Determination of Track Lateral Resistance from Lateral Pull Tests

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Final Report

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### Determination of Track Lateral Resistance from Lateral Pull Tests

**Abstract**

Thermal buckling of railroad tracks in the lateral plane is an important problem in the design and maintenance of continuous welded rail (CWR) track. The severity of the problem is manifested through the number of derailments which are attributable to track buckling, indicating a need for developing, among other things, better control on the allowable safe temperature increase for CWR track.

The work reported here is a part of a major investigation conducted by the Transportation Systems Center (TSC) for the Federal Railroad Administration (FRA) on the thermal buckling of CWR tracks in the lateral plane.

In this report, the measurement of track lateral resistance, which is an important parameter in the assessment of buckling safety, is considered. It is shown that the nonlinear resistance can be exactly determined from the load-deflection data obtained in the Track Lateral Pull Test. The results are presented in a convenient graphical form for a quick evaluation of the lateral resistance using the Track Lateral Pull Test data.

### Key Words
- Track Buckling
- Lateral Resistance
- Continuous Welded Rail
- Track Lateral Pull Test

### Distribution Statement

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PREFACE

Under the sponsorship of the Federal Railroad Administration (FRA), the Transportation Systems Center (TSC) is conducting research to develop the engineering basis for more effective track safety guidelines and specifications. The intent of these specifications is to ensure safe train operations while allowing the industry maximum flexibility for cost-effective track engineering and maintenance practices.

One of the major problems under investigation is track buckling. Track lateral resistance is an important parameter in the assessment of buckling strength of continuous welded rail track. This document presents a means of measuring this parameter by using the Track Lateral Pull Test (TLPT). Although TLPT is not a convenient tool for routine measurement of track lateral strength, it has been a valuable technique in the buckling research work conducted by TSC. It can also serve as a standard, against which other simpler methods such as Single Tie Push Test (STPT) currently under development, can be tested.

Analytic support in developing the track resistance measurement and data reduction scheme was partly provided by Foster-Miller, Inc. (FMI) under DTRS-57-85-C-00071.
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- 1°F = 1.8°C (approximately) for other exact conversions and more detailed tables see NBS Misc. Pub. 206, Units of Weight and Measures, Price $3.25. See Catalog No. C13 in 206.
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EXECUTIVE SUMMARY

The increased utilization of continuous welded rail (CWR) in U.S. tracks has resulted in a number of accidents attributable to derailments induced by thermal buckling of railroad tracks. In an effort to improve the safety of CWR track, experimental and analytic investigations are being conducted by the Transportation Systems Center (TSC) supporting the safety mission of the Federal Railroad Administration (FRA).

This report describes a part of these investigations dealing with the measurement of track lateral resistance which is considered to be an important parameter in the analytic assessment of buckling strength of CWR tracks.

A brief background on the subject of track resistance measurement is presented. A measurement technique called the Track Lateral Pull Test (TLPT) is described which involves the application of a concentrated lateral load to the track and the measurement of lateral load and deflection. A method is proposed here for the exact evaluation of the nonlinear track resistance from the test data. Numerical results are presented in a convenient graphical form for a quick evaluation of track resistance from TLPT data.

The Track Lateral Pull Test is shown to provide a convenient tool for obtaining a realistic average resistance for use in the buckling analyses and track lateral strength assessments.
LIST OF SYMBOLS

x  longitudinal distance from point of load application
E  Young's modulus for rail steel
A  rail cross sectional area
I  rail area moment of inertia about vertical axis
AT  rail temperature (above the stress-free temperature)
P  applied lateral force in track resistance test
Y  lateral deflection
Y₁  lateral deflection in constant resistance region
Y₂  lateral deflection in linear region of resistance
'  primes denote derivatives with respect to x or ξ
α  coefficient of thermal expansion
F₀  constant lateral resistance
Y  yield deflection for lateral resistance characteristic
l  length of track with constant lateral resistance
k  lateral stiffness
N  axial force in the rails
τ  torsional stiffness of fasteners in lateral plane
1. INTRODUCTION

The Transportation Systems Center provides technical support to the Federal Railroad Administration in the development of performance-based safety standards for continuous welded rails (CWR). A significant amount of theoretical and experimental work concerning buckling of tracks has been conducted by TSC to assess the static and dynamic buckling strength of CWR tracks [1, 2, 3].

One of the parameters required for theoretical evaluation of CWR buckling strength is the track lateral resistance. This parameter has a much stronger influence on the buckling strength than other track stiffness related parameters, namely, the longitudinal and the torsional resistances, track vertical modulus and the track bending rigidity.

Several different techniques have been used in the U.S. and abroad for measuring track lateral resistance, which are reviewed in Section 2. The data in general are not consistent, and some of the techniques result in varying degrees of scatter.

One of the methods established as a viable research tool for the measurement of the lateral resistance, is the Track Lateral Pull Test (TLPT). In this method, a concentrated lateral load is applied to the rail at a specified location and the lateral force-deflection relationship at the load point is measured. The lateral resistance is determined from this relationship, on the basis of the algorithm presented here.
The purpose of this report is:

- To present a brief background on lateral resistance measurement schemes using single tie and panel pull tests

- To present an algorithm for evaluating the resistance from the load deflection curve obtained in Track Lateral Pull Test, and

- To present a sensitivity analysis of the results with respect to temperature and torsional stiffness of the track.
2. TECHNICAL BACKGROUND

Lateral resistance measurement schemes are briefly reviewed and an assessment of the literature is described in this section.

2.1 LATERAL RESISTANCE DEFINITION

Lateral resistance is the reaction offered by the ballast to the rail-tie structure against the lateral displacement of the structure. This definition is used in track stability assessments and in buckling analyses and will be retained throughout this work. The resistance may be measured per single tie or per unit length of track, and its characteristic may be idealized as shown in Figure 1. The value of the lateral deflection at the initiation of constant movement, $\bar{y}$, (sometimes called yield point), is usually smaller than 0.39 inches (10mm). This makes the measurement of the initial portion of the force deflection curve difficult and uncertain in practice. However, the initial part of the curve is usually not important in the buckling analysis. It is the constant value of the resistance, $F_0$ (Figure 1) which has a significant influence on the safe temperature increase and, therefore, is the most important parameter to be determined.

2.2 LATERAL RESISTANCE MEASUREMENT SCHEMES

The lateral resistance is measured by mobilizing one or more ties. In the former case, the test is called a Single Tie Push Test (STPT); in the latter case, it is a panel pull test.
Figure 1. Lateral Resistance Characteristic
2.2.1 Single Tie Push Test (STPT)

In this test, the rails are unfastened from the test tie, and the tie alone is pushed (or pulled) laterally by means of a hydraulic loading jack.

In the United States, STPT has been used by ENSCO [4], Southern Railway, TSC and the State University of New York at Buffalo (SUNYAB) [5], but no correlations to the panel resistance have been performed. In Europe, STPT has been used by almost all the railroad organizations [6], and some empirical correlations have been made with panel tests.

The advantages of STPT over the panel pull tests are:

- The test is less destructive to the track
- The test is easy and relatively rapid
- The hardware is portable
- Minimal data reduction is required.

A possible disadvantage of the STPT is the scatter in the results which is discussed in Section 2.3.

2.2.2 Panel Pull Tests

Rigidized and nonrigidized cut-rail panel pull tests [6, 7, 8] have been used for the measurement of lateral resistance. In these tests, the rails are usually cut to form the panels, a lateral load is applied and the deflection is measured. Cutting rail and rewelding is an expensive and cumbersome process that makes the test impractical for other than research use. For this reason, cut panel pull tests will not be considered here, although a large number of European organizations have used them in the preparation of their data bases [6].
2.2.3 Track Lateral Pull Test (TLPT)

In this test, a concentrated lateral load is applied to the rail at a center point on the whole (uncut) section of the track. By applying reasonably large loads if necessary, the ties on either side of the point of load application are mobilized to some extent. It is important to measure the lateral force-deflection relationship at the load point. From this result, the resistance characteristic is determined by the algorithm presented here. (For a schematic representation of the test, see Figure 2.)

The advantages of the method are:

- Several ties are mobilized, which is similar to a buckling scenario. The resistance is a more realistic average than the value from the STPT measurement.

The disadvantages are:

- Destructive in nature
- Large forces (20 kips) are required to displace the track
- The hardware is bulky and needs an external reaction
- Often it is difficult for the external reaction to access locations of interest.

Applications of TLPT for resistance measurement have been recently performed in the United States during buckling experiments at The Plains, VA in 1981 [1] and at the Transportation Test Center (TTC) in 1983 and 1984 [3].
BULLDOZER USED AS LOAD ANCHOR
LOAD GAUGE CELL
RODS
LOADING RIG CABLE

GAGE RODS LOADING RIG CABLE LOAD CELL BULLDOZER USED AS LOAD ANCHOR

DATA

LOAD, P (kips)

LATERAL DEFLECTION (inches)

Figure 2. Setup for Track Lateral Pull Test (TLPT)
2.2.4 **Opposing Rail Push Test (ORPT)**

In this test, alternate spikes and tie plates are to be removed (see Figure 3) and the rails are to be pushed in opposing directions in the lateral plane by a hydraulic jack wedged in between the two rails. The lateral force and the two rail displacements need to be measured. The data can be reduced to yield equivalent lateral resistances in the two directions by using the algorithm developed here.

The advantages of the system are:

- Hardware simpler than in TLPT
- No external reaction point is needed
- The test yields a more realistic average than STPT.

The disadvantages seem to be:

- It is more "destructive" to track than STPT
- It has no previous data and experience.

This method has been only recently conceived by the authors in the context of a track resistance characterization program.

2.3 **ASSESSMENT OF LITERATURE**

It is important to understand that most of the U.S. literature on the lateral resistance placed the emphasis on the ballast properties and used the resistance measurement as a means of quantifying the degree of consolidation along long lengths of track. The determination of the complete bilinear resistance curve as in Figure 1 has not been the main aim of these works. Rather, it has been considered adequate to define the resistance value at some reference deflection, usually taken as 4mm (.15"). For the track buckling analysis, this is inadequate. It is necessary to know the steady value $F_o$.
Figure 3. Setup for Opposing Rail Push Test (ORPT)
which is independent of the lateral deflection. It is desirable to completely define the characteristic of the lateral resistance as in Figure 1.

In References [5, 9], STPT data has been analyzed and the general conclusion reached is that the scatter is large. Because of the simplicity and practicality of STPT, there is a clear need to reduce the scatter associated with STPT. The reliability of STPT data can be improved by calibrating the STPT rig with realistic average values obtained through TLPT.

The data reduction scheme developed here for the determination of the bilinear resistance characteristic, makes TLPT a valuable reference for comparison with other techniques. The importance of a rational data reduction scheme when multiple ties are mobilized in the TLPT and similar panel pull tests cannot be overemphasized. In this context, mention must be made about the panel tests carried out by the Chessie Systems at Sabot, and the AAR in the track laboratory at Chicago. In the former [8], a cut-rail panel 39' long was laterally pulled at the center, and the force value at a fixed deflection was considered as the resistance. Due to the bending effects in the panel and the nonlinear resistance characteristic involved, the force value is not the true lateral resistance. For the same reasons, the data collected in the AAR/Chicago tests [10], do not represent the actual lateral resistance as defined here in the context of track buckling.
3. TLPT ANALYSIS

As described in the previous section, TLPT yields a load-deflection curve as shown in Figure 2. To deduce the nonlinear lateral resistance characteristic of ties (Figure 1) from the TLPT data, we formulate and solve the differential equations of lateral bending of CWR track. We will make the following assumptions in the analysis to follow in Section 3.1:

- The two rails act as a single Euler-Bernoulli beam with known flexural rigidity
- The rails are at the neutral temperature, while the load deflection data is being collected, i.e., there are no longitudinal forces in rails
- The torsional stiffness of fasteners is negligible.

As far as development of the analysis is concerned, the last two assumptions are not necessary. However, the rail neutral temperature may not be precisely known; therefore, we propose to consider temperature deviations from rail neutral temperature, as a source of error in the present analysis. Likewise, a nonzero torsional stiffness can be considered as a source of error. In Sections 3.2 and 3.3, analysis will be carried out to determine errors in the computed lateral resistance due to rail temperature variations and finite torsional stiffness in the fasteners.

3.1 ANALYSIS AND DATA REDUCTION

Figure 4 represents the deflected shape of railroad track, when a lateral concentrated load $P$ is applied. We assume that the deformation is symmetric about the load point, a situation that will generally exist, if the ballast resistance
LATERAL DEFLECTION

LOAD, P

REGION I --- REGION II

LATERAL RESISTANCE

Figure 4. Deflection - Resistance Distribution in Track Lateral Pull Test
is fairly uniform in the vicinity of load point. It is convenient to divide the deflected track length on the positive x side (Figure 1) into two regions:

Region I $\ 0 < x < l$

Region II $\ x > l$

where $l$ represents the transition point, as yet unknown. Up to this transition point from the load point in Region I, the track deflection is more than $y$ (the "break free point" of lateral resistance, Figure 1), and therefore, the lateral resistance is constant $F_0$ in this region. Beyond the transition point in Region II, the resistance is proportional to the displacement and hence equals $ky$. The differential equations for Region I are:

$$EIy_1'''' = -F_0$$ (1)

The primes here denote the derivatives with respect to $x$ and $EI$ is the flexural rigidity of the track. For Region II, it is convenient to shift the origin to the transition point by introducing the new coordinate:

$$\xi = x - l$$

The differential equation for Region II is:

$$EIy_2'''' + ky_2 = 0$$ (2)
Here \( k \) is the slope of the initial part of the resistance curve (Figure 1). Both \( F_0 \) and \( k \) will be determined from the load-deflection data obtained in TLFT.

The primes on \( y_2 \) are to be understood as referring to the derivatives with respect to \( \xi \). The boundary conditions on \( y_1 \) at \( x = 0 \) are:

\[
y_1'' = 0 \tag{3}
\]
\[
E1y_1''' = P/2 \tag{4}
\]

The boundary conditions on \( y_2 \) as \( \xi \to \infty \) are

\[
y_2, y_2', \ldots \to 0 \tag{5}
\]

The continuity conditions at \( x = l \) are:

\[
y_1 = \bar{y} \tag{6}
\]
\[
y_2 = \bar{y} \tag{7}
\]
\[
y_1' = y_2' \tag{8}
\]
\[
y_1'' = y_2'' \tag{9}
\]
\[
y_1''' = y_2''' \tag{10}
\]
It is not difficult to deduce that the following are required solutions of differential equations (1) and (2):

\[ y_1 = -\frac{F_0 x^4}{24EI} + \frac{P x^3}{12EI} + \frac{M_0 x^2}{2EI} + C_0 \]  \hspace{1cm} (11)

\[ y_2 = e^{-\lambda \xi} (C_1 \cos \lambda \xi + C_2 \sin \lambda \xi) \]  \hspace{1cm} (12)

in which \( M_0, C_0, C_1, \) and \( C_2 \) are constants, and

\[ \lambda = \frac{4}{\sqrt{4EI}} = \frac{4}{\sqrt{\frac{F_0}{4EIy}}} \]  \hspace{1cm} (13)

It can be seen that conditions (3) to (5) are satisfied by the proposed expressions.

Conditions (6) to (10) are adequate to determine the five unknowns: \( M_0, C_0, C_1, C_2, \) and \( \lambda \). These conditions yield the following equations:

\[ \frac{M_0 \xi^2}{2EI} + C_0 = \bar{y} + \frac{F_0 \xi^4}{24EI} - \frac{P \xi^3}{12EI} \]  \hspace{1cm} (14)

\[ C_1 = \bar{y} \]  \hspace{1cm} (15)
After eliminating $C_1$, $C_2$, and $M_o$, it can be shown that:

\[ P = \left[ 2F_0 \beta (2 \beta^2 + 6 \beta + 3) + 24EI \lambda^4 \bar{y}(1+\beta) \right] \]
\[ + \left[ 3 \lambda (1+\beta)^2 \right] \]  

From Equation 11, the maximum deflection, $Y_1(\text{max})$, is seen to be $C_0$.

\[ C_0 = Y_1(\text{max}) = (1-\beta^2)\bar{y} \frac{-F_0 \beta^3}{EI \lambda^4} \left( \frac{58-12}{24} \right) \]
\[ + \frac{P \beta^2}{EI \lambda^3} \left( \frac{28+3}{12} \right) \]
Here:

\[ \beta = \lambda l \]

Equations (19) and (20) are the required expressions. It is clear that the relationship between \( P \) and \( y_1(\text{max}) \) is implicit through the parameter \( \beta \). For known values of \( F_0 \) and \( \bar{y} \), the relationship between \( P \) and \( y_1(\text{max}) \) can be obtained by assuming values of \( \beta \) (values of \( l \)). For the special case when \( \bar{y} = 0 \), it can be shown that:

\[ P = 4 \sqrt{\frac{2048 EIF_0^3}{9}} y_1(\text{max}) \]  \hspace{1cm} (21)

**Load-Deflection Plots**

A set of graphs giving the relationship between the lateral load and deflection is prepared using the equations 19, 20 and 21 for some values of \( \bar{y} \) in the range 0 to 10 mm, and of \( F_0 \) in the range 30 to 100 lb/in. These are presented in the Appendix. It is only necessary to superimpose the experimentally determined \( P \) versus \( y_1(\text{max}) \) relationship on the graphs, and select the best curve that matches with the experimental data. The values of \( \bar{y} \) and \( F_0 \) can then be read off the curve.

Another fairly rapid method of data reduction is through use of log-log plots. It is found that log-log plots of curves presented in the Appendix are straight lines, each of which can be uniquely defined by the intercept on vertical axis and the slope. The graph in Figure 5 is prepared on this basis; from this, the values of \( F_0 \) and \( \bar{y} \) can be read using the intercept and the slope of the log-log plot determined experimentally. This method is especially convenient to use if the experimental data are discrete.
Figure 5. Chart Depicting Evaluation of Lateral Resistance ($AT = 0$)
Examples:

As examples, the data obtained on tangent and curved track tests carried out in 1983 at the Transportation Test Center, Pueblo, will be presented. The log-log plots of the discrete experimental data are shown in Figure 6.

Using the intercepts and the slopes, from Figure 5, the resistance characteristic values are found to be as in Table 1.

For the deduced values of $F_0$ and $\bar{y}$ (Table 1), the load-deflection curve is reconstructed using the analysis presented in Section 3.1. The theoretical correlations are excellent as seen from Figure 7 for the tangent track and from Figure 8 for the curved track.

3.2 EFFECT OF TEMPERATURE

In the foregoing analysis, it is assumed that there is no axial force in the rails. This means that the Track Lateral Pull Test is to be done when the rail temperature is at its neutral value. If, however, at the time of testing, the rail temperature differs from its neutral value by $\Delta T$, there will be a corresponding error in the lateral resistance as computed in the previous analysis.

To study the effect of temperature, we need to incorporate the axial force term in the differential equations. The axial compressive force is given by:

$$N = AE_0\Delta T$$  \hspace{1cm} (22)

Here:

$$\Delta T = T - T_{\text{neutral}}$$
Table 1. Test Data for Tangent and Curved Track

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<thead>
<tr>
<th>Track</th>
<th>Intercept</th>
<th>Slope</th>
<th>$F_0$ (lb/inch)</th>
<th>$\bar{y}$ (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangent</td>
<td>4.091</td>
<td>0.302</td>
<td>54</td>
<td>0.3</td>
</tr>
<tr>
<td>Curve</td>
<td>4.081</td>
<td>0.265</td>
<td>48</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Figure 6. Log-Log Plots of Experimental Data for Tangent and Curved Track
Figure 7. Tangent Track Results
Figure 8. Curved Track Results
The differential equations are (Figure 4):

Region I
\[ EIy_1'''' + NY_1'' = -F_o \]  

Region II
\[ EIy_2'''' + NY_2'' + ky_2 = 0 \]

The solution of the equations can be written as:

\[ Y_1 = A_1 \sin \sqrt{\frac{N}{EI}} x + A_2 \cos \sqrt{\frac{N}{EI}} x \]

\[ \frac{-F_o x^2}{2N} + B_1 x + B_2 \]  

\[ Y_2 = \frac{P \lambda^2}{2kab} e^{-b \varepsilon} (C_1 \cos a \varepsilon + C_2 \sin a \varepsilon) \]

where:

\[ a = \sqrt{\lambda^2 + N/4EI} \]
\[ b = \sqrt{\lambda^2 - N/4EI} \]
\[ \lambda = 4 \sqrt{k/4EI} \]
Using the boundary and continuity equations, the required expressions for the unknowns in equations 24 and 25 are obtained. For given values of $F_0$, $y$ and $\Delta T$, the nonlinear equations can be solved to yield the relationship between the lateral force and deflection.

Figures 9 and 10 show the theoretical load versus deflection behavior for varying values of lateral resistance and $\Delta T = 20^\circ F$ and $\Delta T = -20^\circ F$, respectively, with $y = 0.0$ inches.

To estimate the percent of error in the computed lateral resistance when temperature effect is neglected or not known (due to unknown neutral temperature) consider the load versus deflection curves shown in Figure 11. The figure shows the three load deflection curves for $\Delta T = -20, 0, +20^\circ F$, yet each represents the same value of lateral resistance, namely, 60 lb/in. Therefore, if temperature were neglected, the curve corresponding to $\Delta T = -20^\circ F$ would yield a lateral resistance value of 68 lb/in. rather than the actual 60 lb/in. resulting in an error of approximately 15%. Likewise, the curve for $\Delta T = 20^\circ F$ gives a value of 53 lb/in. when temperature is neglected, which results in an error of the same order of magnitude. By roughly interpolating in a linear manner, it can be shown that the change in temperature from the neutral should not exceed $\pm 15^\circ F$ if the maximum permissible error is to be on the order of 10%. From these results, it can be clearly seen that $+\Delta T$ implies compressive force that helps the externally applied lateral force in deflecting the track whereas $-\Delta T$ gives a tensile force that opposes the lateral force. Hence, the lateral resistance will be underestimated for $\Delta T > 0$ and overestimated for $\Delta T < 0$. 
Figure 9. Theoretical Load - Deflection Curves
($\Delta T = -20^\circ F$)
Figure 10. Theoretical Load - Deflection Curves
($\Delta T = 20^\circ F$)
Figure 11. Load Deflection Curves for Different Temperatures
3.3 EFFECT OF TORSIONAL STIFFNESS

In the analysis presented in Section 3.1, the torsional stiffness of fasteners in the lateral plane is neglected. For wood tie track with cut spike construction, this is a reasonable assumption. For fasteners with some torsional stiffness, the analysis presented in Section 3.1 needs to be modified.

We assume that the torsional stiffness $\tau$ is constant and defined by:

$$\text{Torque} = \tau \text{(angle of twist)}$$

$$= \tau \frac{dy}{dx}$$

The differential equations for the two regions (Figure 4) are:

Region I: \[ EIy_1'''' - \tau y_1'' = -F_0 \]  
Region II: \[ EIy_2'''' - \gamma_2'' + ky_2 = 0 \]  

Solutions of the above equations can be developed on lines similar to those in Section 3.1. Inspection of equations (23) and (28) reveals that the effect of $\tau$ is the same as an equivalent axial tensile force $N = \tau$. Therefore, it can be concluded that if $\tau = 100$ inch kips/inch of track length, the error is the same as that due to $\Delta T = -20^\circ F$, i.e., about 15%. 

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4. CONCLUSIONS

The following conclusions can be drawn from the investigations carried out on the Track Lateral Pull Test (TLPT)

- The nonlinear response of lateral track deformation under a concentrated lateral force can be determined easily, if the lateral resistance of ties is idealized as a bilinear characteristic. Conversely, if the nonlinear response is known (determined experimentally using track lateral pull test rig) the bilinear resistance curve can be deduced uniquely using the solution technique proposed here.

- The data reduction scheme proposed here yields reasonably accurate values, if the rail neutral temperature is known. If the neutral temperature is uncertain by ±15°F, then there may be an error of about 10% in computed lateral resistance. Similarly, if the fastener has torsional stiffness, this will contribute to some error in the computed resistance, unless the stiffness is included in the differential equations governing track lateral deformation under TLPT. For cut spikes, the error is estimated to be below 10%.

- The Track Lateral Pull Test provides a good means of determining the lateral resistance in buckling test scenarios, since this can also be used to set misalignments usually required in buckling tests. However, the test is "destructive" to track and very cumbersome in practice requiring a bulldozer and other bulky hardware. Therefore TLPT is to be used as a research tool to calibrate other simpler schemes of measuring the lateral resistance, such as STPT and other nondestructive techniques.
APPENDIX:
LOAD-DEFLECTION PLOTS
(136 lb rail, $\Delta T = 0$)
Figure A-1. TLPT Theoretical Results ($\bar{y} = 0.0$ inch)
Figure A-2. TLPT Theoretical Results ($\bar{y} = 0.1$ inch)
Figure A-3. TLPT Theoretical Results ($\bar{y} = 0.2$ inch)
Figure A-4. TLPT Theoretical Results ($\bar{y} = 0.3$ inch)
Figure A-5. TLPT Theoretical Results ($\bar{y} = 0.4$ inch)
Figure A-6. TLPT Theoretical Results ($\bar{y} - 0.5$ inch)
REFERENCES


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