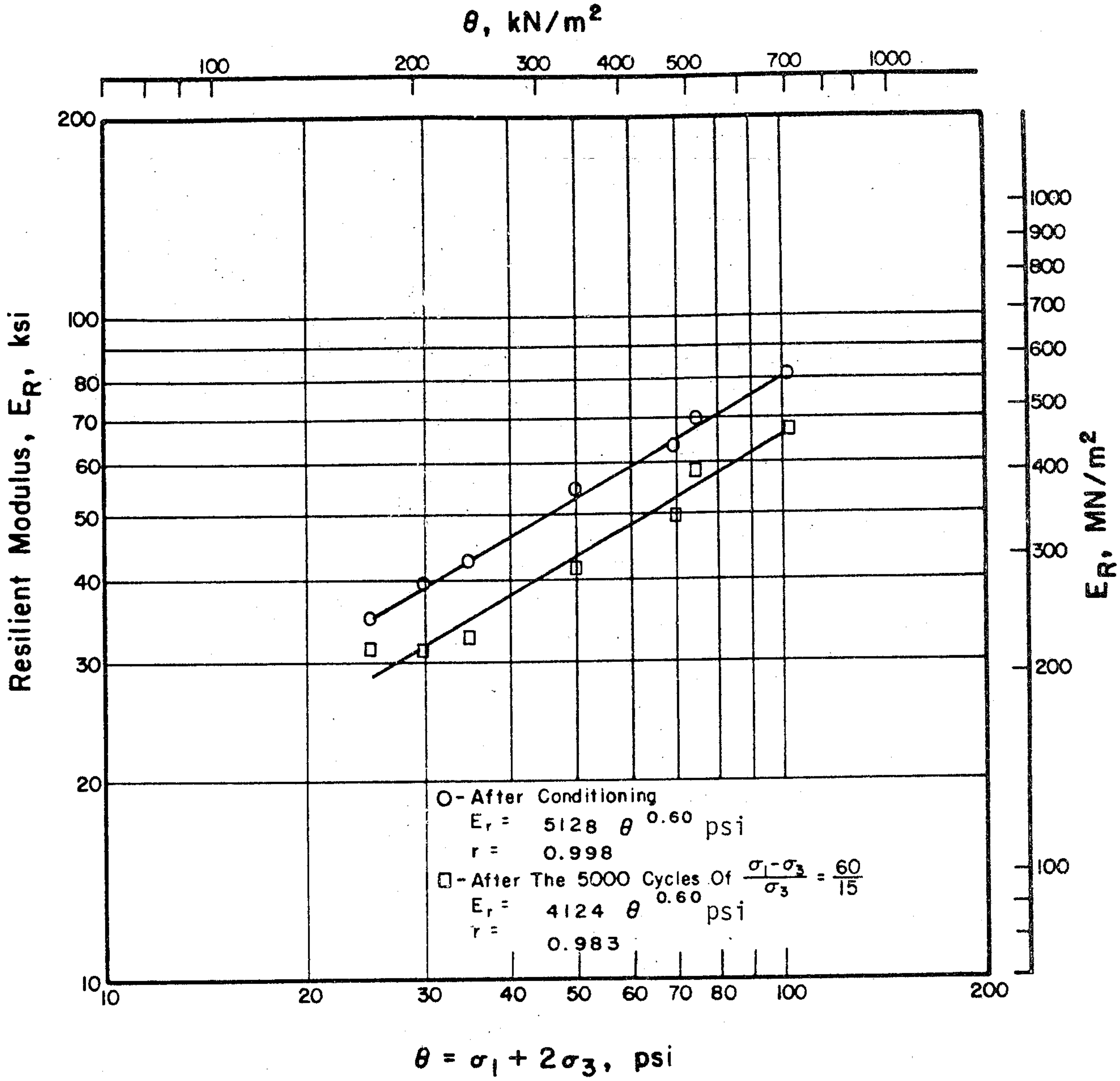


Figure 32. Influence of Total Stress, Θ, on Resilient Modulus.

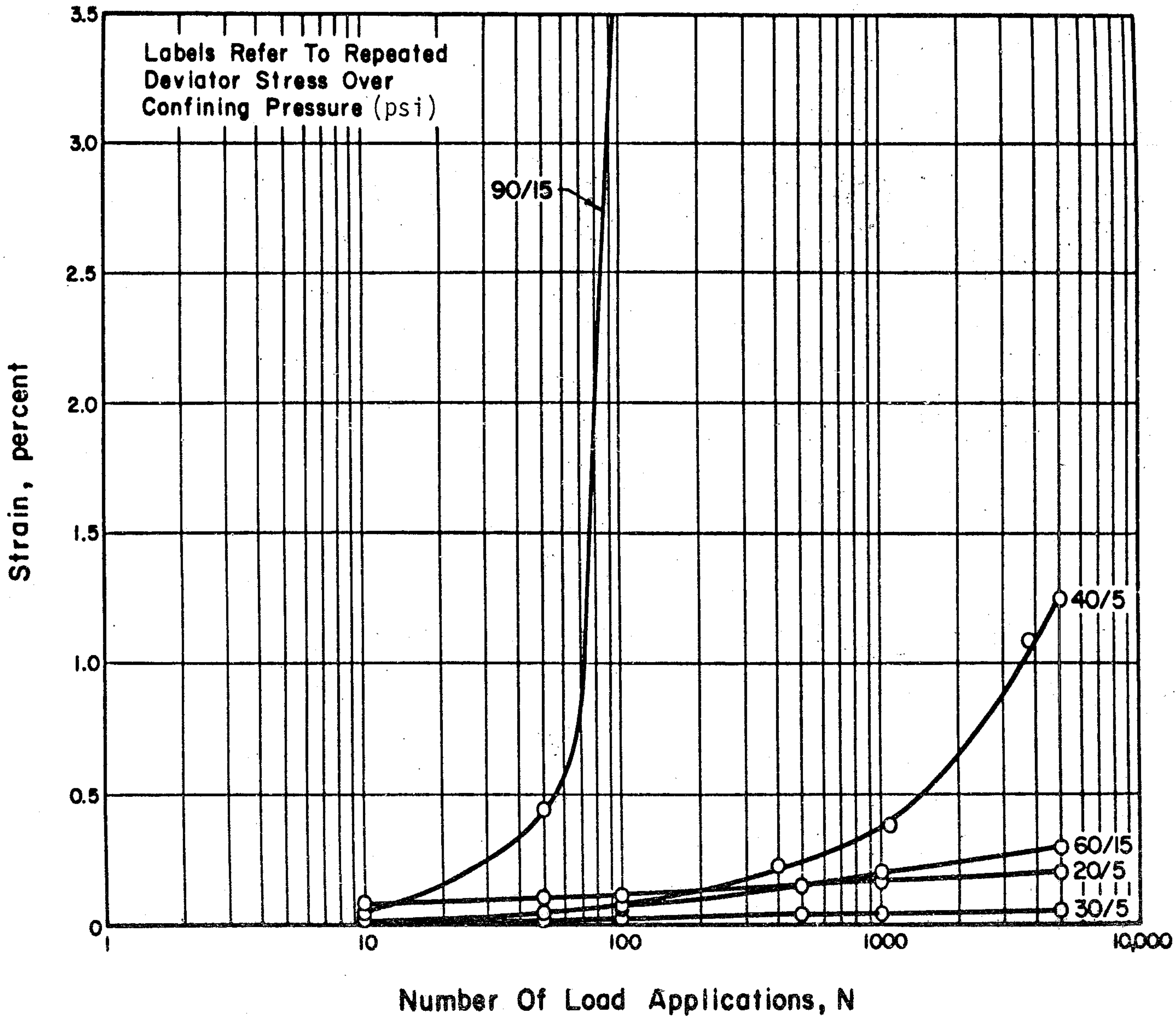


Figure 33. Influence of Number of Load Repetitions and Deviator Stress Ratio on Cumulative Permanent Strain.

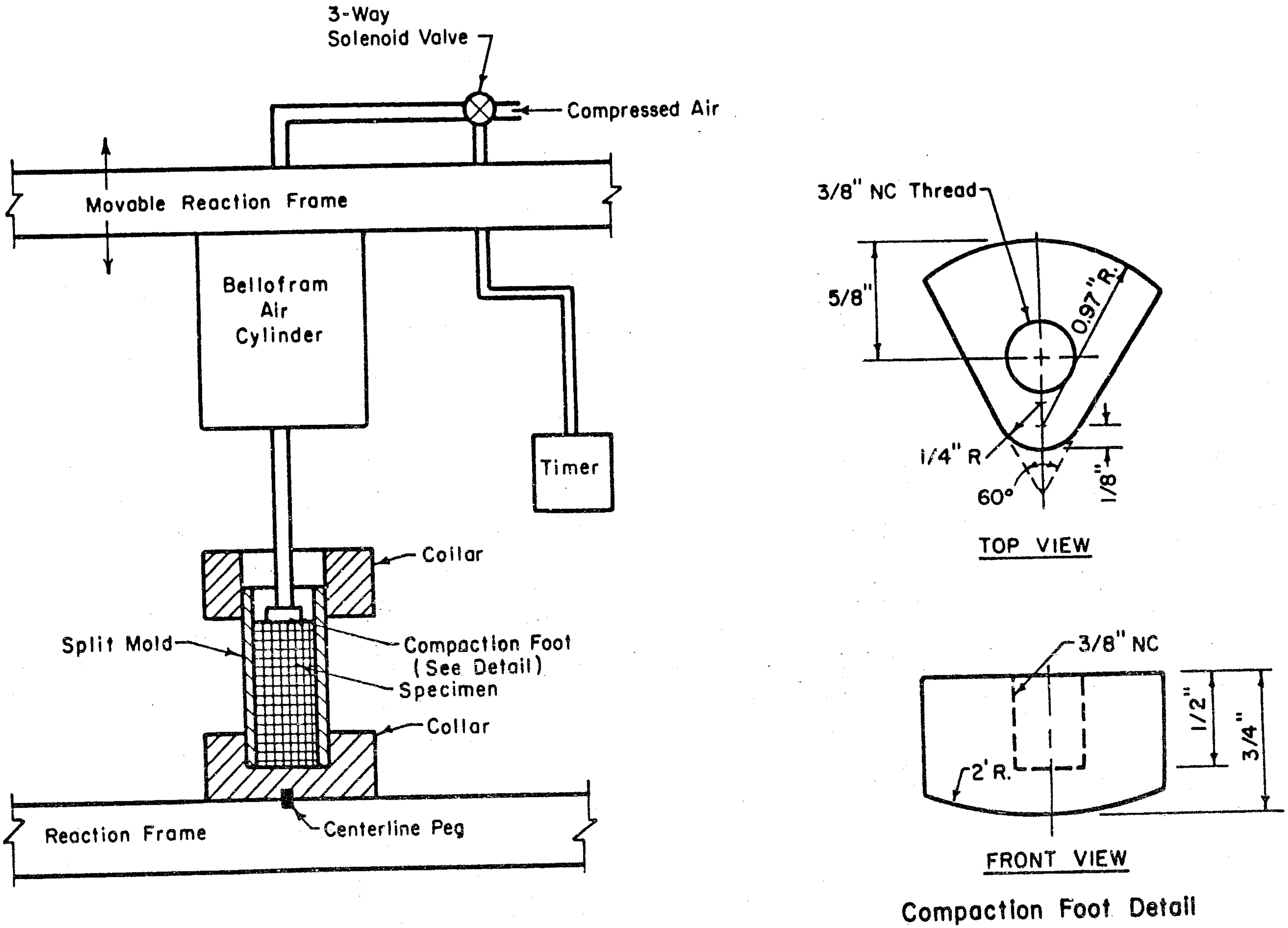


Figure 34. Schematic Diagram of Kneading Compaction Apparatus

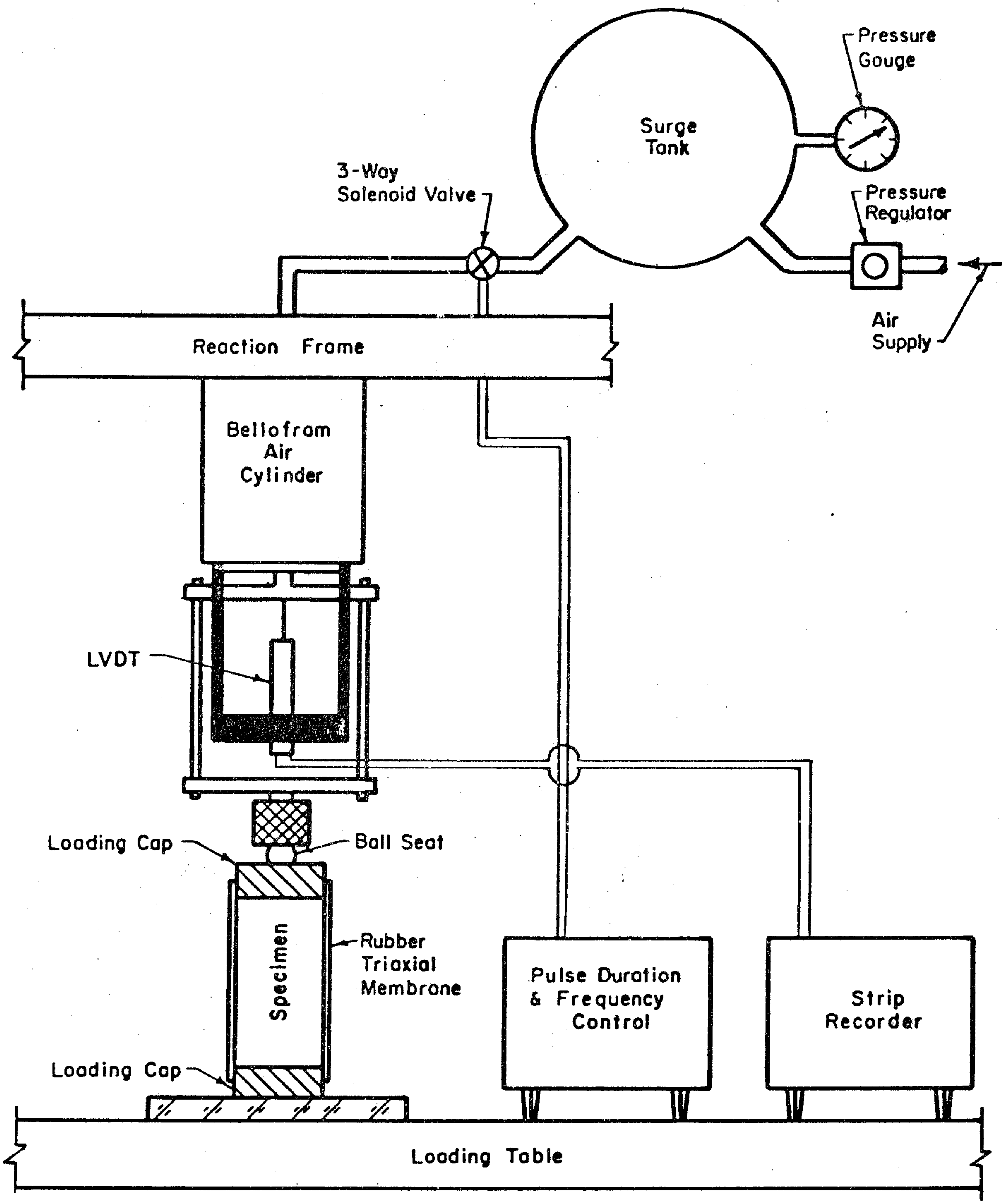


Figure 35. Schematic Diagram of Resilience Testing Equipment Used with Fine-Grained Soils (From Ref. 53).

soils has been found to be very slight and is typically less than "between specimen" testing variability (3).

Additional repeated load testing for characterizing the permanent deformation behavior is conducted in which a constant confining pressure is imposed.

The repeated axial stress that is imposed during the resilient and permanent deformation testing generally ranges from about 3 psi (21 kN/m^2) to about 50 psi (345 kN/m^2).

Typical results from the two phases of the repeated load testing of fine-grained soils are depicted in Figures 2 and 30.

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