REPORT NO. UMTA-MA-06-0025-74-4

ASSESSMENT OF DESIGN TOOLS AND CRITERIA FOR URBAN RAIL TRACK STRUCTURES

Volume II. At-Grade Slab Track

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APRIL 1974

FINAL REPORT

Prepared for

DEPARTMENT OF TRANSPORTATION

URBAN MASS TRANSPORTATION ADMINISTRATION
OFFICE OF RESEARCH AND DEVELOPMENT
Washington DC 20590

NOTICE

NOTICE

Technical Report Documentation Page

1. Report No. UMTA-MA-06-0025-74-4	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle ASSESSMENT OF DESIGN TOOLS AND CRITERIA FOR URBAN RAIL TRACK STRUCTURES Volume II. At-Grade Slab Track		5. Report Date April 1974
		6. Performing Organization Code
7. Author(s) Howard C. Meacham, Robert H. Prause John Waddell*		8. Performing Organization Report No. DOT-TSC-UMTA-74-5
9. Performing Organization Name and Address Battelle-Columbus Laboratories 505 King Avenue Columbus OH 43201		10. Work Unit No. (TRAIS) UM404/R4734
		11. Contract or Grant No. DOT-TSC-563
12. Sponsoring Agency Name and Address Department of Transportation Urban Mass Transportation Administration Office of Research and Development Washington DC 20590		13. Type of Report and Period Covered Final Report March 1973 - March 1974
		14. Sponsoring Agency Code
15. Supplementary Notes *Under contract to: Department of Transportation Transportation Systems Center Cambridge MA 02142		Vol. I is subtitled "At Grade Tie-Ballast Track"

16. Abstract

The development of techniques and criteria for track design is an important part of the Rail Supporting Technology Program that is being managed for the Urban Mass Transportation Administration by the Transportation Systems Center (TSC). This report presents the results of a critical review of the technical factors which govern the design and performance of at-grade slab track for urban rail systems. The assessment of current design practice is based on a review of the literature and discussions with experienced track design personnel. The evaluation includes descriptions of slab structures now in use in four countries, followed by review of design and analysis procedures used to characterize the subgrade and its support characteristics; the reinforced concrete slab itself, and the subgrade-support system. With a few exceptions, most of the work reported in the literature is based on highway or runway applications, where the mechanism of load transfer into the slab is completely different than in a rail support slab. Further research on the mechanisms of load transfer from rail fasteners into a reinforced concrete slab is needed, and the newly developed finite element approach appears well-suited. Continued study of settlement and failure criteria is needed for soil and base materials subjected to cyclic loading. The relative merits of various types of reinforced concrete slabs for example, pre-stress or post-stress considerations and joints are recommended for further study.

17. Key Words Track, track design, track loading, rapid transit systems, ballast, fastenings, rail rail joints, stresses, soil mechanics, slab, reinforced concrete.		18. Distribution Statement DOCUMENT IS AVAILABLE TO THE PUBLIC THROUGH THE NATIONAL TECHNICAL INFORMATION SERVICE, SPRINGFIELD, VIRGINIA 22151.	
19. Security Classif. (of this report) UNCLASSIFIED	20. Security Classif. (of this page) UNCLASSIFIED	21. No. of Pages 90	22. Price

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

PREFACE

This report presents the results of a program to review the technical factors which govern the design and performance of at-grade urban rail track structures. The report has been prepared by Battelle-Columbus Laboratories (BCL) under Contract DOT-TSC-563 for the Transportation Systems Center (TSC), Systems Manager for the Urban Mass Transportation Administration's Rail Supporting Technology Program. The program was conducted under the technical direction of Dr. Leonard Kurzweil, Code TMP, at the Transportation Systems Center.

The report is presented in two volumes. Volume I gives results related to the design and performance of tie-ballast track construction and Volume II is an evaluation of the technical requirements for designing track structures incorporating reinforced concrete slabs.

The cooperation and assistance provided by Dr. Leonard Kurzweil and Dr. Herbert Weinstock of TSC and Mr. Ronald Melvin of BCL is gratefully acknowledged.

This report is necessarily quite dependent on previous work which has been reported in the literature, and the authors are grateful to the several authors and publishers who granted permission for the use of copyrighted material.

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1.0 INTRODUCTION

The development of urban rail systems is an important factor in meeting the transportation requirements of large cities in the coming years. In addition, the rapidly changing economic and technological environment makes it necessary to continually re-evaluate the criteria used to design all components of the transportation system.

The objective of this program was to evaluate the technical factors which govern the design and performance of urban rail track structures. This report includes information that is useful for the engineering design of track as well as recommendations to be used for the development of both short and long-range program plans for future track research.

These results will be used to develop standard techniques and criteria for track design as a part of the Urban Mass Transportation Administration's Rail Supporting Technology Program that is being managed by TSC.

The results of this program are presented in two volumes. This report, Volume II, covers present design practices and problems relating to at-grade track structures incorporating reinforced concrete slabs. Volume I covers present design practices and problems associated with conventional at-grade tie-ballast track structures.

The major conclusions and recommendations resulting from this work are summarized in Section 2 of this report. Section 3 contains a description of existing slab tracks while Section 4 includes a detailed review of present design practices pertinent to slab track. Section 5 includes brief preliminary calculations illustrating how present design techniques can be applied to the proposed section of slab track to be included in the UMTA transit track at the Pueblo High Speed Ground Test Center.

2.0 SUMMARY AND RECOMMENDATIONS FOR FUTURE RESEARCH

Several experimental sections of reinforced concrete slab-type track structure are now in operation, both in the United States and abroad. Four of these sections are well documented in the literature: those in England, Germany, Japan, and the United States. The experimental slab track in England was constructed on a previously used track roadbed, and was formed by slip form pavers. Several different fastenings are used. In Germany, two different designs are being tried, both of them using prestressed steel reinforcing rods both longitudinally and laterally. Several different variations of slab track are in use on the Japanese National Railways, predominantly in tunnel applications. In some, wood ties are embedded in the concrete to enable rail gauge to be changed easily. Other sections include precast concrete longitudinal beams supported periodically by precast concrete piers. In the United States, the Santa Fe Railroad and FRA have cooperated to build a concrete track structure in Kansas. Two of the sections are longitudinal beams, while a third is a cast-in-place slab. The reinforcing is not prestressed in any of the Kansas designs.

Problems which have been experienced with many concrete slab applications indicate that much is yet to be learned about application of reinforced concrete to the railroad load environment. The logical design of slab type track involves the determination of the pressures and strains imposed on all elements of the track structure, followed by a determination of the allowable stresses and strains that the various elements can withstand. A problem which has existed historically in the design of slabs (whose development has been mainly based on highway and airport applications) is that the soil characteristics needed for a mathematical model of the slab and its support are completely different than the soil mechanics data historically used in civil engineering to define soil properties. The design of the slab and the subgrade support has usually been based on the mathematical models of a beam of infinite length supported continuously along its length on an elastic foundation. A common representation of the foundation is one in which the vertical pressure is assumed to be proportional to the vertical deflection of the slab. The subgrade is characterized by a subgrade modulus k.

The modulus can be obtained by loading the soil with a plate, and dividing the plate load by the elastic deflection and area, giving units of $lbs/inch^3$.

The second basic method of representing the beam subgrade support is to assume that the base course and subgrade form a semi-infinite, elastic, homogeneous, isotropic solid, and assume that the applied load is uniformly distributed over a small area. The equations relating pressure to depth and load were developed by Boussinesq, and this approach especially is applicable to tie-type track in which point or line loading is more nearly approached than in the case of a slab.

A refinement in either method is to assume that there is more than one layer; this more nearly represents the actual case where a base, subbase, and subsoil layer may all have different properties. Burmister analyzed the stresses and strains in a layered system, consisting of an elastic slab infinite in the horizontal plane, placed on a semi-infinite solid of lower modulus of elasticity. In tie type track, the two layers can represent the ballast and sub-ballast or subgrade, or in slab track, one layer can actually represent the slab and the other the base course beneath it. There are a number of assumptions which are usually made in an analysis of this type, and the net result is that the assumptions vary sufficiently from the case of a slab type railroad track structure that the results are questionable. Therefore, further research is needed to develop accurate procedures for calculating pressures beneath slab track structures, and/or to evaluate the errors in the current method of calculation.

Even if embankment stresses could be calculated accurately, there is meager information regarding the design stresses for embankment materials subjected to cyclic loading. The most applicable recent work along these lines appears to be that done by British Rail. The resultant design method is based on achievement of a "balance" which is obtained when the ballast is sufficiently deep that the calculated maximum principal stress difference induced in the subgrade by the heaviest commonly occurring axle load is equal to the "threshold" principal stress difference determined by cyclic laboratory tests. Although this method was checked out on tie type track, the same basic conclusions are applicable to slab type track structures.

Throughout the years, much effort has been expended toward determining the modulus of subgrade reaction as specifically applied to slab track design.

Considering the beam-on-elastic-foundation model, it has been found that different values of subgrade modulus must be used in formulas to predict deflection than those which give accurate results for the bending moment in the slab, and still another must be used to obtain good agreement for pressures. Procedures have been developed whereby the value of k which applies directly under the slab load can be modified as a function of distance away from the loading to give an accurate prediction of deflections, subgrade pressures, and slab bending moments. This is based on a highway slab, however, and here again, further research is needed along these lines for the specific application of a reinforced slab for a track structure which, due to its non-symmetrical placement of reinforcing steel from top to bottom, and the different mechanism of load transfer into the slab, may deviate considerably from the relationships applying to highway slabs.

Newly developing techniques of structural analysis incorporating finite-element techniques may provide a breakthrough in the analysis of complicated slabs such as that represented by a reinforced concrete slab with loads introduced at the rail fastener locations. Finite-element techniques were used in the design of the Kansas test track, but continued development and application of this approach is needed. One advantage of the finite-element approach is that the stresses across the width of the slab can be calculated, rather than being restricted to the two-dimensional beam-on-elastic-foundation problem in which the various parameters are assumed to be uniform across the width of the slab and vary only along the length of the slab.

Preliminary calculations pertaining to the urban transit rail slab track proposed for the High Speed Ground Test Center at Pueblo were made, using axle loads and spacing corresponding to the State-Of-The-Art Car (SOAC). The calculations were made to illustrate the application of the various design techniques used for slab design to a particular problem--rather than to develop a specific slab design for the Pueblo facility. An uncracked section was assumed for design purposes, and a depth of slab of 9 inches was calculated as being required, with a width of 9 feet. Stiffness of the slab was based on the criterion of transmitting only one pressure pulse to the subgrade for each truck passage, rather than one pressure pulse per axle passage (two per truck) inherently obtained with tie-type track due to the relatively low bending stiffness provided solely by the rails.

A much more thorough analysis is required before the design is finalized (on the basis of minimizing life-cycle costs), including design studies of basic types of construction to be used.

Additonal analyses of concrete slab track structures should be based on realistic rather than idealized design configurations. These design studies should include detailed evaluations of rail fastener attachments. A comparison of prestressed and post-stressed slabs with continuous reinforced and nonreinforced concrete should be made to further evaluate the effect of slab joints. High tensile strength concrete should be evaluated to determine if such materials would be advantageous for urban rail slab track structures. Hopefully, analyses such as these, together with some laboratory tests of full scale slabs, can be made as a part of the design of the Pueblo track.

There is also a real need for data on the performance of subgrade materials under cyclic loading by a slab structure. The Kansas Test Track has a comprehensive array of instrumentation for the slab structure and the subgrade, and the data and experience from this installation should be evaluated thoroughly before constructing a slab track at the HSGTC.

3.0 EXISTING SLAB-TYPE TRACK STRUCTURES

Slab track brings to mind different concepts to different people and therefore the first decision in the design of a slab track is to select the general configuration of the track. To this objective, it is useful to review some of the systems which either are being, or have been, tried.

3.1 England

The British have selected a test site between Bingham and Radcliffe-on-Trent stations on the Grantham-Nottingham line. The experimental slab track has been constructed on a previously used track roadbed. (See Reference 1.)

The ballast is a combination of ash and slag laid on Keuper Marl with pockets of aluvium. The old foundation was trimmed to give the required levels for concreting the base slabs. Six different fastening systems are being tried at this site.

A modified slip form paver by Guntert and Zimmerman was used to lay the slabs. The first stage of construction was a wet, lean concrete base about 150 mm thick over the whole length of the site. The main slab reinforcement, consisting of two layers of longitudinal and transverse deformed alloy-steel bars with a yield strength of 60,000 lb/in^2, welded into cages, was then laid and the main slab poured. The wet lean mix was a 15 to 1 nominal mix with gravel aggregate, while the structural slab was a 5.5 to 1 nominal mix with limestone aggregate for ease of subsequent drilling.

A cross section of one of the slabs, the BR direct laying system is shown in Figure 1. Slabs for other fastenings (except the Channel Tunnel section) were similar in cross-section to the BR direct laying, but had horizontal seating and generally a crowned slab center to drain water outward.

The rail fastening hardware for the BR system is shown in Figure 2. The rail is <u>continuously</u> supported on a flexible rubber-bonded cork pad 10 mm. thick.

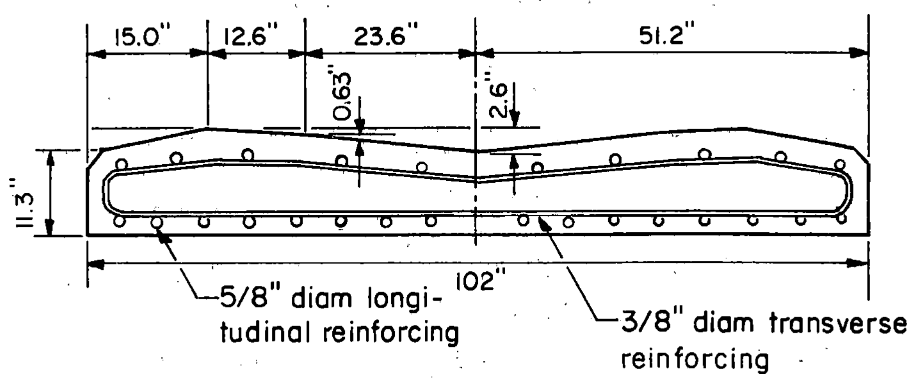


FIGURE 1. CROSS-SECTION OF SLAB FOR THE BR DIRECT LAYING SYSTEM (From Reference 1, page 548)

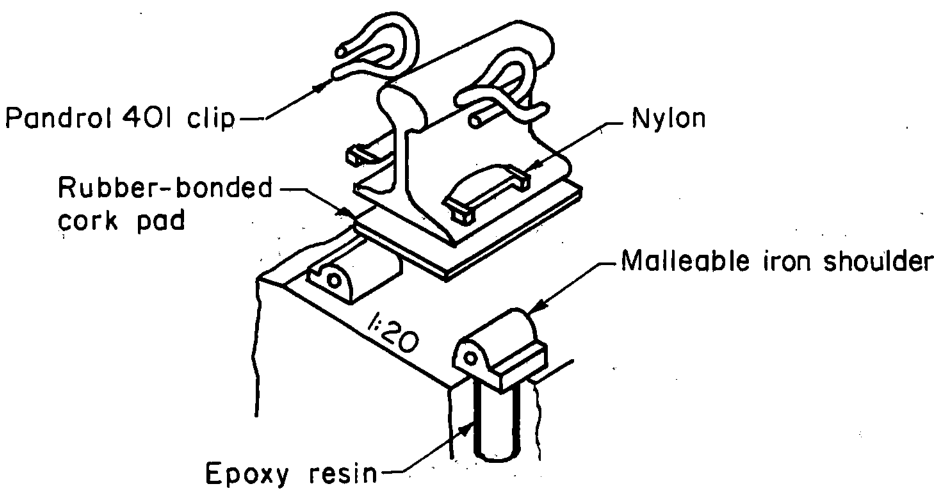


FIGURE 2. FASTENING USED IN THE BR DIRECT LAYING SYSTEM, EMPLOYING PANDROL 401 CLIPS (From Reference 1, page 548)

The proposed Channel Tunnel system at Radcliffe is shown in Figure 3. Large precast base units were grouted onto the cast-in-place slab. Prestressed longitudinal track beams were placed into the channels in the base units and were supported on long microcellular rubber pads. Polysulphide material was poured in two strips between the sides of the longitudinal track beams and base units to give lateral support. The rails were continuously supported on pads of conventional rubber bonded cork 4.5 mm (0.18 in.) thick.

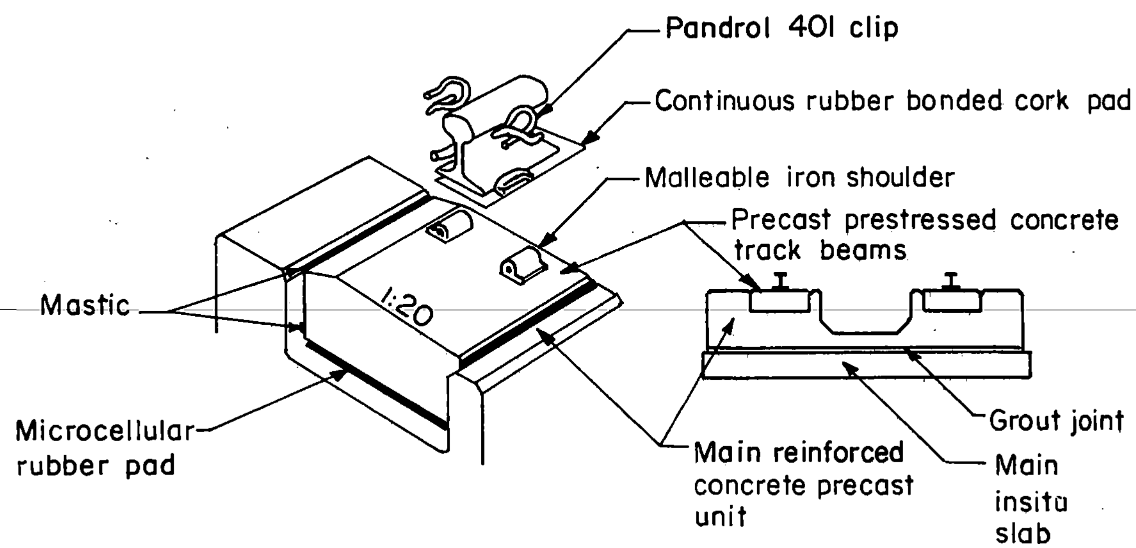


FIGURE 3. PROPOSED FORM OF TRACK FOR USE IN THE CHANNEL TUNNEL (From Reference 1, page 549)

3.2 Germany

The German Federal Railway built a section of slab track in 1967, incorporating three different designs developed as a joint effort by the DB, the concrete industry, and the Munich Technical University. The first German design used slabs prestressed longitudinally and cross-wise (Figure 4). The prestressing bars fit into dowels in the next adjacent slab. The slabs rest on a layer of Styropor lightweight concrete (sp. gr. 0.48; compressive strength 20 kg/cm^2 = 286 psi) for thermal insulation purposes; this is adequate for temperatures down to -23°C (-9°F) according to the designers.

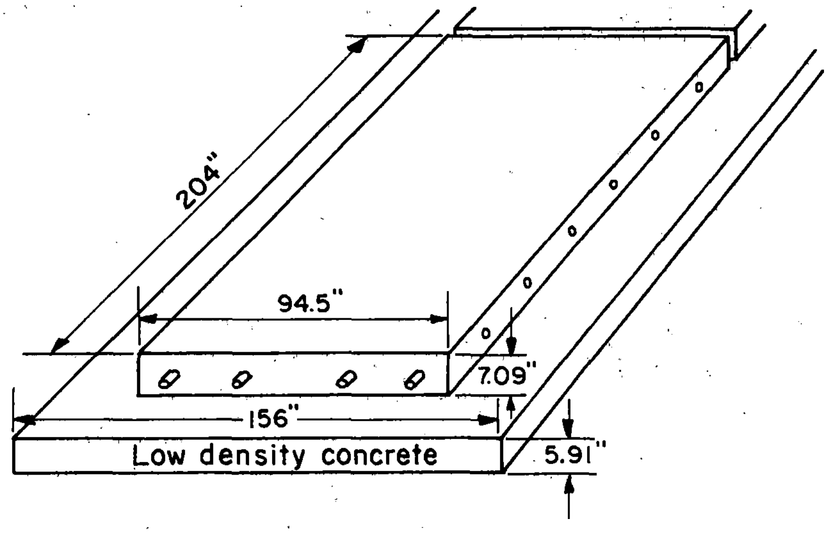


FIGURE 4. GERMAN SLAB-TYPE TRACK STRUCTURE, USING PRESTRESSED CONCRETE SLAB CONSTRUCTION IN WHICH THE LONGITUDINAL PRESTRESSING BARS FIT INTO DOWELS IN THE ADJACENT SLAB. (From Reference 2, page 309)

The second design also consists of bearing slabs prestressed transversely and longitudinally, which are connected by full-load-carrying Thermit steel pipe muff joints (Figure 5). The 30 cm (11.8 in.) thick layer of sandy gravel is made to act as low strength concrete in its upper part by the addition of a small amount of concrete.

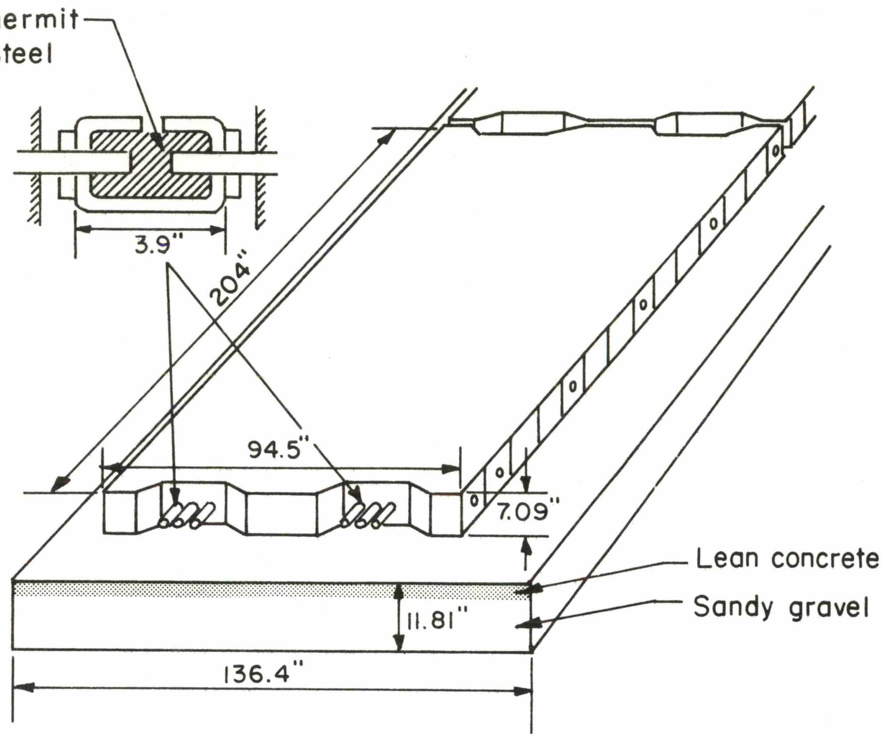


FIGURE 5. GERMAN SLAB-TYPE TRACK STRUCTURE, USING PRESTRESSED CONCRETE SLABS LINKED BY THERMIT MUFF JOINTS ROUND THE PRESTRESSING BARS. (From Reference 2, page 809)

The third design is composed of 6.48 m (255 in.) long prestressed longitudinal beams restrained intermittently by transverse beams. These beams are laid on a sandy gravel, the upper part of which is a weak concrete (Figure 6). The individual grid sections are joined by prestressed steel fishplates, and the space between the beams is filled with ballast, the top layer being bitumenised gravel.

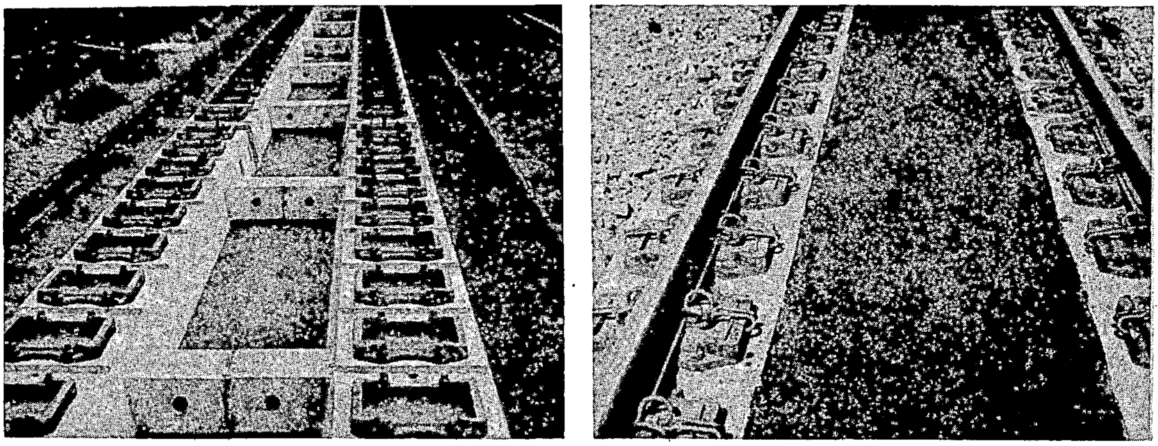


FIGURE 6. GERMAN LONGITUDINAL BEAM-TYPE TRACK STRUCTURE, USING PRESTRESSED CONCRETE BEAMS JOINED AT INTERVALS BY TRANSVERSE BEAMS (From Reference 2, page 309)

Rail fastenings are of three varieties, with narrow base plates, K screws, spring washers and clamps, and also with narrow and wide ribbed base plates, each with two kinds of spring clip. The laminated rubber pad with a vertical stiffness of about 5 tonne/mm (280,000 lb/in.) is situated in the outer steel holder. A certain degree of height correction is possible by using wood shims under the foot of the rail and by using plastic foil shims under the rubber pad.

The slabs experienced a uniformly moderate settlement of 5 mm (.2 in.) after 12 million tons. Dynamic settlement of the slabs or beams amounts to some 0.6 mm (0.024 in.)

3.3 Japan

There are two kinds of standard concrete track being used by the Japanese National Railways, both of which were designed primarily for tunnels [3]. Both types used concrete beds, one with concrete tie blocks embedded, the other with wood ties embedded in the slab. The concrete ties are used for straight track, and the wood ties for curves. The reason for the use of wood ties on the curved track is that this construction is considered more adaptable to increasing train speeds, it is easier to adjust cant and gauge when wood blocks are used. In both cases a rubber pad is inserted under the rail and another under the baseplate so as to provide a spring constant of about 35 ton/cm (178,000 lb/in.) in the vertical direction, which the Japanese consider to be about the same elasticity as the ballast provides.

The Japanese National Railways have constructed some experimental concrete track sections on the Tokaido trunk line extension (Tokaido Shin Kansen)--one is referred to as the JRM track, the other as the JRA track. The JRM track is composed of precast concrete supports fixed in the roadbed and precast longitudinal concrete track beams complete with rails (Figure 7a). Adjusting rubber mats are inserted between the support and the beam. This track is very expensive, but may be applicable to tunnels and bridges. The JRA track has a 5 cm (2 in.) thick asphalt layer under a concrete slab. Figure 7b shows an example of this design. The asphalt concrete, having a strength of 6 to 10 Kg/cm^2 (86-140 psi), supports the vertical load, and lateral loads are restrained by shear. The slab is precast reinforced concrete 16 cm (6.3 in.) thick under the rail. Adjustment of the track is achieved by raising the slabs and injecting quick hardening filler.

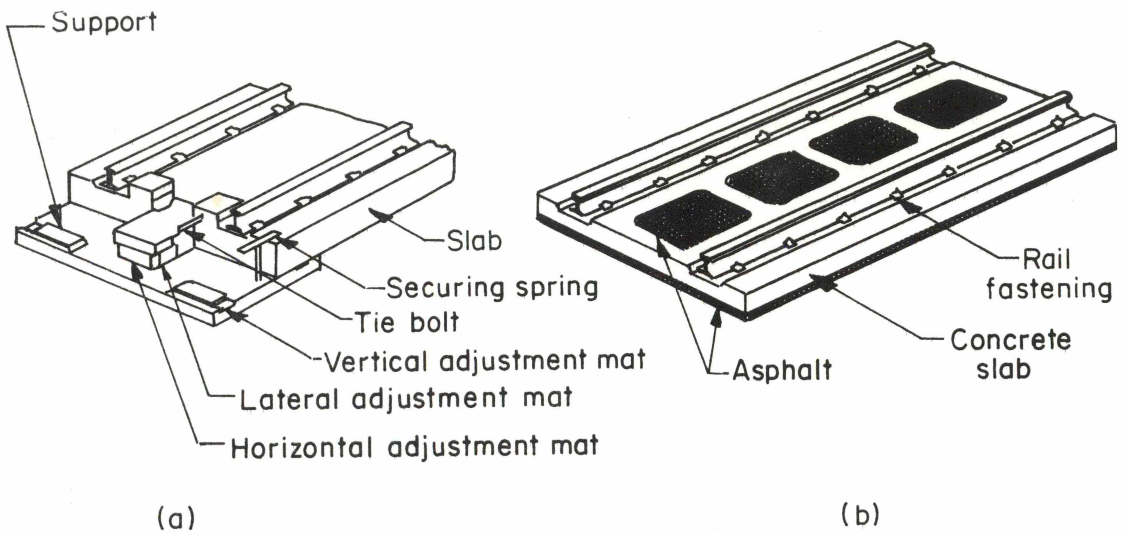
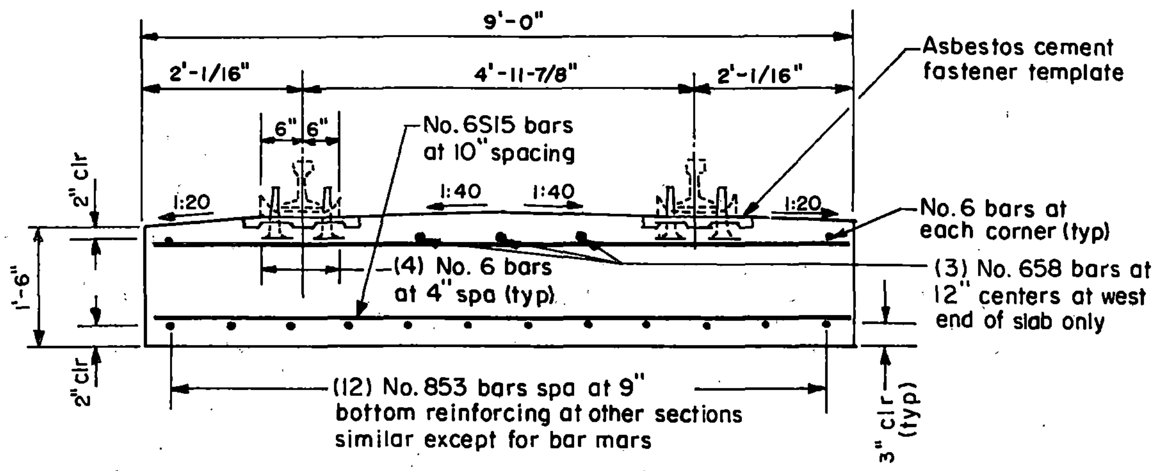


FIGURE 7. EXPERIMENTAL CONCRETE SLAB TRACK BEDS ON THE TOKAIDO SHIN KANSEN: (a) JRM TRACK WITH RUBBER ADJUSTMENT MATS, (b) JRA TRACK WITH A CONCRETE SLAB BEDDED IN ASPHALT (From Reference 3, page 261)

3.4 U. S. A.

The latest slab track installations in the United States are at the Kansas Test Track at Aikman and Chelsea, Kansas on the Santa Fe Railroad [4]. Three types of sections were installed on this track. Two of the sections were longitudinal beams, one precast and the other cast-in-place, both connected by lateral pipe gauge members. The third section is a cast-in-place slab (Figure 8). All sections contain steel reinforcing and rest on ballast and subgrade. Fastex track fasteners (supplied by Illinois Tool in conjunction with Johns-Manville) are used throughout (Figure 9).



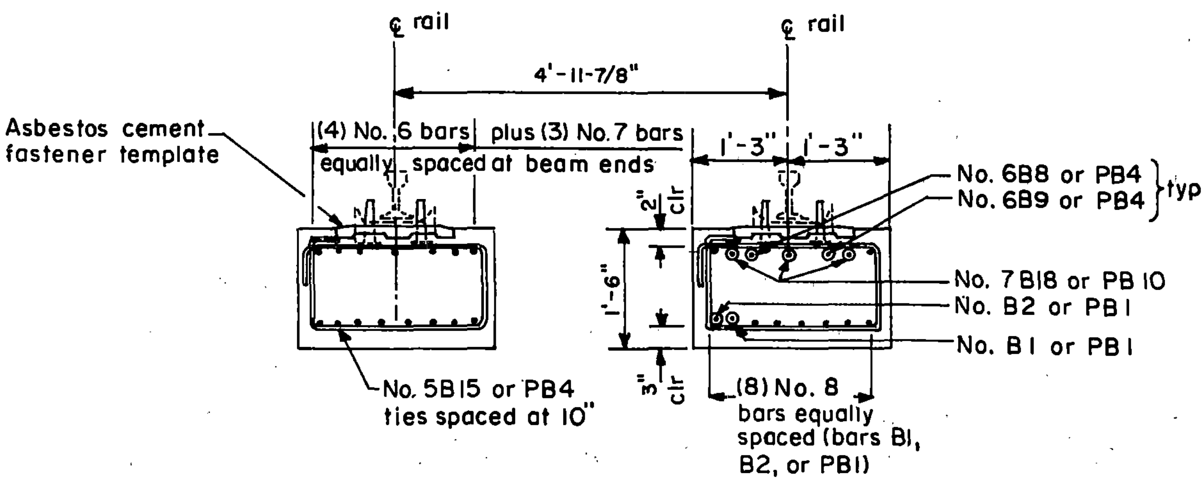


FIGURE 8. REINFORCED CONCRETE SLAB AND BEAMS FOR KANSAS TEST TRACK (From Reference 4, page 71)

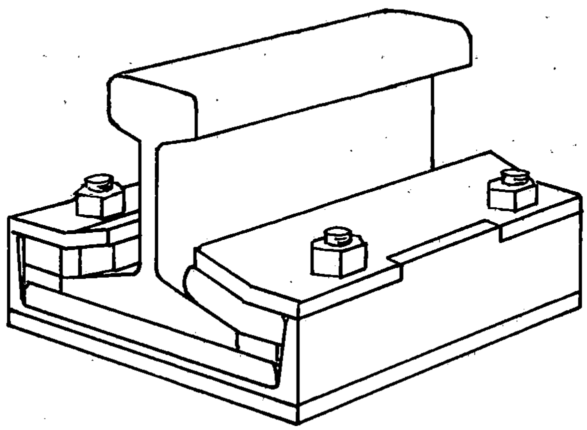


FIGURE 9. FASTEX FASTALL TRACK FASTENER USED IN KANSAS TEST TRACK (From Reference 4, page 20)

4.0 DESIGN OF SLAB-TYPE TRACK STRUCTURES

The actual design techniques needed for a slab-type track vary considerably from those required for conventional track using individual cross-ties. Even the terminology is different--with the classical ballast/sub-ballast terminology common to track designers replaced by the base/sub-base terminology common to highway designers. In both cases, however, the design procedure is similar, in that a logical procedure involves the determination of the pressures and strains imposed on the track support structure, and a determination of the allowable stresses and strains that the materials used in the support structure can withstand.

4.1 Basic Requirements

Before going into these two aspects of the design, some of the basic requirements of the support structure should be mentioned. Whereas the ballast in tie-type track provides an adjustment capability--both vertically and laterally, in the slab-type track this function can most practically be incorporated into the fasteners. The slab also eliminates direct entrance of water, from rain or snow, over the width of the slab. Therefore, the requirements of the material beneath the slab are not identical to those of the ballast beneath the ties. The material directly under a highway slab is usually referred to as base course, and may be crushed stone, slag, soil aggregates, or stabilized soil. These materials are not entirely suitable for ballast in a tie track, and in effect ballast can be considered as a high-class base material beneath a slab.

There are many ways in which the function of the base course is the same as conventional ballast. For example, to prevent pumping and provide drainage of the subgrade, the base course should be well graded and free of excessive fines. Trench type construction should be avoided, especially in clays where it would tend to cause water pockets and resulting weak spots in the structure. Moreover, it is essential that the base be free to drain for the life of the track. One method of accomplishing this is to use a thin layer of sand to prevent intrusion of fines into the base course.

The base course should be designed for stability. Therefore, it should have a high internal friction and shearing resistance. The base course should have the ability to withstand abrasion and crushing because of the possible generation of unwanted fines under load and during initial compaction. Any aggregate which breaks down under freezing conditions should not be used.

The thickness of base used depends primarily on strength and construction requirements. The base should be thick enough to allow proper compaction (a nominal 6-inch layer should suffice on stiff subgrade, according to Yoder) [5].

In slab-type track, the requirement of the base material to provide protection from frost heave is of utmost importance. Freezing of the subgrade is undesirable, because upon freezing the subgrade will expand, causing high stresses and possible rupture or yielding of the slab. Moreover, upon thawing, a saturated region with low strength and subgrade modulus tends to develop between the slab and the still frozen subgrade. With the embankment in this state, large deformations in the subgrade can occur in a relatively short time.

Frost heave requires three factors to be present simultaneously. The soil must be frost-susceptible, there must be slowly depressed air temperatures, and there must be a supply of water. It is up to the ingenuity of the designer to eliminate at least one of the factors in the particular application. The frost-susceptible soil may be either replaced or treated to change its characteristics. An air-entrained concrete base can be used to provide insulation. (In calculations of frost penetration, the concrete slab itself is considered roughly twice as effective as ballast or typical base courses as an insulating medium.) The use of drains, bitumen, and/or PVC films can be used to keep water out. Other methods are conceived and used in certain situations to provide protection of the embankment from freezing.

A section of railroad is usually not limited to a single type of soil; and as the right-of-way proceeds, large changes in the nature of the parent material can occur abruptly. Common practice is to use a uniform thickness of slab, rather than to "tailor" the thickness of each section to the subgrade properties. Therefore, one goal of the embankment design for a slab track is to provide uniform support to the slab with only gradual variations, because it is not the absolute settlement, but the differential settlement, which leads to failure.

The soil encountered in preparing the right of way is often deemed unsuitable for meeting all these requirements. In these cases, the designer can decide either to remove and replace this soil, or to alter it using some type of stabilization. A summary table of various admixtures which are used for stabilization is presented by Yoder [5].

Before a design strength for the embankment material(s) is selected, a determination of both the interaction of the slab with the embankment and of the proper method to evaluate the soil properties must be made. Unfortunately, these are areas where there is much disagreement and little definitive data. Also, much of the data which have been published are often not applicable to the type of loading imposed upon a railroad track slab.

A basic problem is that the soil characteristics needed for a mathematical model of the slab and its support are completely different than the soil mechanics data historically used to define soil properties. The design of the slab and its (embankment) support can perhaps best be based on mathematical models of a beam of infinite length or a slab forming an infinite horizontal plane, resting on an elastic foundation. The accepted representation of the foundation supporting the plane or beam is that of a Winkler foundation, [6] in which the pressure is assumed to be proportional to the vertical deflection of the slab. The subgrade is, therefore, characterized by a subgrade modulus k with units of lbs/in.^{2}/in. of deflection. These units can be thought of as a way of expressing the results of a field test in which a force is applied to the soil with a (circular) plate. The modulus is obtained by dividing the plate load by the elastic deflection and area.

However, the subgrade modulus parameter k is not normally obtained from tests on a laboratory soil sample. Furthermore, the correct value to use for k is open for much debate. Regardless of the test which is used to determine the value for k, a linear function is being fitted to nonlinear data in order to obtain a number for the range of interest. The value of k also changes with soil moisture content, with rate of loading, and with the number of loads applied. Based on research in this area, it is also proposed that the value of k may be a function of the thickness of pavement upon the subgrade, and that different values should be used depending on whether deflections, pressures, or moments are being predicted. Because the effective foundation modulus is affected by the properties of the slab, and the slab stresses and deflections are affected by the foundation modulus, the design of a slab-type track structure is an iterative procedure in which all components must be included.

With an elastic foundation model such as the beam-on-elastic foundation, variations in the subgrade modulus k representing the foundation can be made. However, the only output regarding stresses (pressure distribution) in the foundation that can be obtained is the contact bearing pressure between the slab and the foundation, or the overall compression of the foundation, both assumed to be proportional to the deflection of the spring(s) representing the foundation. Therefore, in a comprehensive study of slab-embankment interaction, this model must be complemented by models of the embankment which give as outputs the pressure distribution versus depth, and enable the effects of base and subbase layers of different thicknesses and strength to be analyzed.

Assuming deflections and pressures can be calculated accurately, the other gap in data needed for the embankment design involves data on stress-strain characteristics of materials commonly used in embankments. What few data are available are discussed in following portions of the text.

With this background, the following sections of the report discuss briefly the determination of stresses in the embankment, allowable stresses, and value of the subgrade modulus to be used for modeling slab-type track structures.

4.2 Stress Distribution in the Embankment

Much work relating to stress distribution in ballast beneath cross-ties has been done, and provides the basis for stresses in the base course beneath slab-type track. The basis for calculations of stress or pressure distributions in ballast beneath ties is the Boussinesq elastic theory. The assumption is made that the ballast and subgrade form a semi-infinite, elastic, homogeneous isotropic (uniform properties in all directions) solid, and the applied load is considered to be uniformly distributed over a small contact area. The equations for larger distributed loads are obtained by integrating Boussinesq's equations for point loads over the contact area [7].

The equations for the vertical and horizontal stresses at any point on the vertical or z axis under a uniformly loaded circular area are:

$$\sigma_z = p\left(1 - \frac{z^3}{(a^2 + z^2)^{3/2}}\right) \qquad (1)$$

$$\sigma_y = \sigma_x = \frac{p}{2}\left((1 + 2\mu) - \frac{2(1 + \mu)z}{(a^2 + z^2)^{1/2}} + \frac{z^3}{(a^2 + z^2)^{3/2}}\right) \qquad (2)$$

σ_z = vertical stress on the axis

$\sigma_y = \sigma_x$ = horizontal stress on the axis

p = applied pressure or intensity of loading at the surface

z = distance of the point from the surface

μ = Poisson's ratio

a = radius of circular area of loading

Note that both expressions are independent of the modulus of elasticity, and that the vertical stress is independent of all elastic constants. This would not be so if the material had varying elastic properties.

The vertical and horizontal stresses on the axis are major and minor principal stresses, respectively. The maximum shear stress at any point is half the difference between the principal stresses. Therefore:

$$\tau_{max} = \frac{\sigma_z - \sigma_x}{2}$$

$$= p\left(\frac{(1 - 2\mu)}{4} + \frac{(1 + \mu)z}{2(a^2 + z^2)^{1/2}} - \frac{3z^3}{4(a^2 + z^2)^{3/2}}\right) \qquad (3)$$

where τ_{max} = maximum shear stress at a point on the axis.

The vertical elastic displacement at the surface under the center of the applied loading is given by:

$$\Delta = \frac{2pa}{E}(1 - \mu^2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

where E = modulus of elasticity of the solid

and Δ = displacement.

When $\mu = 1/2$, equation (4) reduces to:

$$\Delta = \frac{1.5\ pa}{E} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

The maximum vertical stress on any horizontal plane occurs on the vertical axis through the center of the load. The maximum shear stress at any point on a horizontal plane also occurs on the axis, provided the plane is at least 0.71 x radius of the loading area below the surface for circular loading.

If the load is applied by means of a rigid circular plate, the loading is not uniformly distributed and the theoretical displacement of the plate, assuming $\mu = 1/2$, is given by:

$$\Delta = \frac{1.18\ pa}{E} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

The Boussinesq equations have been often used for analysis of the soil pressure under ties, and reasonable correlation with field data has been obtained. For example, in 1958 an investigation was made by British Railways at Kegworth in the London Midland Region [8]. Dynamic stresses in the subsoil under approximately 230 mm (9.1 in.) of clean ballast were correlated with dynamic tie loading. It was found that:

(1) There was a linear relationship between tie loading and subsoil stresses which was independent of speed and wheel arrangement

(2) The measured vertical stress distribution was close to that predicted by simple elastic theory and was not markedly different for timber or concrete ties

(3) Scatter in stress level between nominally identical positions in the track substructure was highly dependent on ballast condition.

Committee D71 of the ORE incorporated a program of stress measurement into the work program [8]. The committee used a computer program to calculate stress in the substructure using elastic theory in order to provide a theoretical basis on which to compare results. Although the results deal with tie on ballast track, they are worthy of mention because they provide a basis for a design method which could be applied to slab track.

It was concluded that single layer elastic theory used with a simplified tie soffit contact pressure distribution provided an adequate means of calculating subgrade vertical stresses for practical engineering purposes, and that in view of the degree of the scatter involved in practice a more rigorous approach did not seem justified (see Figure 10).

A considerable amount of research on ballast pressure distribution has been done at the University of Illinois for AREA. As a result of this work, it was concluded that superposition was valid for small stresses in ballast, and that the stress distribution was independent of the type of ballast used.

One step in the refinement of the method for calculating stresses in an embankment is to consider a two-layer system rather than a single-layer system. In tie-type track, the two layers can represent the ballast and subballast or subgrade, or in slab track one layer can represent the slab. Burmister analyzed the stresses and strains in a layered system, consisting of an elastic slab infinite in the horizontal plane, placed on a semi-infinite

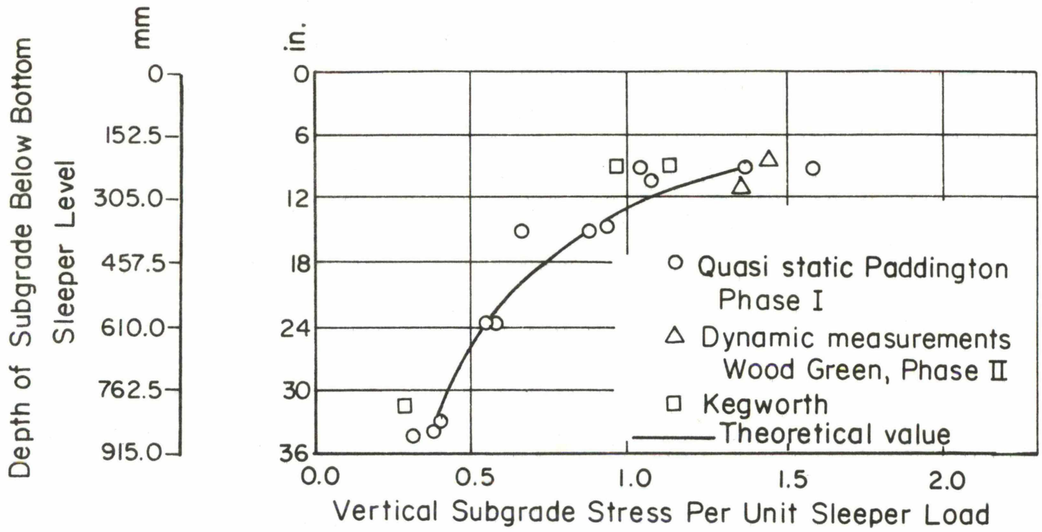


FIGURE 10. VERTICAL STRESSES: SUMMARY OF QUASI-STATIC AND DYNAMIC MEASUREMENTS (From Reference 8, page 255)

solid of lower modulus of elasticity. The system was subjected to a uniformly distributed load of circular area. The results of his computations for a perfectly rough interface (assumes no relative shear displacement between the layers) are shown in Figure 11. [9]

The Mathematics Division of Great Britain's National Physical Laboratory extended Burmister's analysis of the stresses and strains in a two-layer system and computed the stresses for various cases. Some of the results are shown in Figures 12 and 13. Poisson's ratio was assumed to be 0.5 in both layers for all computations. Note that the addition of the lower layer (with much lower E) tends to greatly reduce the depth to which the high stress levels penetrate.

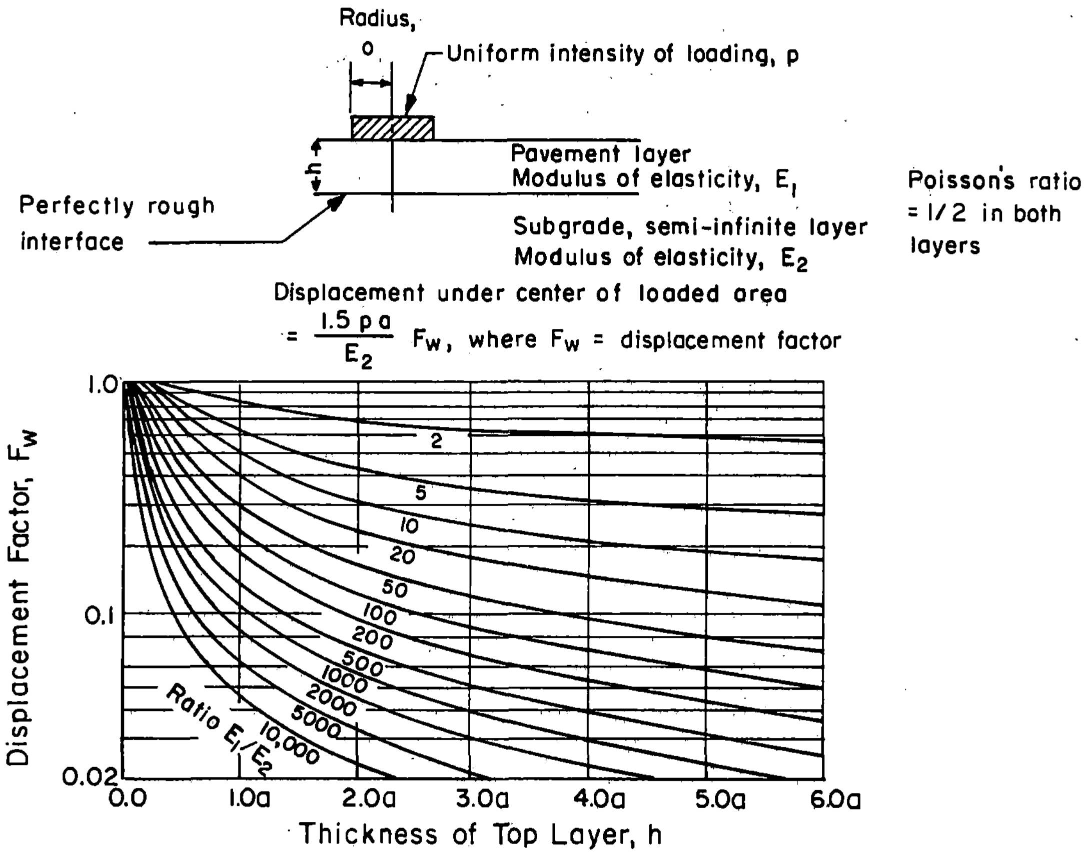


FIGURE 11. THEORETICAL DISPLACEMENTS IN A TWO-LAYER ELASTIC SYSTEM (From Reference 9, page 428)

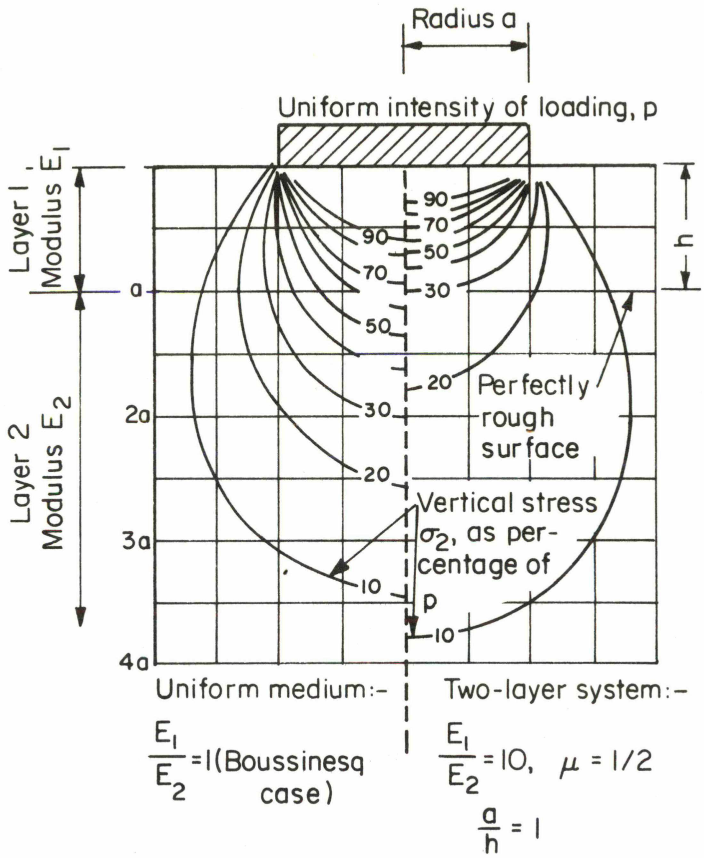


FIGURE 12. COMPARISON BETWEEN THE VERTICAL STRESS DISTRIBUTION IN A UNIFORM MEDIUM AND IN A TWO-LAYER SYSTEM (From Reference 9, page 435)

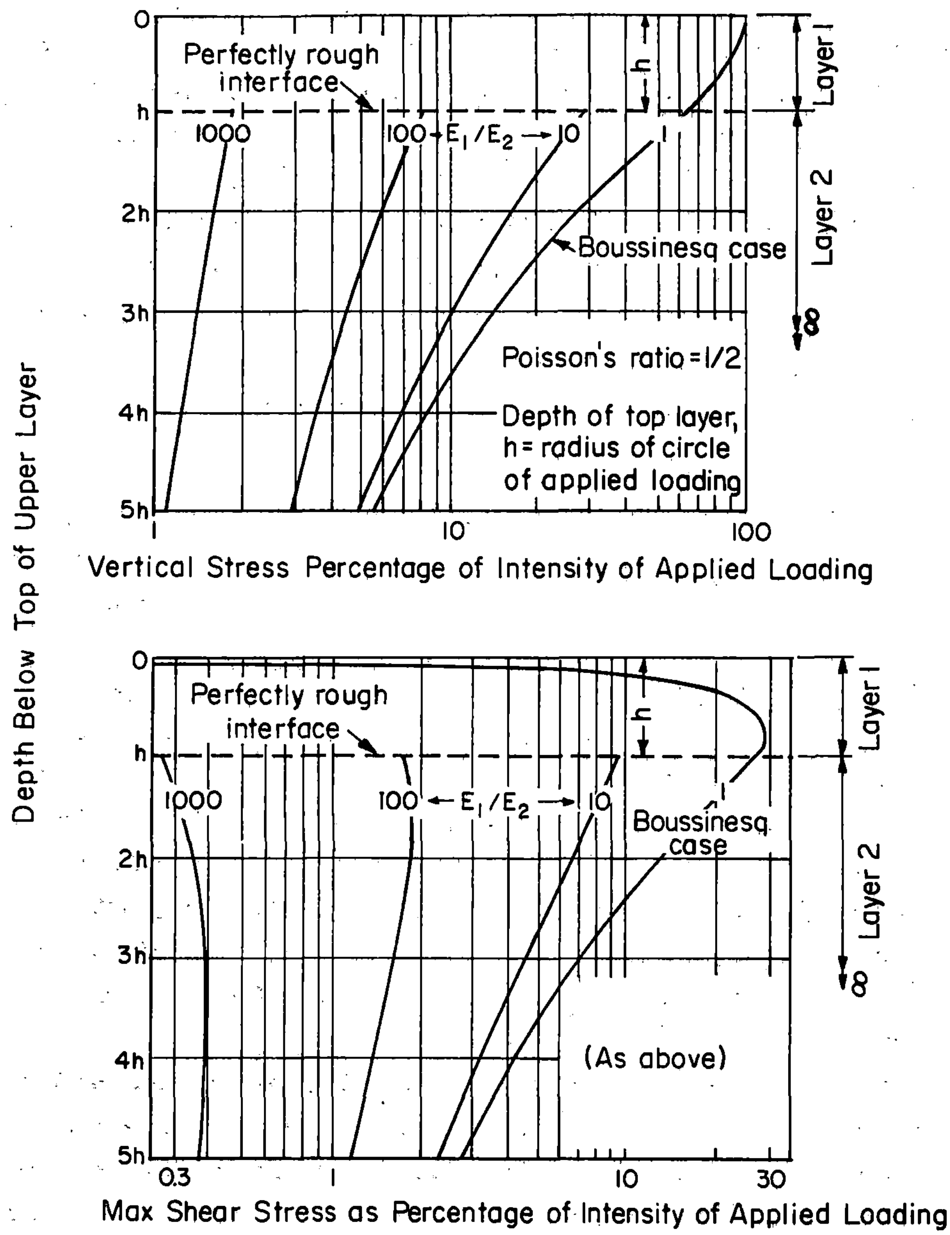


FIGURE 13. STRESS ON THE AXIS IN THE LOWER LAYER OF A TWO-LAYER ELASTIC SYSTEM DUE TO A CIRCULAR UNIFORM LOADING (From Reference 9, page 434)

There are a number of limiting approximations associated with the two layered system model for the slab on its foundation. The representation of the slab as an infinite horizontal plane, loaded over a circular area, is quite different from the actual case of a slab of finite width with the load applied through fail fasteners. Burmister's analysis also assumes a slab without reinforcing. In actuality, the reinforcing in the slab will change the effective modulus of the slab. Moreover, the bending stiffness of the slab may be considerably different in the transverse and longitudinal directions. Research is needed to develop accurate procedures for calculating the pressures beneath track structures and/or to evaluate the error in current methods.

4.3 Allowable Pressures in Embankment Materials Under Cyclic Loading

Even if embankment stresses could be calculated accurately, there would be meager information to use regarding the design stresses for embankment materials. At least there is agreement on the criterion for failure; this is usually agreed to be settlement (or plastic deflection), although the time factor and degree to which the settlement is differential remain debatable.

The soil classifications often used (see Volume 1) are all essentially based on static load properties. However, the soil beneath a railroad track is subjected to repeated cyclic or dynamic loads, and its performance under dynamic rather than static loading is the key to track stability. A track structure subjected to repeated loads will exhibit both plastic and elastic deformations, and even though the elastic deformation of the subgrade is small, the accumulated plastic deformations may be of such a magnitude as to cause failure.

Repeated loading results in three principal effects on the ballast or base-course. First, some of the material will break down when exposed to the repeated stress conditions. Second, there is mutual intrusion of the subgrade into the ballast and of the ballast into the subgrade. Third, there is pumping or blowing of the base course or ballast.

The soil is affected in many ways by the repeated loading. The subgrade modulus tends to increase with repeated loading. However, some soils tend to break down completely after repeated loading. Seasonal variations must not be neglected, for an embankment can be quickly destroyed during times of thawing and/or heavy rains when pumping is most likely to result from repeated loading.

It is also important to note that the stress-strain characteristics will vary with the rate of load application. The rate of increase of modulus of deformation* for soils under repeated loading is a function of moisture content and density. Soils compacted at a lower than optimum moisture content or at relatively high densities show high increases, whereas those compacted at a moisture content higher than optimum or at low densities show lesser increases in modulus of deformation, as shown in Figure 14.

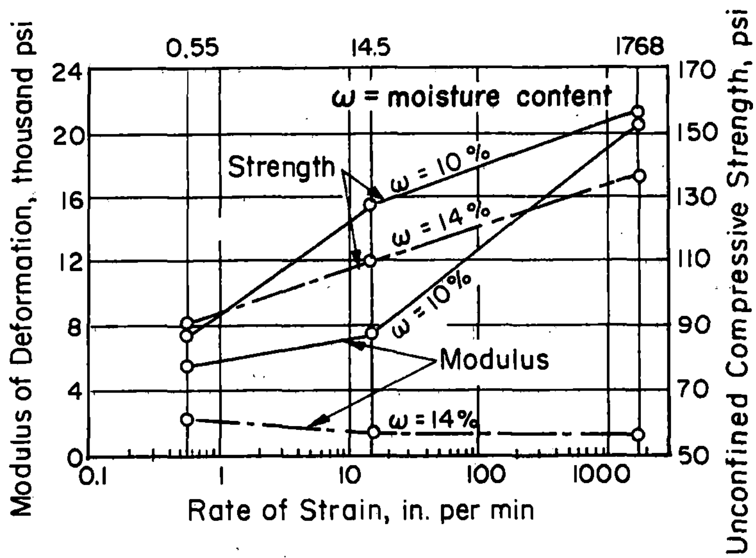


FIGURE 14. EFFECT OF RATE OF LOADING ON UNCONFINED COMPRESSIVE STRENGTH AND MODULUS OF DEFORMATION OF A SILTY CLAY. SOILS WERE COMPACTED USING MODIFIED AASHO COMPACTIVE EFFORT. (From Reference 5, page 106)

* The modulus of deformation is obtained in a lab test, and is similar to E, Young's modulus. It also is indicative of subgrade modulus.

These results merely indicate that the soil exhibits damping as well as spring characteristics under dynamic loading. It is therefore apparent that fatigue limits and material parameters from dynamic tests should be used wherever possible.

In recent years, researchers attempting to develop improved or advanced track structures have used various approaches for quantitatively relating the embankment settlement to track stability. For example, the TRW-Systems Group [10] attempted to describe the relationship between differential and absolute settlements. They proposed that the extent of differential settlement in a given distance depends on the type of soil, magnitude of stress, log of number of stress applications, and a hyperbolic function of distance considered.

In Battelle-Columbus' "Studies for Rail Vehicle Track Structures" [11] a criterion for evaluating the deterioration of the subgrade was developed. The objective was to derive a relation between the imposed cyclic pressures and the settlement of the soil. The criterion proposed that information needed to predict the damage to the soil by the passage of a freight or passenger vehicle is the magnitude of the pressure pulses, the frequency content of the pulses, and the number of pulses. On this basis, a slab track was designed which gave a subgrade pressure-time "signature", for a train passing over it at 160 mph, equivalent to that of a conventional tie-type track with the same train traveling over it at 53 mph. The key to this is the ability of the slab track to eliminate the pressure pulse for each axle, giving instead one pulse per truck--or if extremely stiff, one pressure pulse per pair of trucks (rear of one car and front of following car). The concept here was that a more stable track structure could be designed for high-speed trains without quantifying ballast and subgrade settlement or deterioration with time, based on development of an equivalent pressure-time history for conventional track.

Some definitive quantitative research correlating settlement with measured stresses and soil properties has been conducted. The investigation by the British Railways at Kegworth mentioned previously included studies of soil properties and development of a related design procedure. Laboratory soil tests were conducted on clay soil samples; a triaxial compression test was chosen to obtain the soil parameters. Only the major axis was pulsed, while the radial principal stresses were held constant. A square wave of 30 cycle/min. and constant amplitude was used for loading. Drainage of the samples was provided. Examples of the results are shown in Figure 15. These results indicate an effective endurance limit for soil loading such that when the endurance limit is exceeded the settlement proceeds at a rapid rate. However, when the applied load is less than the endurance load, the settlement reaches equilibrium and is relatively stable. (This would, of course, be sensitive to changing soil conditions.) Of particular note is the gross change in soil performance obtained by reducing the pressure from 10.1 psi to 9.4 psi.

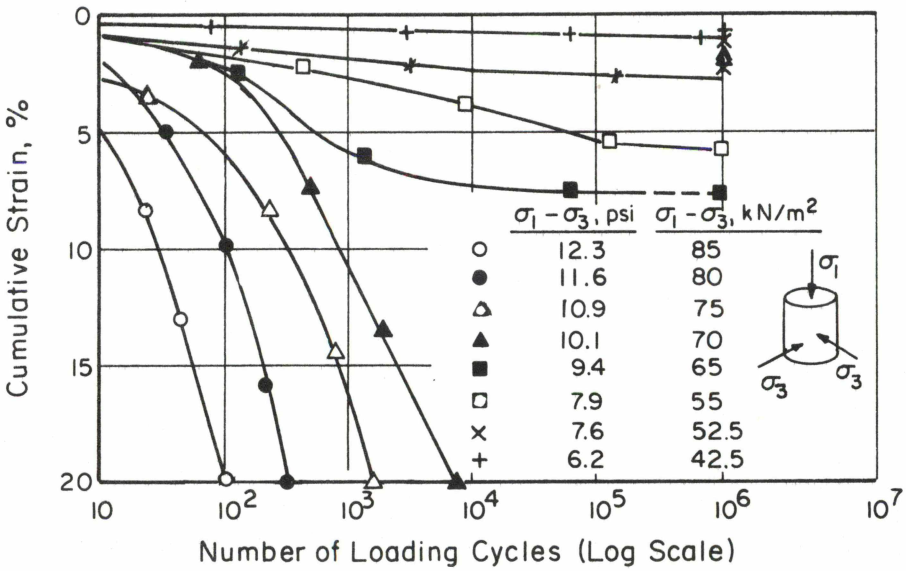


FIGURE 15. EXAMPLES OF CUMULATIVE STRAIN RESULTING FROM LOADING TESTS ON CLAY SOIL SAMPLES (From Reference 8, page 259)

Based on these results, the committee proposed a design method for track structures; this should be applicable to slab track. The inherent assumptions are:

(1) That the threshold stress parameters for the subgrade soil may be obtained using the standard repeated loading test

(2) That simple elastic theory can be used to predict the stresses in the subgrade from traffic loading

(3) That the significant stresses are those produced only by the static effect of the heaviest commonly occurring axle load (dynamic load can be included as an effective static load)

(4) That the water table is at the top of the subgrade.

The design method is based on the achievement of a "balanced" design which is obtained when the ballast is sufficiently deep so that the calculated maximum principal stress difference induced in the subgrade by the heaviest commonly occurring axle load is equal to the average "threshold" principal stress difference as predicted from laboratory tests. This threshold stress is the stress corresponding to the limiting strain determined in the soil loading tests. Above this level of applied strain, the soil deformation is continuous--below it, deformation reaches a stabilized value, as shown in Figure 15. The theoretical subgrade maximum principal stress differences for various axle loads can be calculated, and the threshold stress/depth relationship superimposed on these curves, making it possible to obtain the depth of ballast at which the threshold stress is equal to the stress induced by a given axle load. The results for this procedure for a particular case are shown in Figure 16, which illustrates the increase in required ballast depth with increasing load and decreasing threshold strength of the subgrade material.

An assessment of this design method as applied to tie-ballast track was made by performing a series of laboratory and field observations. Good correlation between low settlement rates and ballast depths equal to or greater than the design depth was obtained. While there are some limitations to this method, extension of this technique to other soil types appears very desirable. For designing slab track, the maximum principal stress differences caused by loading the slab would be used.

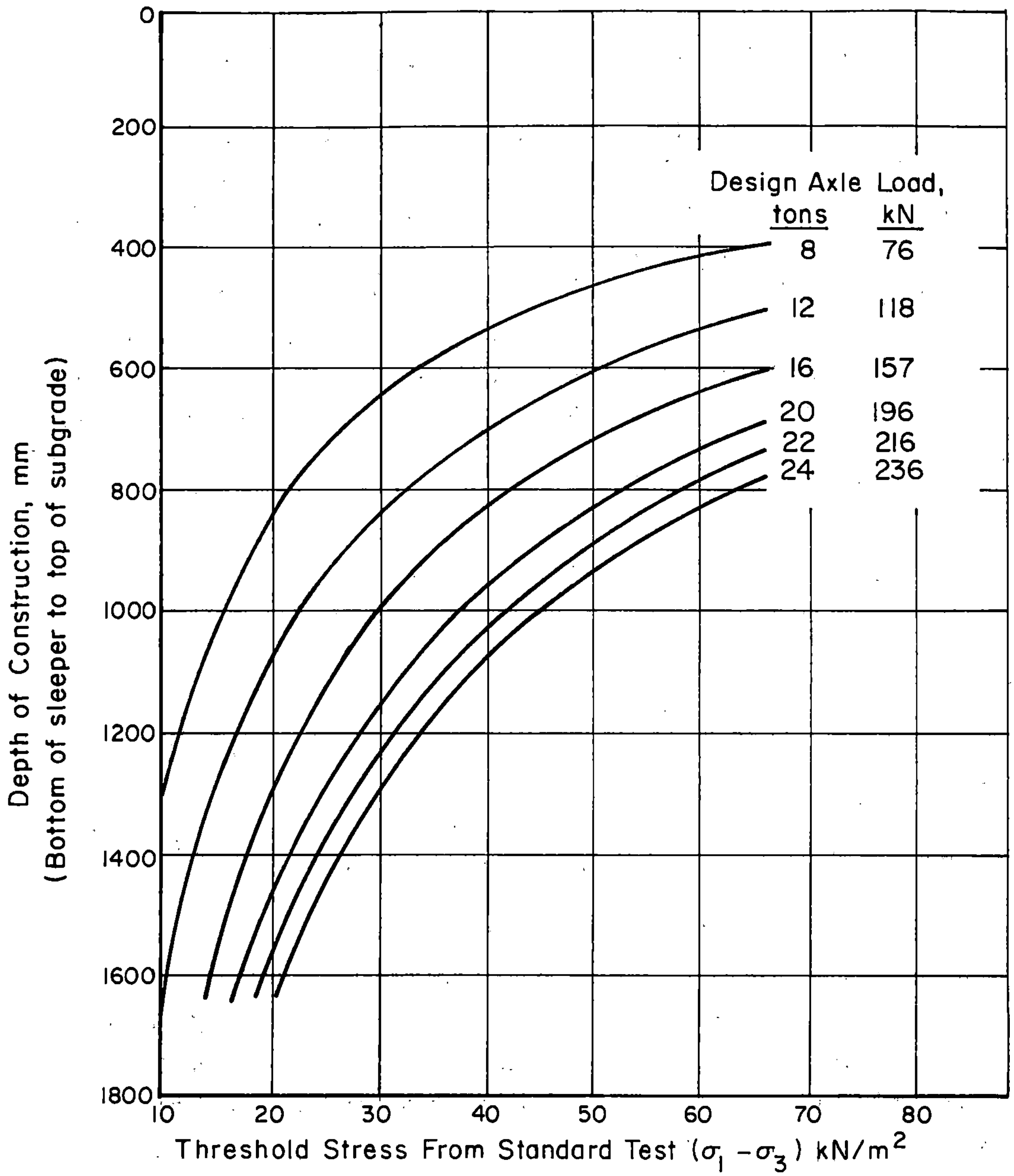


FIGURE 16. RELATIONSHIP OF AXLE LOAD AND SUBGRADE STRENGTH TO DEPTH OF BALLAST

4.4 Characterization of the Embankment, Subgrade Modulus

It is the purpose of this section to consolidate a number of the different methods used to predict the modulus of subgrade reaction and attempt to show the interrelation between them as specifically applied to slab track design. There are numerous methods used for the design of the actual slabs for highways and airport runways. Much of this information is directly applicable for designing a railroad slab track for rail vehicles, but the rail fastener introduces significant effects into the slab which must be accounted for in the track structure application. Some of the differences between highway (or runway) and rail applications are as follows:

(1) The load is completely channeled and no edge loads on the slab are applied in rail applications

(2) The rail distributes a wheel load to several fasteners

(3) The fastener inserts cause a discontinuity in the surface of the slab

(4) The slab surface is not subjected to abrasion.

The design methods for highways and airports, for the most part, incorporate the Boussinesq or Burmister soil stress distributions discussed earlier. The design methods have varying degrees of empirical and theoretical support. Until design techniques are specifically developed for railroad slab tracks, the use of the beam-on-elastic-foundation model appears to be a necessary step in the preliminary design stage of the track structure. From this model, the required bending stiffness and/or allowable bending moments can be calculated. Next, the general design of the slab in terms of width, depth, amount of reinforcing, etc., can be established. Precast versus cast-in-place designs can be evaluated, together with the relative merits of prestressing, poststressing, or not stressing at all.

The beam-on-elastic-foundation representation requires a subgrade modulus, k. These models assume the subgrade acts as a continuous bed of springs and that each "spring" or subgrade element has no effect on its neighboring elements (Winkler foundation). The value of this model is its relative ease of application and computation. The other applicable model is the elastic isotropic solid (as used in the Boussinesq and Burmister models). This is difficult to apply to a slab model under loading, and the Winkler foundation k is usually used. There is much evidence which shows agreement between the observed response of pavements and track using the Winkler model; however, there is also evidence that an elastic-isotopic solid model of the soil will closely predict the response of soils to load.

The correct formula to use for determining the subgrade modulus k is still under debate. The problem is complicated by the large variations in properties which soil undergoes during the course of the year. The designer can minimize this by providing good drainage and other features to maintain a stable subgrade. However, instead of trying to determine a single value for k, it is more feasible to determine the high and low values which bound the expected variations, and to work within this range of values.

In 1888, Zimmerman [12] defined "k" as a constant depending on the type of subgrade, in his analysis of railway ties and rails. During the subsequent development of theory of beams and slabs resting on soil, this concept prevailed although it was recognized that "k" depended upon the size and shape of the loaded area. One example of this dependence is the halving of the subgrade modulus under railroad ties to account for the subgrade coupling from one tie to another, as suggested in Hetenji [13]. It is, of course, possible to experimentally determine the value of k by using a plate bearing test, but even the applicability of the directly measured value is controversial. The subgrade modulus can also be calculated from other measured soil properties. Regardless, the values will at best only apply to a limited range of deflections under a given slab.

During the 50's, a controversy arose over the Winkler model and the theories based on it. A compromise seems to have been reached in which the Winkler model is retained because of the reasonable estimates which it provides, but values of k are based on results for an elastic-isotropic solid model.

In 1961 Vesic [14] proposed the following formula for k so as to obtain a good approximation of both bending moments and deflections for a flexible beam of infinite length and finite width. The pressure across the width of the beam at any given station along its length is assumed uniform.

The value of k is given by

$$k = \frac{0.65}{B} \sqrt[12]{\frac{E_s B^4}{E_b I}} \quad \frac{E_s}{1 - (\mu_s)^2} \tag{7}$$

where k = subgrade modulus, lb/in^3

B = width of beam, in.

E_s = Young's modulus of the soil, lb/in^2

μ_s = Poisson's ratio of the soil

I = Beam moment of inertia about bending axis, in^4

E_b = Young's modulus for beam, lb/in^2.

Vesic and Saxena [15] extended the beam analysis to a slab of infinite width and length resting on an elastic isotropic solid with no slippage at the interface, which they consider to be a better model of a highway pavement. They considered a slab of thickness, h, characterized by a modulus of deformation, E, and a Poisson's ratio μ under a vertical point load Q. The slab deflection w at a distance r from the load Q is

$$w = \frac{Q \ell_o^2}{D} \quad I_w \tag{8}$$

and the contact pressure, p, is

$$p = \frac{Q}{\ell_o^2} \quad I_p \tag{9}$$

where the characteristic length, ℓ_o, is

$$\ell_o = \sqrt[3]{\frac{2D\ (1 - \mu_s^2)}{E_s}} \tag{10}$$

and the flexural stiffness, D, is

$$D = \frac{Eh^3}{12\ (1 - \mu^2)} \tag{11}$$

and where μ_s and E_s refer to the soil as before. I_w and I_p are evaluated and shown in Figure 17 by the solid lines.

The pressure divided by the deflection at any point is defined to be k, and the value of this ratio directly under the load is defined as k_o, which is also equal to $D/\ell_o^{\,4}$. Note that the contact pressure decreases faster with distance than does the deflection, indicating a decrease in k with distance from the load. By using the Hertz-Westergaard theory for the same case, with a constant coefficient of subgrade reaction equal to k_o, one obtains relatively poor agreement of pressures and deflections, as shown by the dotted lines in Figure 17. To improve agreement of pressures according to the two theories, one can select some other value of k, and a comparison was made by Vesic and Saxena using a Winkler subgrade with constant coefficients of subgrade reaction. They found from theirs and other analyses (Timoshenko and Woinowsky-Krieger, Ref. 16) that if one uses a constant value of $k = 2.37\ k_o$, one obtains a good agreement for pressures; that the use of $k = 0.42\ k_o$ will give good agreement with deflections; and that $k = k_o$ will give good agreement of bending moments in the vicinity of the load. If the latter substitution is made,

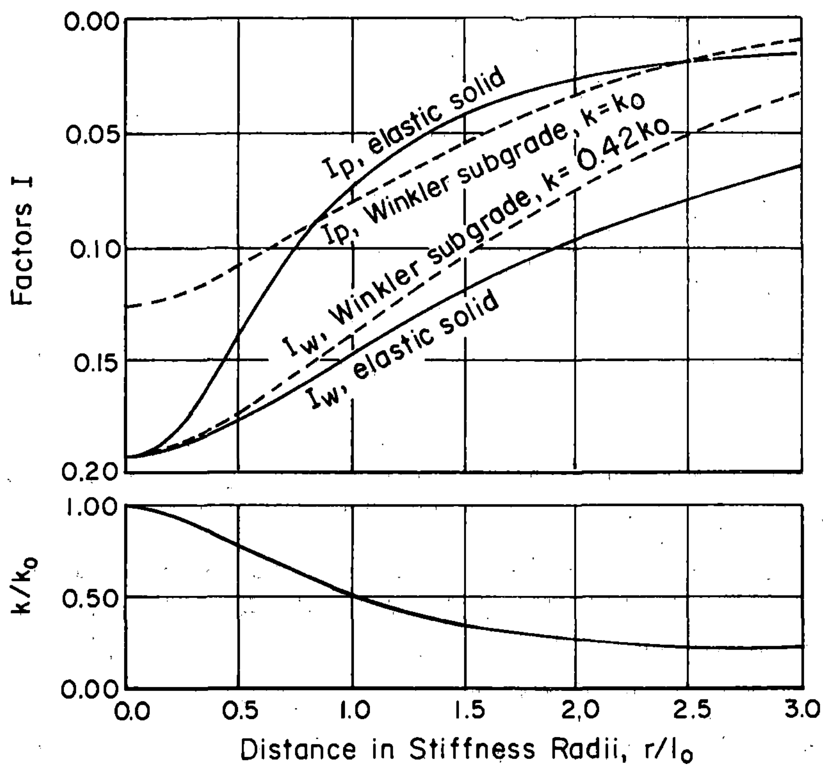


FIGURE 17. VARIATION OF FACTORS, I, AND COEFFICIENT, k, WITH RELATIVE DISTANCE, r/l_o. (From Reference 15, page 5)

and setting μ and μ_s, equal to 0.15 and 0.50, respectively, the following equation is obtained:

$$k = k_o = D/\ell_o^4 = \sqrt[3]{\frac{E_s}{E}} \quad \frac{E_s}{(1-\mu_s^2)h} \quad . \tag{12}$$

This equation for slabs resembles Equation 7 for beams, but is definitely not the same equation. It indicates that k depends on the relative flexibility of the slab with respect to the subgrade, and should be inversely proportional to the thickness h.

Although Equation 12 should give good agreements of bending moments in the slab, to obtain a good agreement of deflections, a value of k 2.4 times lower should be used; namely,

$$k = k_w = 0.42 \sqrt[3]{E_s/E} \quad \frac{E_s}{(1-\mu_s^2)\ h} \quad . \tag{13}$$

Vesic and Saxena extended this analysis to account for a finite depth, H, of compressible subgrade, placed on an incompressible medium. For this case, the deflection w at distance r from the load Q again is given by Equation 8, that is:

$$w = \frac{Q\ \ell_o^2}{D} \quad I_w \tag{8}$$

where ℓ_o and D are as before (given by Equations 10 and 11), but I_w is a different function, and is shown in Figure 18.

Similarly, the bending moment is shown in Figure 19. A study of this figure reveals that if the analysis of a slab resting on an elastic-isotropic solid of finite depth is made with k_o calculated from Equation 12, the agreement of deflections improves as the thickness of the subgrade layer decreases. When the subgrade becomes about 2-1/2 (stiffness) radii thick, close agreement of both deflection and bending moments is obtained. As the thickness continues to decrease, this agreement of bending moments and deflections continues to exist, with a unique value of coefficient of subgrade reaction, k, which is generally higher than k_o. According to Vesic and Saxena, this unique value is defined as follows:

$$k = 2.5\ (\ell_o/H)k_o = \frac{1.38\ E_s}{(1-\mu_s^2)H} \tag{14}$$

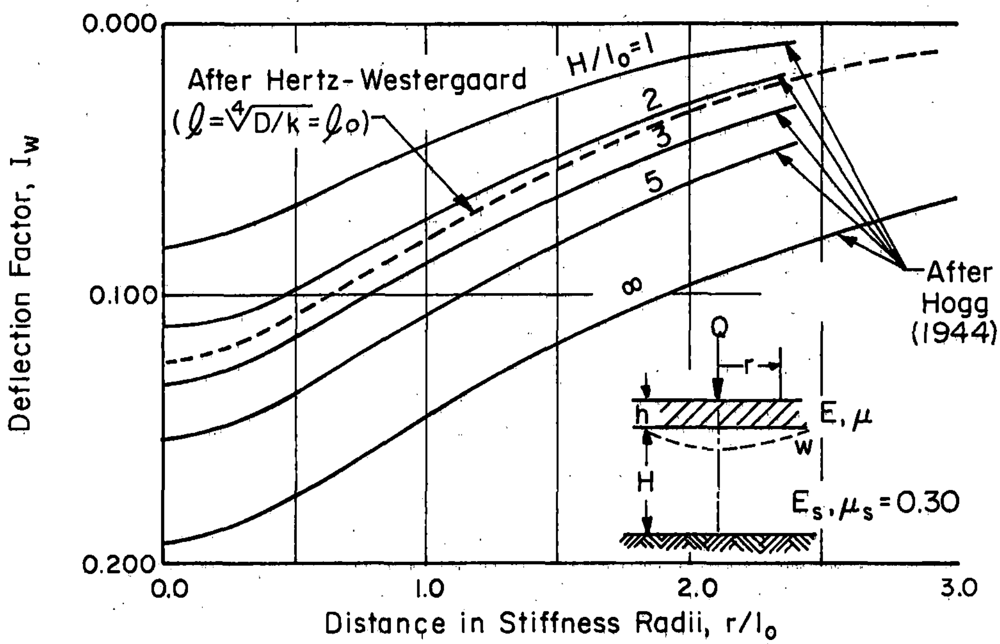


FIGURE 18. DEFLECTION FACTORS, I_w, FOR AN INFINITE SLAB RESTING ON AN ELASTIC SOLID OF FINITE DEPTH. (From Reference 15, page 7)

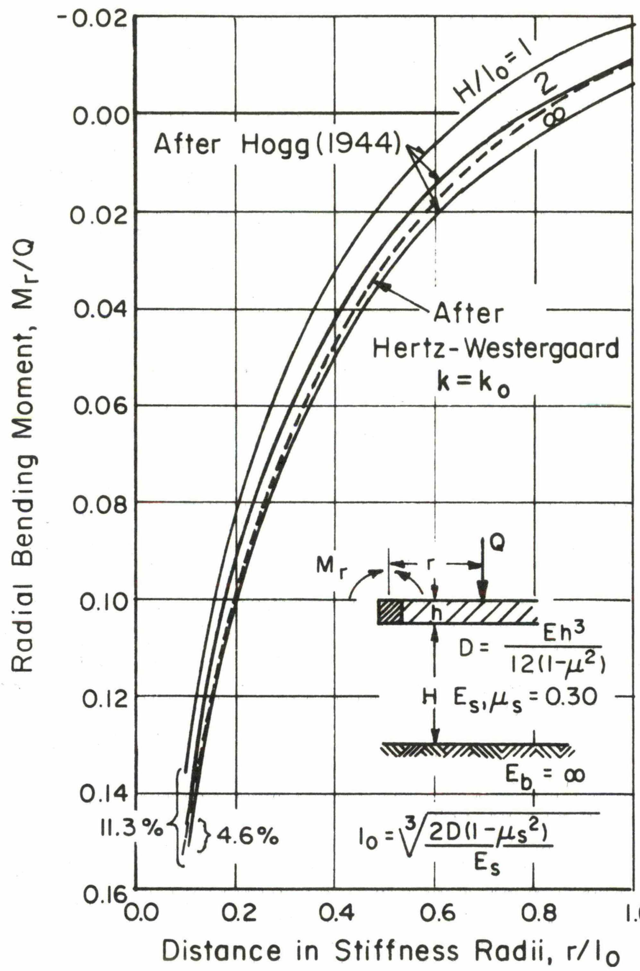


FIGURE 19. BENDING MOMENTS IN AN INFINITE SLAB RESTING ON AN ELASTIC SOLID OF FINITE DEPTH. (From Reference 15, page 7)

which is valid for $H/\ell_o < 2.5$ or $H/h < 1.38 \sqrt[3]{E/E_s}$.

The following conclusion is taken directly from Reference 15: "It may be concluded that Winkler's hypothesis is for all practical purposes satisfied also for slabs resting on an elastic-isotropic solid subgrade of finite thickness, H, as long as the thickness does not exceed about 2.5 stiffness radii, ℓ_o (defined by Equation 10). The coefficient of subgrade reaction, k, of such slabs can be estimated by using Equation 14.

When the thickness of compressible subgrade exceeds about 2.5 stiffness radii, Winkler's hypothesis can still be used for analysis. However, the value of coefficient of subgrade reaction, $k = k_o$, that yields good agreement of pavement stresses will not provide good agreement of deflections as well. The deflection patterns will agree only if a lower value $k = k_w$ is selected".

The formula used by Westenhoff and Novick [4] for the Kansas Test Track for subgrade modulus was the Barkan [17, 18] formula which assumes the slab acts as a rigid footing on an elastic isotopic half space. This formula is shown in Table 1. The formula for the horizontal spring constant was obtained by **assuming** a uniform distribution of shearing stress on the contact area and computing the horizontal displacement.

TABLE 1. SPRING CONSTANTS FOR RIGID RECTANGULAR FOOTING RESTING ON ELASTIC HALF-SPACE (From Reference 17, page 350)

Motion	Spring Constant	Reference
Vertical	$k_z = \frac{G}{1 - \mu} \beta_z \sqrt{4cd}$	Barkan [18]
Horizontal	$k_x = 4 (1 + \mu) G\beta_x \sqrt{cd}$	Barkan [18]
Rocking	$k_\psi = \frac{G}{1 - \mu} \beta_\psi 8cd^2$	Gorbunov-Possadox [19]

(Note: Values for β_z, β_x, and β_ψ are given in Figure 20 for various values of d/c).

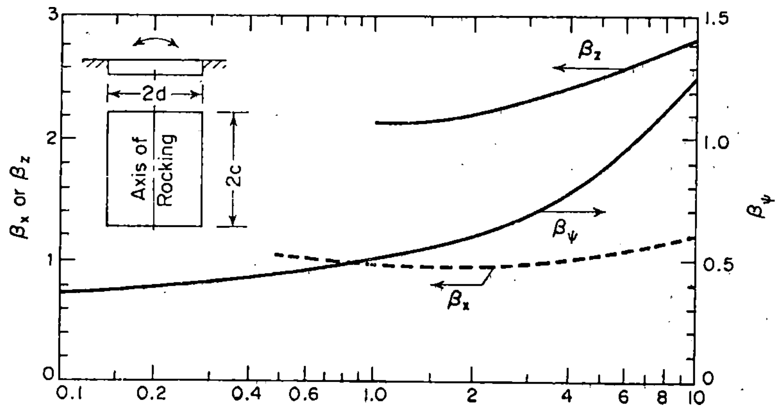


FIGURE 20. COEFFICIENTS β_z, β_x, AND β_ψ FOR RECTANGULAR FOOTINGS (AFTER WHITMAN AND RICHART, 1967)

Westenhoff and Novick also spent considerable effort in an attempt to obtain appropriate values for the subgrade parameters of the Kansas Test Track (KTT) site, based on soil and embankment data measured at the site. The embankment for the KTT was designed by Shannon and Wilson, Inc. [21]. A maximum design stress was chosen which lies between the stresses calculated for point and line loading. A safety factor of 2.5 was assumed, from which an unconfined compressive strength of 3.9 kg/cm^2 for the embankment material was derived. This value was compared with a plot of strength of various materials, and optimum water content from which a standard section for the Santa Fe was selected. It was felt that a standard section would make future comparison easier, and that "the relative advantages of slight modifications in embankment geometry are not readily discernible".

A Vibroseismic Survey was performed by the Waterways Experiment Station of the U. S. Army Corps of Engineers [22] to determine the embankment properties after the construction of the embankment was completed. According to the survey report, "The purpose of the vibroseismic survey was to provide seismic information pertinent to the soil conditions and elastic properties of the test embankment. Specifically, compression-wave velocities; depth to interfaces; and shear-wave velocities, shear and Young's moduli, and Poisson's ratios as a function of depth were to be determined. In addition, attenuation tests were conducted to determine the frequency and damping characteristics of the soil such as log decrement, damping ratio, and amplitude decay as a function of distance from the source".

A plate bearing test was also performed on the embankment in order to determine the values of static vertical subgrade modulus. That value was compared with dynamic values of subgrade modulus calculated using the Barkan formula and an influence length (for a single load) of 30 feet. The values for shear modulus and Poisson's ratio for the embankment were taken from the report covering the vibroseismic survey.

The subgrade modulus data were used to determine spring rates for a Winkler foundation model. The use of this model involves four assumptions: (1) shearing resistance is zero in the embankment model; (2) linear springs are used; (3) the resistance is concentrated at discrete points; and (4) the subgrade acts in tension as well as compression (this is valid if the dead load of the structure equals the dynamic load).

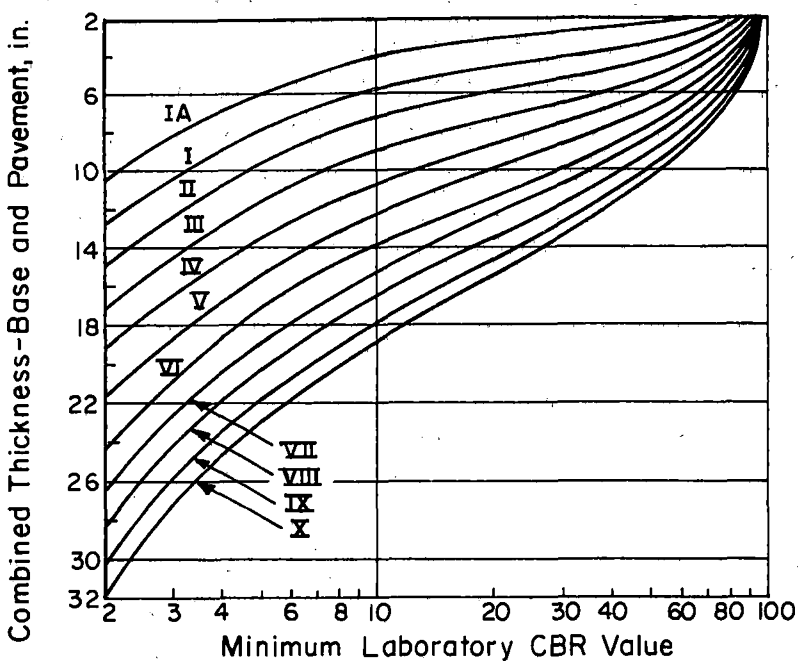
The theoretical embankment stresses were calculated using Boussinesq, Westergaard, and Burmister techniques for a locomotive, passenger car, and an empty hopper car. Burmister techniques were chosen to simulate the effect of a rigid base 7 feet below the slab.

The Kansas Test Track, therefore, represents perhaps the most thoroughly analyzed railroad slab track ever built. In view of this and the fact that considerable instrumentation was installed in the embankment, a good program of data collection and analysis is recommended to further knowledge on slab track for railroad application.

In the highway field, an example of a present slab design technique is the California Bearing Ratio (CBR) method [23] which is mentioned here because of its potential development for railroad application. This method is based on the properties of the embankment material as determined from the CBR test--a test originally developed by the California Division of Highways (and described in Volume 1 of this report). Once the California Bearing Ratio is found, layer thicknesses are found by reference to an empirical design chart such as Figure 21, which is the chart of the Kentucky Highway Department. However, this gives the total thickness of the combined base and pavement, and the thickness of the concrete slab must be found by other methods. This method is usually used for asphalt pavement, and the use of concrete reduces necessary base thickness somewhat--perhaps as much as 20 percent. The development of a similar simplified design procedure for railway slab track, using rail fastener load rather than the Equivalent Wheel Load (EWL), appears worthy of investigation.

4.5 Design of the Reinforced Concrete Slab

If the slab track is modeled as a beam-on-elastic-foundation, one of the outputs of the computation could be the required bending stiffness (EI). It is necessary to convert this bending stiffness to the thickness of slab, with the amount and location of reinforcing. However, even before the beam-on-elastic-foundation model can be used, it is necessary to choose a slab width so that the subgrade support can be converted to the proper value of spring rate to be used under the beam in the mathematical representation. Although not directly applicable, it is interesting to note how this has been approached for tie-type track. J.D.W. Ball in Reference [24] determined the optimum length of the tie for 4'8-1/2" gauge conventional track to be 9 feet. In arriving at this conclusion, he assumed continuous contact under the tie with the pressure being proportional to deflection, and defined the optimum condition as being when the ends and middle of the tie deflected equally, thus overcoming most of the tendency to displace ballast. As the transverse bending stiffness of the tie increases, the tie width becomes less important in limiting the lateral displacement of ballast because of the corresponding decrease in relative deflection along the tie.



Kentucky CBR Procedure

1. Specimen molded at optimum moisture as determined by AASHO T-99.
2. Compaction-5 layers, 10 blows per layer of 10-lb hammer falling 18 in.
3. Soaking - 4 days.

Directions for Selecting Design Curve

1. Relate actual wheel loads in both directions for design life to EWL (equivalent 5000-lb wheel loads) using Tabulation 1.
2. Select proper curve from Tabulation 2.

Tabulation 1

Actual Wheel Load, lb	No. of EWL per Wheel Load
4500-5500	1
5500-6500	2
6500-7500	4
7500-8500	8
8500-9500	16
9500-10,500	32
10,500-11,500	64
11,500-12,500	128

Tabulation 2

Total EWL, millions	Curve No.
<0.5	IA
0.5-1	I
1-2	II
2-3	III
3-6	IV
6-10	V
10-20	VI
20-40	VII
40-80	VIII
80-160	IX
160-320	X

FIGURE 21. PAVEMENT DESIGN METHOD OF THE KENTUCKY DEPARTMENT OF HIGHWAYS, BASED ON THE CBR METHOD. (Source: HRB Bulletin 288) (From Reference 23 page 440)

The slab acts as a two-way beam and Mr. Ball's analysis does not strictly apply, but it should be a good approximation. Support for this assumption is given by the choice of a 9-foot width for the KTT. Figures 22 and 23 show the variation of pressure as predicted by Westenhoff and Novick for the KTT. (The figures also illustrate the advantages of slab structures over beam structures under lateral loading. Note that the slab minimizes pressure differential due to lateral loading, as well as providing the low initial pressure because of the increased bearing area.)

4.5.1 Elastic Bending Stresses

Assume now the width has been used in a beam-on-elastic foundation model, and that a required moment or bending stiffness for the beam has been calculated. The general procedure for designing a reinforced concrete beam knowing either the moment or stiffness involved the concept of a "transformed area". This idea is a useful simplifying idea in the design of concrete and steel subject to the same deformations. The area of the steel, A_s, can be replaced in the design or analysis with an equivalent area of concrete equal to nA_s, where n equals the ratio of the moduli of elasticity = E_s/E_c. The steel stress, f_s, will equal nf_c. Although concrete that is not prestressed can withstand some tension stresses, the nonprestressed concrete in tension is usually assumed to be cracked in the bending calculations. With the slab track, both positive and negative bending occurs, and the designer has a choice of assuming the concrete in tension eventually will crack--in which case the steel will take all tension forces, or designing so that the design tensile stresses are below the allowable concrete tensile stresses, and assuming the concrete does not crack. In both cases, the model can be used for design, with the tension areas being retained or deleted as necessary. A transformed area for a double reinforced beam under assumed elastic action is shown in Figure 24.

The elastic type of analysis is correct only for the effect of short-term loads, including stresses generated by train passage. The elastic stresses are complicated by the presence of shrinkage stresses and creep in the concrete. The inelastic effects are significant in compression steel which tends to pick up

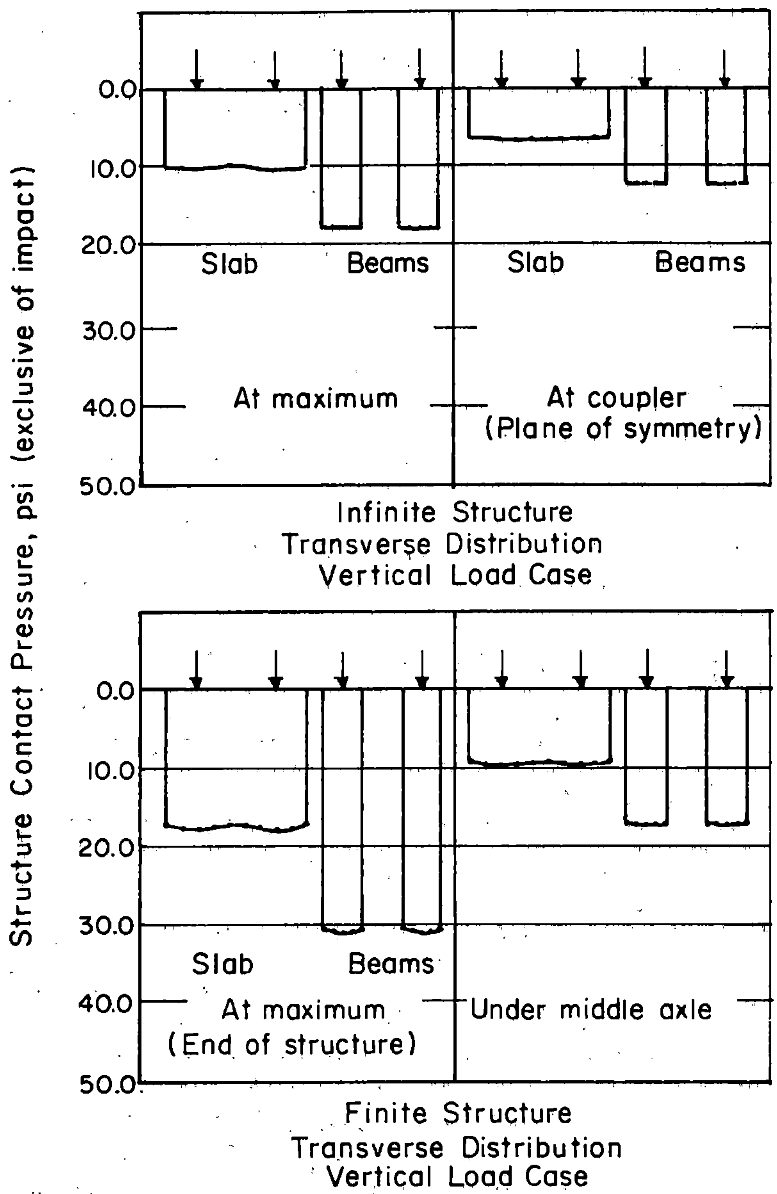


FIGURE 22. CALCULATED BEARING PRESSURE BENEATH CONCRETE SLAB AND BEAMS WITH VERTICAL LOADING ONLY

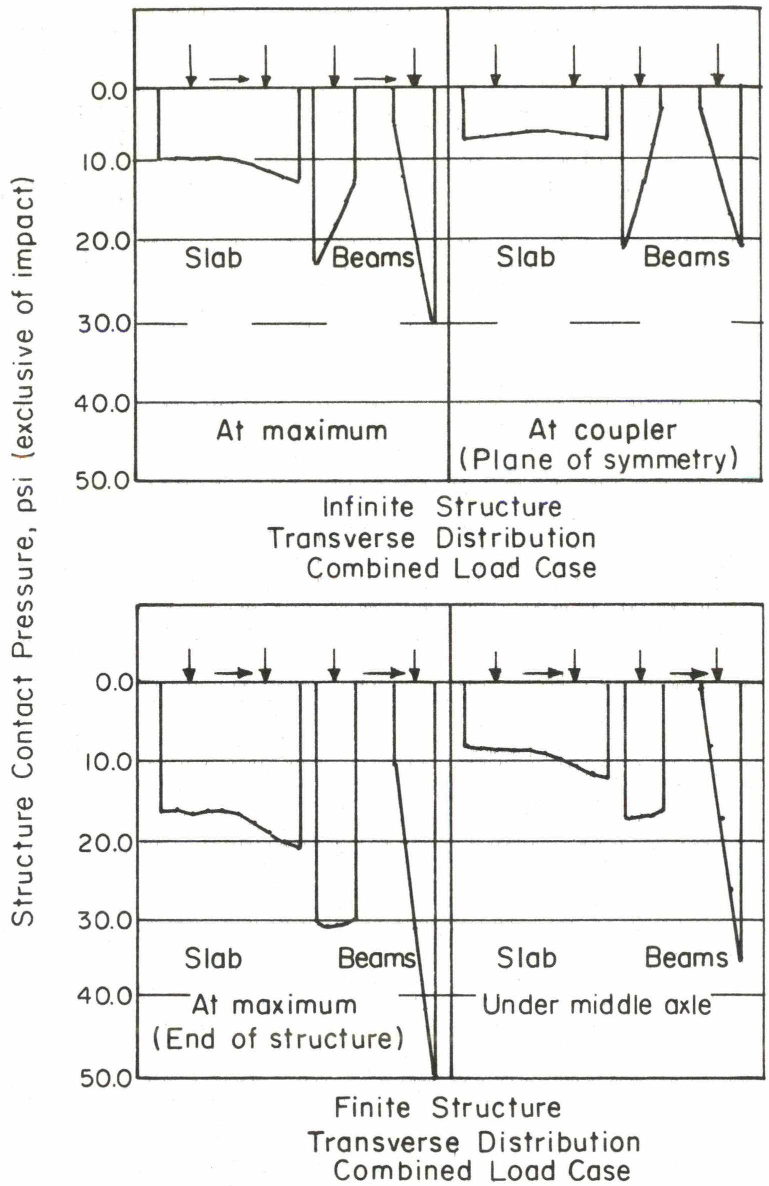


FIGURE 23. CALCULATED BEARING PRESSURE BENEATH CONCRETE SLAB AND BEAMS WITH COMBINED VERTICAL AND LATERAL LOADING

stress much in excess of nf_{cs}. For example, as an approximation for building designs, the ACI code suggests that f_s' may be taken as $2nf_{cs}$. This is equivalent to an effective compressive stress f_s' of $(2n-1)\ f_{cs}'$ and a transformed area of $(2n-1)A_s'$. This approach is sometimes referred to as the plastic approach, and while recommended for building designs, may not be applicable for the short-term loading produced in track structures.

The design stresses in the concrete and steel and n can now be selected using techniques such as the one given in Chapter 8 of the AREA manual. This leaves the depth of the beam, the depth to the reinforcing, the distribution of reinforcing, and the width of reinforcing to be decided. The location of the desired neutral axis (see Figure 24) can be determined using similar triangles.

$$\frac{kd}{d} = \frac{\text{max. compressive stress}}{\text{max. compressive stress + max. tensile stress}} \tag{15}$$

and for this case

$$\frac{kd}{d} = \frac{f_c}{f_c + f_s/n} \quad . \tag{16}$$

If one is designing for moment, there are four stresses to consider:

(1) Tension in steel

(2) Tension in concrete

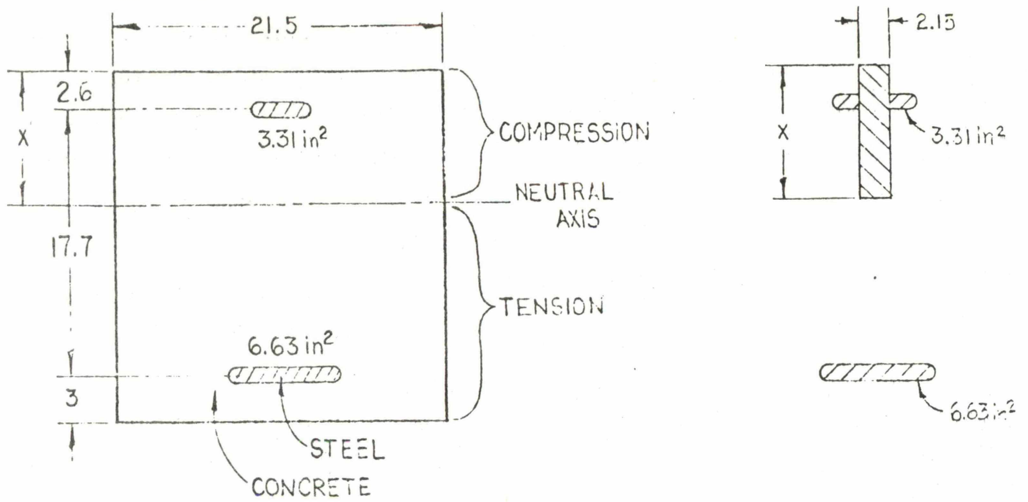
(3) Compression in steel

(4) Compression in concrete.

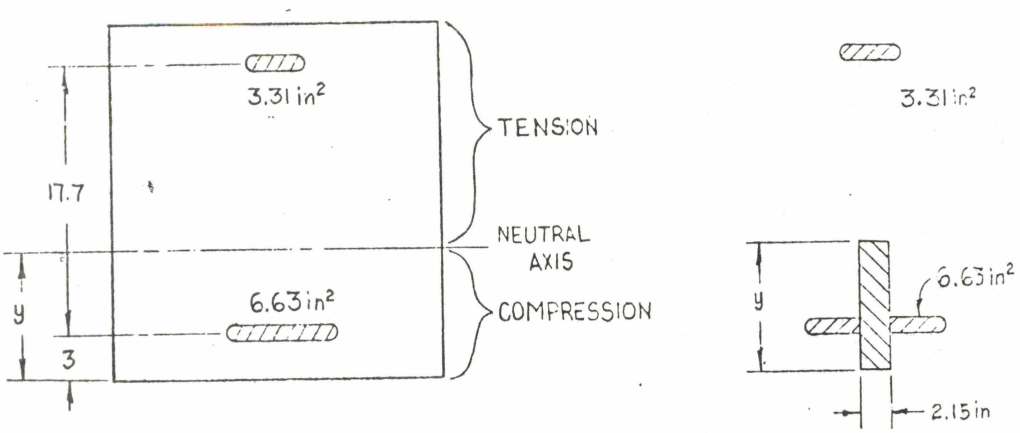
In terms of engineering design, the most efficient reinforced concrete beam is a so-called balanced design, in which the stresses in the concrete and in the steel reinforcing bars are the same percentage of their allowable stresses. In other words, if the load on the beam were progressively increased, the concrete and steel would fail simultaneously. A balanced design thus makes maximum use of the strength of both of the materials and may result in some economy.

A balanced design may not, however, be the most economical design. This is particularly true when the beam is being designed on the basis of stiffness and not on the basis of strength. Other factors, such as the relative costs of concrete and steel, may make it more economical to use only a minimum amount of steel and make up for it by using more concrete; or, on the other hand, the labor costs involved in placing forms for concrete may be substantially higher than the labor costs for placing reinforcing bars, and this may result in a more economical design being one where steel is used in large quantities.

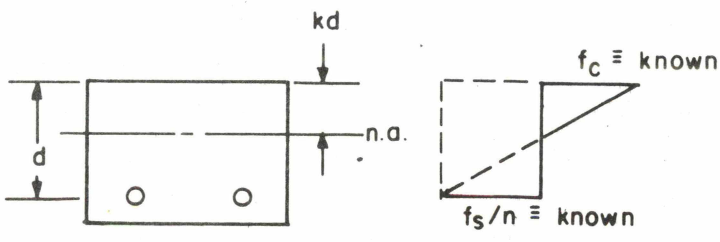
Reinforced Concrete Beam Equivalent Steel Beam



(a) Positive Bending



(b) Negative Bending



(c) Determination of Neutral Axis

FIGURE 24. REPRESENTATION OF REINFORCED CONCRETE BEAMS FOR STRESS CALCULATION

When designing the beam on the basis of a balanced design with known applied moments, equations can be written enabling the overall dimensions of the beam to be solved. The resulting equations can then be solved for the area of steel.

Designing the slab for a specified bending stiffness is similar to the moment method except that after the neutral axis is found, the resulting equations are solved for the specified EI. Note that when using a transformed area, the E for concrete is used throughout.

For the design of concrete highway slabs having only minor reinforcement to control cracking, Westergaard's [25] analysis is usually used. The formulas provide a method for calculating the stresses within a given thickness of slab. Westergaard's analysis is based on the following assumptions:

(1) The concrete slab is a homogeneous, isotropic, and elastic solid

(2) The reaction of the subgrade is vertical and proportional to the deflection of the slab

(3) The applied load is uniformly distributed over the contract area.

The formula he obtained for the maximum tensile stress in the concrete due to a wheel load at some distance from the edge of the slab is

$$\sigma_i = \frac{0.275P}{h^2} \quad (1 + \mu)\log_{10} \left\{\frac{Eh^3}{kb^4}\right\} \tag{17}$$

where

σ_i = maximum tensile stress in the concrete at the bottom of the slab at an interior location, directly under the center of an applied load (lb/sq. in.)

E = modulus of elasticity of concrete (lb/sq. in.) (3 to 6 x 10^6 lb/sq. in.)

μ = Poisson's ratio for concrete (0.1 to 0.35)

k = Modulus of subgrade reaction (lb/sq. in./in.)

h = Thickness of concrete slab (in.)

P = Total load exerted by one wheel (lb.) or fastener

a = Radius of equivalent circular loading area (in.)

$$b = \sqrt{(1.6a^2 + h^2)} - 0.675h \text{ when } a < 1.724\ h$$

$$b = a \quad \text{when } a > 1.724\ h.$$

In using the formula one must remember that:

(1) The wheel load is distributed over several fasteners, and the degree to which adjacent loads influence each other depends on the slab stiffness.

(2) No evaluation of subgrade stresses is made.

(3) Interior loading stresses are somewhat high, and Westergaard suggests a modified form of the formula may be used. It is

$$\sigma_i = \frac{0.275P}{h^2}\left[1 + \mu\right]\left[\log_{10} \frac{Eh^3}{kb^4} - 54.54\ Z \left\{\frac{\ell}{L}\right\}^2\right] \tag{18}$$

where L is the maximum radial distance in inches from the center of the load within which a redistribution of subgrade reactions is made, Z is a ratio of reductions of the maximum deflections and ℓ is the radius of relative stiffness given by

$$\ell = \left\{\frac{Eh^3}{12\ (1 - \mu^2)k}\right\}^{1/4} \tag{19}$$

The experimental determination of L and Z is discussed by Teller and Sutherland [26]. Both quantities vary with pavement and subgrade stiffness, but where conditions are not known Teller and Sutherland suggest the use of $Z = 0.2$ and $L = 5\ell$ which will usually give slightly conservative stresses.

4.5.2 Temperature Stresses

A temperature gradient through the slab will cause the slab to tend to warp. The weight of the concrete will tend to hold the slab in its original position and thus induce stresses into the slab. For rigid concrete slabs, the stresses caused by warping were also analyzed by Westergaard. Bradbury used Westergaard's analysis to develop the coefficients shown in Figure 25 for use in the following equations [23].

Warping stress along the edge of a slab

$$\sigma_{xe} = \frac{C_x E \varepsilon_t \Delta_t}{2} \tag{20}$$

Warping stresses in the interior of a slab

$$\sigma_x = \frac{E\varepsilon_t \Delta_t}{2}\left(\frac{C_x + \mu C_y}{1 - \mu^2}\right) \tag{21}$$

$$\sigma_y = \frac{E\varepsilon_t \Delta_t}{2}\left(\frac{C_y + \mu C_x}{1 - \mu^2}\right) \tag{22}$$

Symbols in these formulas are defined as follows:

σ_{xe} = maximum temperature warping stress at the edge of the slab in the direction of slab length, psi

σ_x = maximum temperature warping stress in the interior of the slab in the direction of slab length, psi

σ_y = maximum temperature warping stress in the interior of the slab in the direction of slab width, psi

E = modulus of elasticity of concrete, psi

μ = Poisson's ratio for concrete

C_x and C_y = coefficients that relate slab length and the relative stiffness of slab and subgrade to temperature warping stresses. Values of C_x and C_y are given in Figure 25.

ℓ = radius of relative stiffness (see Equation 19)

ε_t = thermal coefficient of expansion and contraction of concrete per ^{o}F.

Δ_t = difference in temperature between top and bottom of slab in ^{o}F.

For lack of more exact data, a typical value of 3^{o}F per inch of slab is used for the temperature differential. These equations are approximations because the weight and restraint of the rail, fasteners, and reinforcing is not considered. Stresses due to warping from frost heave or other causes (subgrade settling, etc.) can be quite high. For example, assuming a slab deflection in the form of a sine wave with an amplitude of ±3 inches in 100

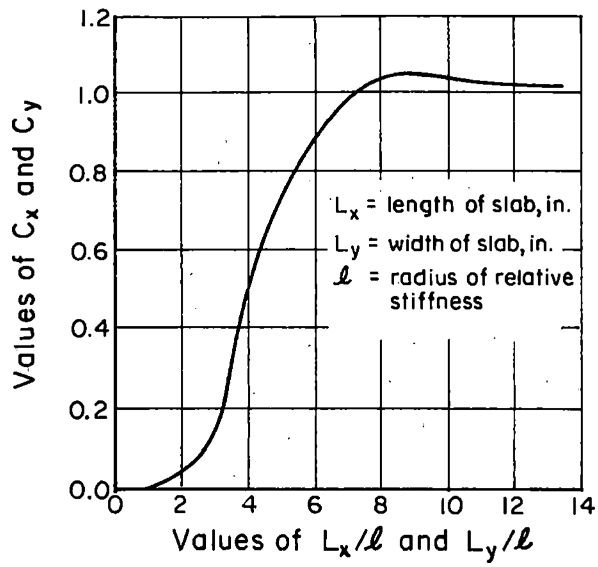


FIGURE 25. WARPING STRESS COEFFICIENTS FOR USE IN FORMULAS FOR FLEXURAL STRESSES IN CONCRETE PAVEMENTS (From Reference 23, page 646)

feet of length, for a slab section with an EI of 2 x 10^{10} lb.-in.^{2}, the bending moment would be 1.6 x 10^6 in.-lb., which is more than an order of magnitude larger than the moment caused by the train loads (see later section).

Stresses also occur in rigid slabs from temperature changes which tend to cause the slab to expand or contract. The stress results from the restraint caused by friction forces. According to Yoder [5], the slab must displace a minimum of 0.05 inch for friction forces to be fully developed. Kelley has proposed the distribution of frictional stresses shown in Figure 26.

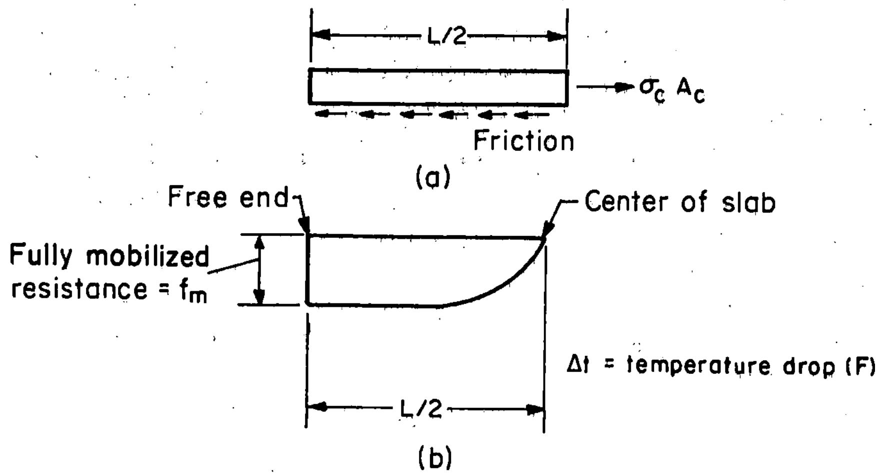


FIGURE 26. STRESSES RESULTING FROM CONTRACTION. (a) FORCES ACTING ON CONTRACTING SLAB; (b) VARIATION OF SUBGRADE RESISTANCE WITH LENGTH. (From Reference 5, page 60)

For the length where the slab is acted on by friction, the following equation applies:

$$\sigma_c = \frac{WLf}{24h} \tag{23}$$

where σ_c = longitudinal stress in slab (psi)

W = weight of slab (psf)

L = length of slab (ft)

f = average coefficient of subgrade resistance

h = depth of slab (inches).

If a better value is not available, f = 1.5 is often used for calculations. Warping of the slab will cause a loss of contact with the subgrade, and the frictional resistance will actually be somewhat less than calculated.

During the design of the KTT, additional work was done to effectively model the ballast-slab interface. Field tests were made to determine the resistance the slab provides to lateral loads. The test data are shown in Figure 27, and indicate that the resistance to displacement is more like a spring force (increasing with deflection) than like a coulomb friction (constant force once breakaway occurs), especially at higher pressure. Again, the difficulty of making accurate design calculations is indicated by this discrepancy between assumed and measured values for various parameters - in this case, the friction force.

4.5.3 Finite Element Slab Model

The design method used by Westenhoff and Novick for the design of the Kansas Test Track slab made use of a finite element program. Reinforcing was apportioned in accordance with the DOT specification for a cracked section stiffness of 4×10^{10} lb-in^2. This value was based on the stiffness required for the transmission of only one pressure pulse per truck (rather than one per wheel) to the subgrade [11]. The resulting design had two different section stiffnesses, one for positive bending, and a second for negative bending. A NASTRAN finite element program with quadrilateral plate-bending elements (QUAD 2) was used by W&N to model the slab. Equivalent homogeneous sections were used to account for the cracked sections.

Several assumptions are inherent in the use of the plate-bending elements chosen as the elements in the finite element program to model the slab. First, the plate is assumed to lie in the center plane of the slab. Second, the stiffness of positive and negative bending is identical. Although this was not the case, it was considered acceptable due to the approximate nature of the values used for several other significant parameters in the analysis. Third, the load is considered to be transferred to the slab by the fastener at a point rather than over an area. A factor of 1.5 was applied to the computer results to account for approximation and modeling convergence.

A conceptual representation of the model is shown in Figure 28. While the plate element model used for the KTT represents a refinement in the method for modeling a slab, the expense may not be justified when one considers the variability and precision of the inputs. The advantage of the finite element

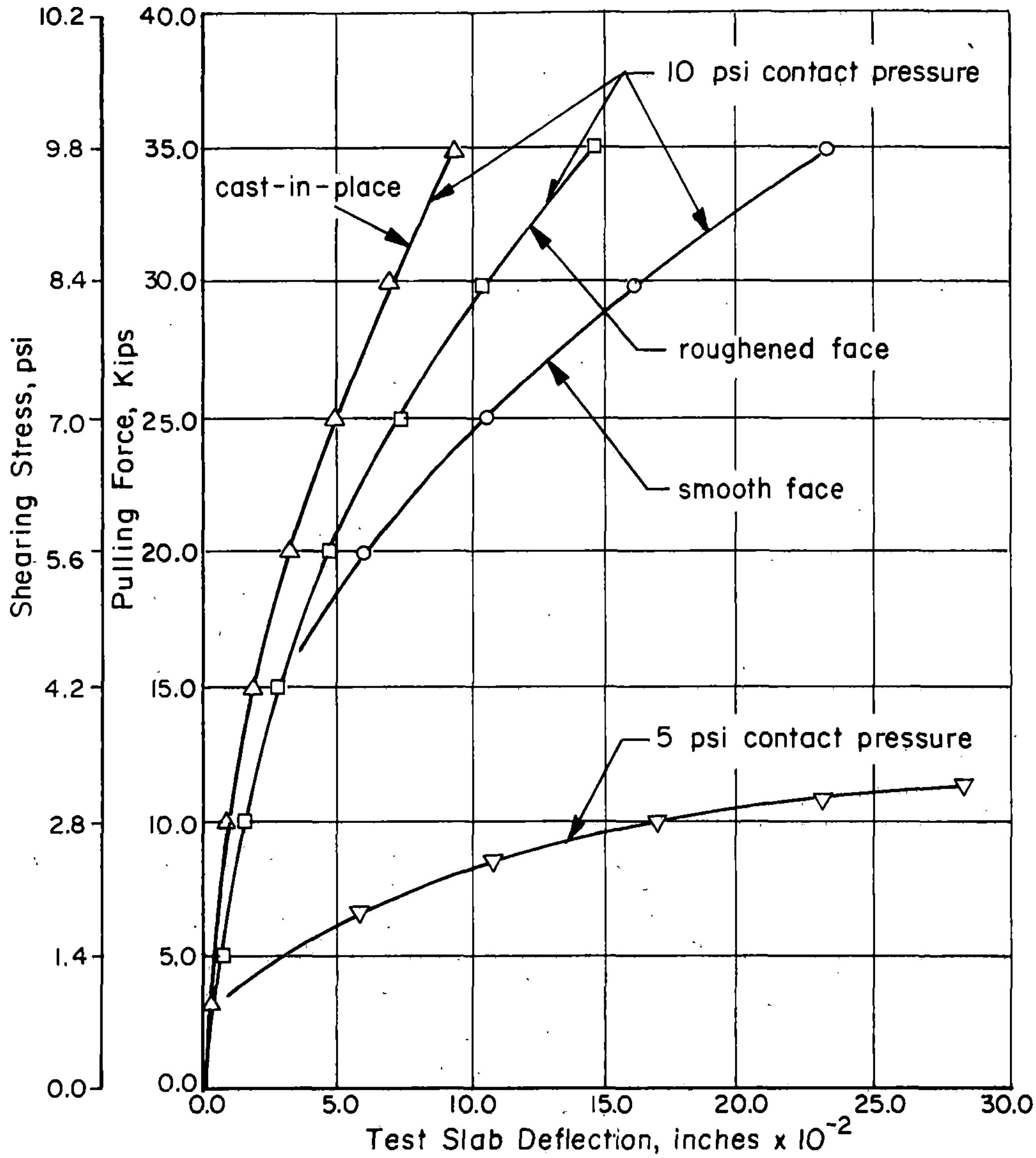


FIGURE 27. BALLAST - SLAB SHEAR TEST RESULTS CAST-IN-PLACE, ROUGHENED FACE IN CONTACT, OR SMOOTH FACE IN CONTACT (From Reference 4, page 33)

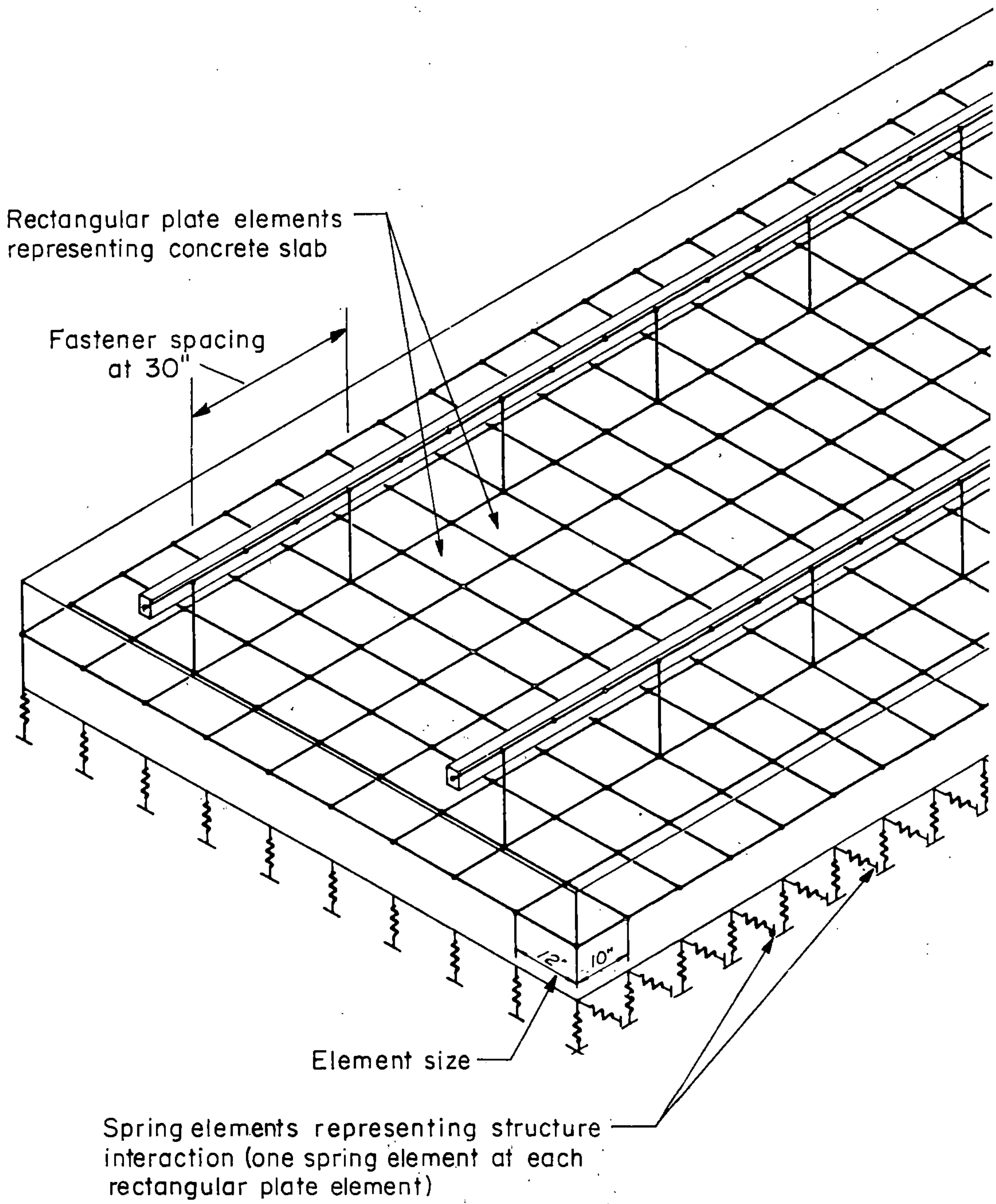


FIGURE 28. FINITE ELEMENT REPRESENTATION OF KANSAS TEST TRACK CONCRETE SLAB AND RAIL (From Reference 4, page 25)

model is that the output contains information concerning the variations across the width of the slab. The pressure variations along the longitudinal axis are probably no better than the much simpler beam-on-elastic foundation models (such as the one described in Volume 1, Appendix B), and the factors used in this particular application seem so coarse as to negate any refinement of stress accuracy offered by this approach.

4.5.4 Joints

The use of joints causes discontinuities in the track structure, and discontinuities cause problems. If a slab having temperature reinforcing only is used, then joints must be provided. If, on the other hand, a double reinforced beam is used, then it may be possible and desirable to eliminate expansion and contraction joints to the extent practical with existing construction techniques.

The joints in concrete pavements are necessary for various reasons, One of the most important reasons is because variations in temperature and moisture content cause volume changes in the concrete. If the volume change is restrained, stresses arise in the concrete, causing cracking. There are a number of types of joints, and these are classified according to the function [27]. Some of these joints are described below and are shown in Figure 29.

(1) Expansion joints. Expansion joints provide space for the expansion of the concrete. There are two types of expansion joints.
    (a) The first contains nonextruding compressible material and dowel bars for load transfer.
    (b) The second contains a nonextruding compressible material and the thickness of the slab along the joint is increased.

(2) Construction joints. Construction joints are those which occur because of the construction operations--for example, between one day's work and the next.

(3) Contraction joints. Contraction joints are used to control cracking of the slab when the concrete contracts during curing.

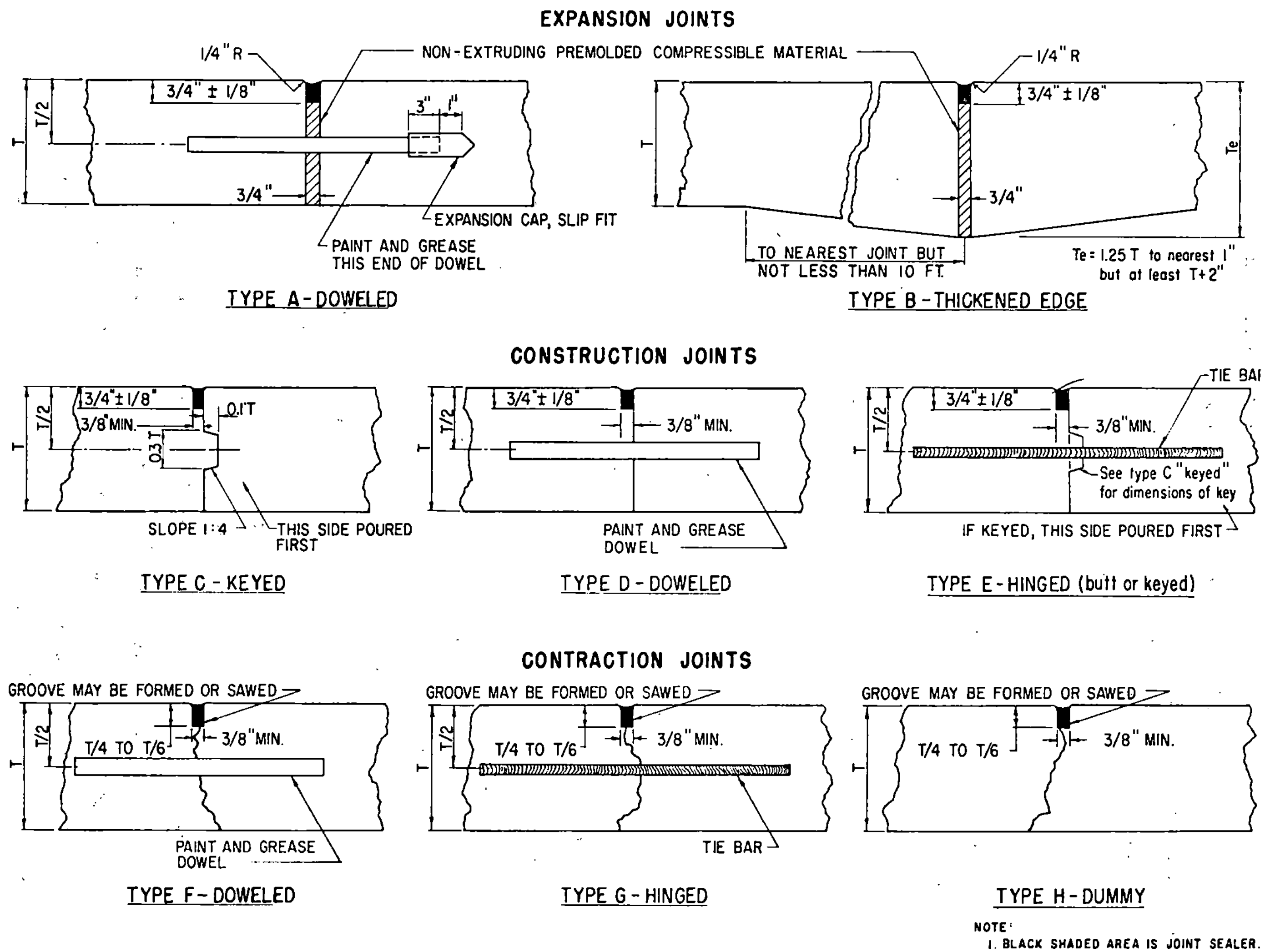


FIGURE 29. DETAILS OF JOINTS IN RIGID PAVEMENTS (From Reference 27, page 38)

(4) Hinge or Warping Joints. These joints are often used with keyed joints to assure that the key transfers load. They are also used with dummy-groove joints to assure that grain lock is maintained.

4.5.5 General Design Guidelines for Reinforced Concrete

When designing concrete slab track, the AREA handbook [28] should be referred to. Although the AREA handbook does not presently contain a section on the design and construction of slab track, most of the information in Chapter 8 on concrete structures and foundations is applicable.

Although the tensile strength of concrete is low, it does have definite tensile strength characteristics, and fatigue characteristics similar to other materials in that its life is inversely proportional to the applied stress. For reversed stress, a stress level of 50 percent of the modulus of rupture is usually used as a limit for infinite life.* A fatigue curve for plain concrete in flexure is shown in Figure 30. Compressive strengths of 3500-5000 psi and infinite life tensile or flexural strengths of 10 percent of compressive are typical. The strength is dependent on the curing condition and time, and therefore the strength is usually specified at 28 days.

A few guidelines outlined by Ferguson in Reference 29 are pertinent to the design of the slab track. The specified strength should be at least 15 percent higher than design. Concrete control cylinders that vary less than 10 percent are very good. The reinforcing will vary 3-5 percent in strength and 3-1/2 percent in weight. The placing of the reinforcing bars is usually to only 1/8 to 1/4 inch. Noting these things, the following precisions are recommended as a rough guide.

Loads to nearest 1 pound per square foot
Span lengths to about 0.01 feet
Total loads and reactions to 0.1 Kip
Moments to 0.1 Kip-inches
Individual bar areas to 0.01 in^2
Concrete sizes to 1/2 inch

* The modulus of rupture typically is on the order of 650-750 psi (see Reference 5, p 114, 455, and 485).

Bar spacings to 1/2 inch

Effective beam depth to 0.1 inch.

The ASCE-ACI committee recommends the following load factors

1.2 for dead load

2.4 for live load

2.4 for wind or earthquake.

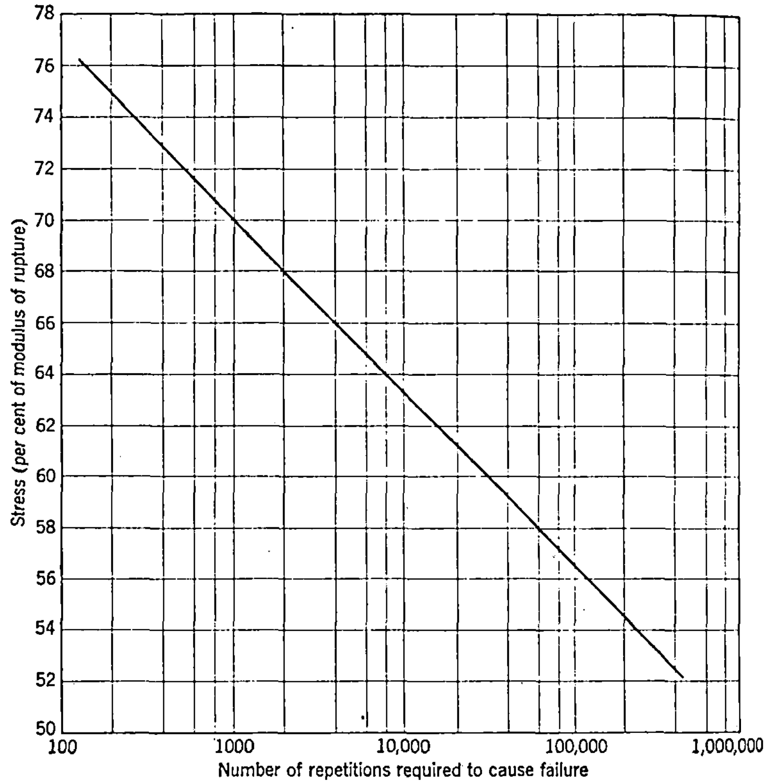


FIGURE 30. FATIGUE CURVE FOR PLAIN CONCRETE IN FLEXURE
(From Reference 5, page 114)

5.0 PRELIMINARY DESIGN CALCULATIONS FOR PUEBLO SLAB TRACK

The urban rail slab track now being planned for the U. S. DOT High-Speed Ground Test Center will provide a proving ground for analyzing the potential of slab tracks for reducing life-cycle costs of urban rail track structures. To illustrate the application of the state-of-the-art techniques previously described, preliminary design calculations for such a Pueblo slab track are presented in this section of the report. The criterion or rationale for the design was considered to be to minimize life-cycle costs of the track structure. Unfortunately, the maintenance costs are difficult to quantify; in terms of the other major cost, the initial construction costs, cost versus performance parameters (for example, bearing pressures) were evaluated.

To provide a reference in terms of a modern tie track, the Washington Metro track was chosen. Some design parameters for this track are shown in Table 2. [30]

In order to determine a loading profile and magnitude, various transit cars shown in Table 3-1, Volume 1 were considered. It was decided that the SOAC (state-of-the-art-car) and the CTA (Chicago Transit Authority) specifications spanned the range of applicable axle spacings. Upon evaluation of the loads, a 30,000 pound wheel load was determined to be an applicable design load, representing a dynamic impact factor of about 2.0 for the SOAC wheel loads.

The next step in the design was to model the track structure as a continuous beam (rail) resting on an elastic foundation (rail fasteners) which is supported by another beam (concrete slab) resting on an elastic foundation (base-subbase). This model is described completely in Appendix A.

The beam-beam model of a slab track has four parameters which characterize the track. They are the bending stiffness of the rail, the modulus of the fasteners, the bending stiffness of the slab, and the modulus of the supporting embankment. The model represented one half a track and the parameter values therefore apply to one rail, half the slab width, etc.. The rail sizes considered were 100 AREA to 132 AREA rails. The corresponding bending stiffness of these rails varies from 1.5×10^9 lb-in.^{2} for 100 pound rail to

TABLE 2. WASHINGTON METRO DESIGN (WOOD TIES) [30]

Parameter	Design Values
Wheel load	Static -- 15,000 lbs Dynamic -- 30,000 lbs
	← 78 in → ← 198 in. →
Wheel spacing	← 84 in. → ← 192 in. →
Rail	115 pound RE
Ballast	12 in. ballast 8 in. subballast
Tie size	7 in. x 9 in. x 8-1/2 in. wood
Tie spacing	27 in.
Rail stress	13,200 psi
Ballast-Tie seat load(a)	65 psi
Subgrade pressure(a)	20 psi
Track modulus(b)	1500 $lb/in.^2$

(a) Calculated by doubling the rail seat load obtained from elastic theory.

(b) The 1500 $lb/in.^2$ modulus was calculated by assuming $U' = 2000\ lb/in.^2$ for 7 in. x 9 in. x 8-1/2 in. ties spaced at 20 inches [31, 32] and

$$U = U' \cdot \frac{\ell_t'}{\ell_t} \cdot \frac{A_b}{A_b'}$$ [See Equation (3-12), Vol. 1]

U = track modulus

A_b = tie bearing area = $b(\ell-60)\left[1 - \frac{0.018\,(\ell-60)}{t^{3/4}}\right]$

ℓ_t = tie spacing

ℓ = tie length

t = tie thickness

b = tie width

2.6×10^9 lb-in.^{2} for 132 pound rail. The fastener modulus varies from 200 lbs/in/in. to 30,000 lbs/in/in. (This range would include relatively soft fasteners such as the "Pandrol" on 18-inch centers, and almost all stiffer fasteners.) The modulus of subgrade reaction typically varies from 100 - 500 lbs/in.^{3}. Assuming the slab track is nominally 100 inches wide (50 inch half width), these values convert to an elastic foundation modulus of 5000 lb/in.^{2} - 25,000 lb/in.^{2}.

The important outputs from the program are the deflections and the moments. Therefore, for the track structure parameters study, the maximum deflection of the rail, the maximum moment in the rail, the maximum deflection of the slab, and both the maximum positive and maximum negative moments in the slab were plotted as functions of the four track parameters.

Figures 31 and 32 show the effect of the variation of the various parameters. In these figures, the following symbols are used:

M_r = maximum moment in rail

$+M_s$ = maximum positive bending moment in slab

$-M_s$ = maximum negative bending moment in slab

Y_r = maximum deflection of rail

Y_s = maximum deflection of slab.

Figures 31 and 32 indicate that the foundation modulus $U = k_o W$ (where W is the slab half-width) greatly affects the deflection and bending moment in the slab. The fastener deflection (relative deflection of the rail and slab) is increased by increasing the foundation modulus. For a slab track, the track modulus is mainly a function of the subgrade modulus because the bearing area of the slab is restrained in the relatively small range of 6-9 feet. For the same bearing area, a doubling of k_o approximately halves the deflection and, therefore, the bearing pressure remains about the same (bearing pressure is $k_o \cdot y_s$). There is a gain, however, because a soil with a higher modulus tends to have a higher bearing capacity. (Also, from a design standpoint, the smaller deflections will conform much better to the linear elastic model.)

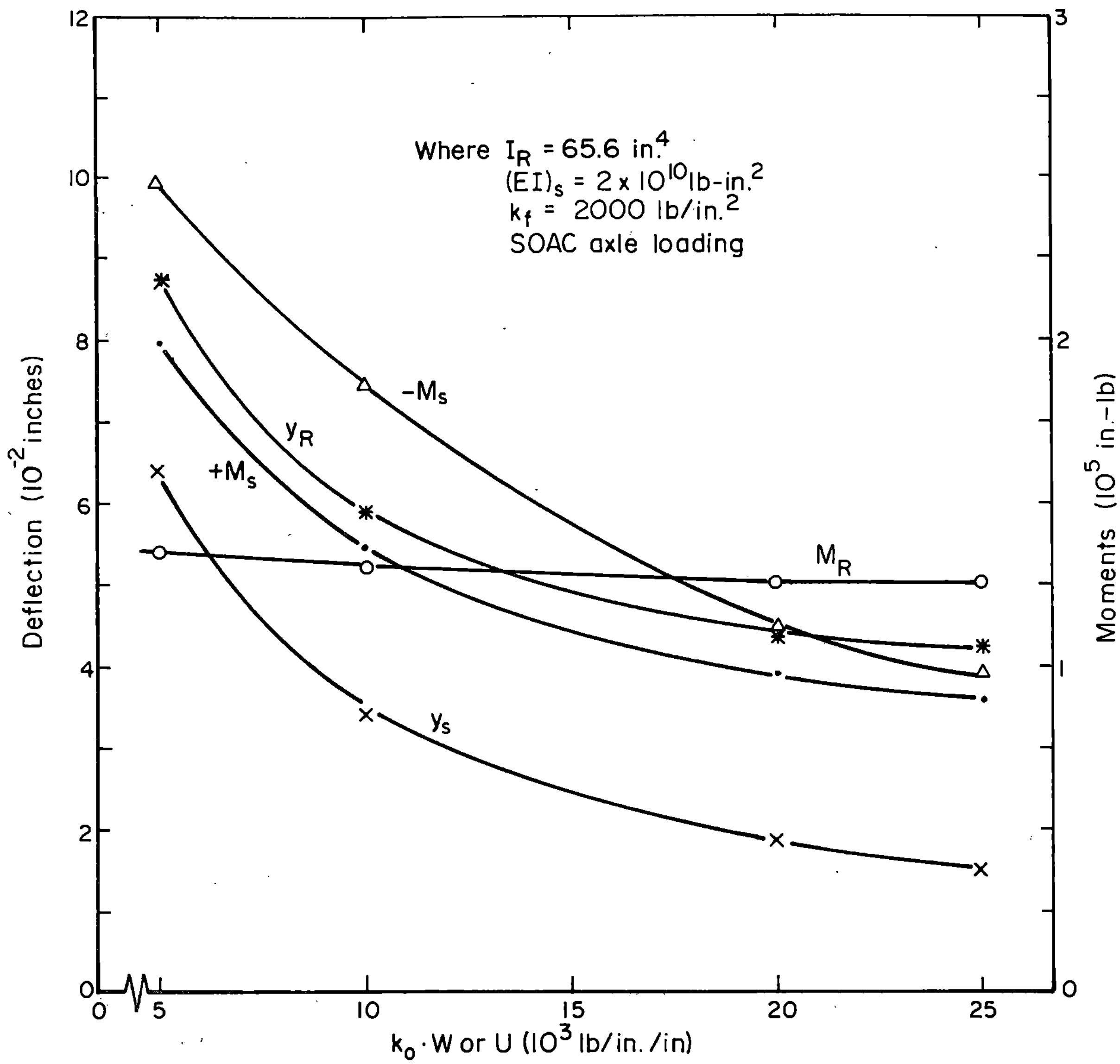


FIGURE 31. RELATIONSHIP OF TRACK STRUCTURE BENDING MOMENTS AND DEFLECTIONS TO SUPPORT MODULUS

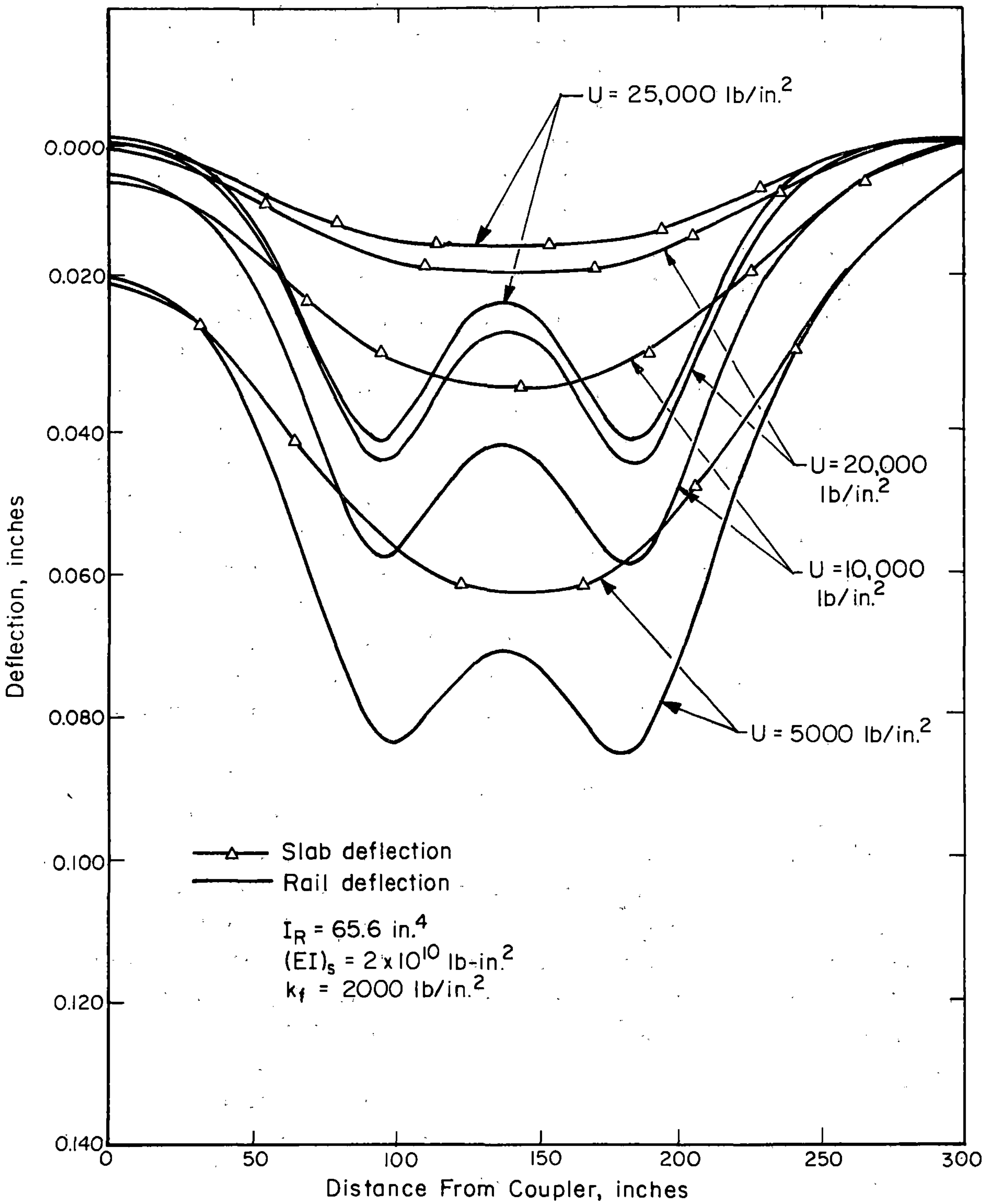


FIGURE 32. RELATIONSHIP OF RAIL AND SLAB DEFLECTION TO SUPPORT MODULUS

Figures 33 and 34 show that varying the bending stiffness of the slab does not have much effect on the moment in the rail or the fastener deflection. The maximum deflection of the slab is decreased by increasing the bending stiffness. There is a major effect on the bending moment of the slab, however. Note that the maximum moment increases with increasing $(EI)_s$. Unlike most situations, this does not necessarily mean that the stress in the slab increases. In fact, in most beam designs, the stress will actually decrease because the section modulus increases faster than the moment increases.

An important design consideration is the location of the maximum moment. The maximum moment is a function of the magnitudes of the foundation stiffness, $k_o \cdot W$, and the slab stiffness, EI_s. For a fixed foundation stiffness the absolute value of the moment of the slab increases with increasing slab stiffness. For a fixed slab stiffness the maximum absolute moment in the beam increases with a decrease in the foundation stiffness. For a fixed foundation stiffness the sign of the maximum absolute moment is positive (top of slab in compression, bottom in tension) for low relative values of slab stiffness, but becomes negative and then positive again as the stiffness increases. (Note: a low relative value can be as stiff as 10^{10} lb-in.^{2} if the subgrade is stiff.) This behavior can be illustrated by studying the deflection profiles of a beam-on-elastic-foundation system as shown in Figure 35. These curves can also be interpreted as the time-history of the bending moment at any fixed location in the slab, during a train pass-by.

The location of the maximum bending moments and associated stresses are important to the interaction of the fastener with the slab. The areas surrounding the loaded fastener inserts will be areas of high local stress concentration, and caution must be exercised in the detailed design of the slab that the concrete does not crack at these points and allow the inserts to pull out. In the Kansas Test Track, serious problems have been experienced with the rail fasteners pulling out of the slab. The existence of the negative bending moment with the attendant tension in the top of the slab may be contributing to this problem.

A balanced design is one in which the bottom of the slab would be as highly stressed as the top, which implies that the maximum positive bending moment would be greater than the negative bending moment. A balanced

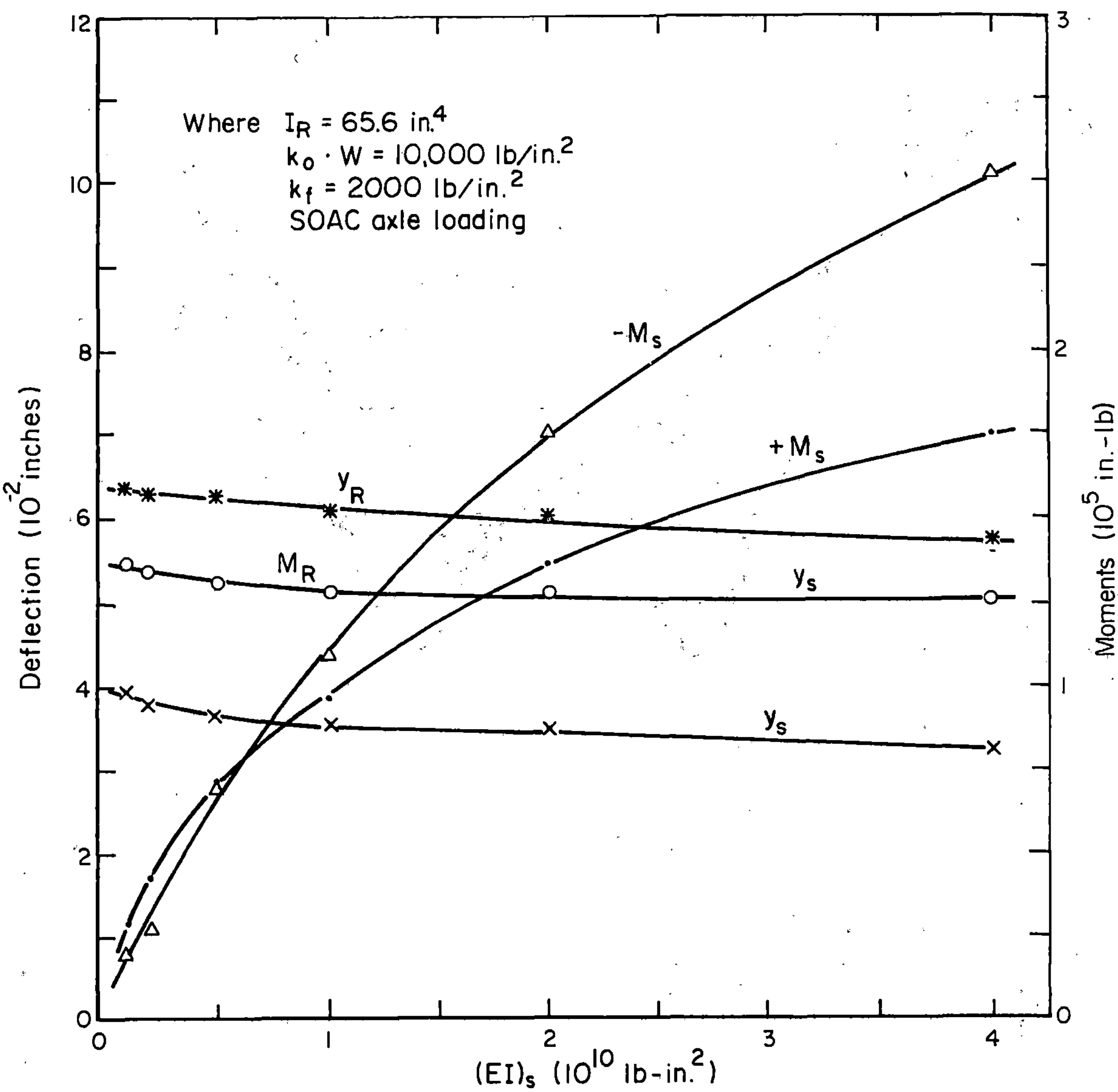


FIGURE 33. RELATIONSHIP OF TRACK STRUCTURE BENDING MOMENTS AND DEFLECTIONS TO SLAB BENDING STIFFNESS

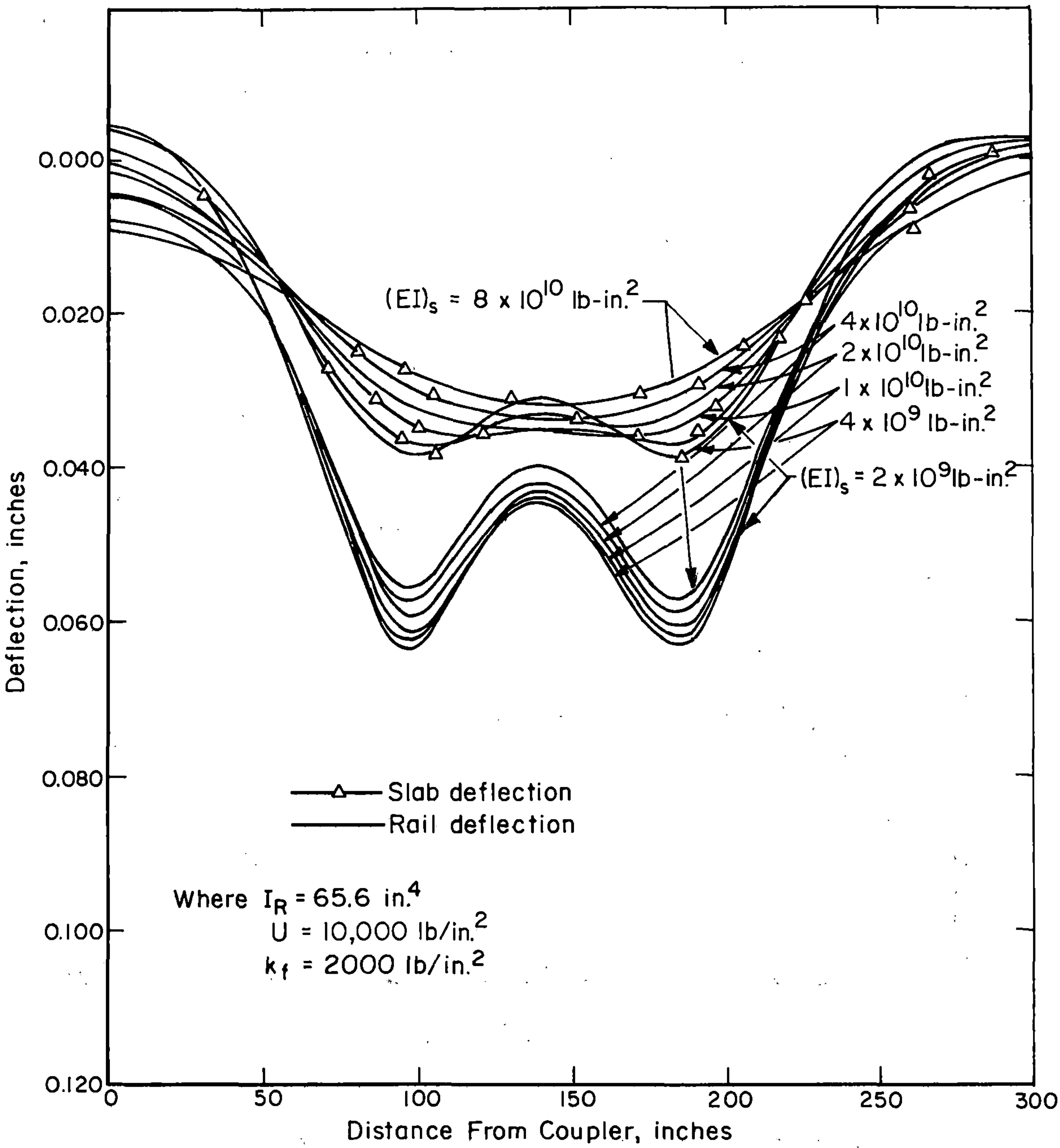


FIGURE 34. RELATIONSHIP OF RAILROAD SLAB DEFLECTIONS TO SLAB BENDING STIFFNESS

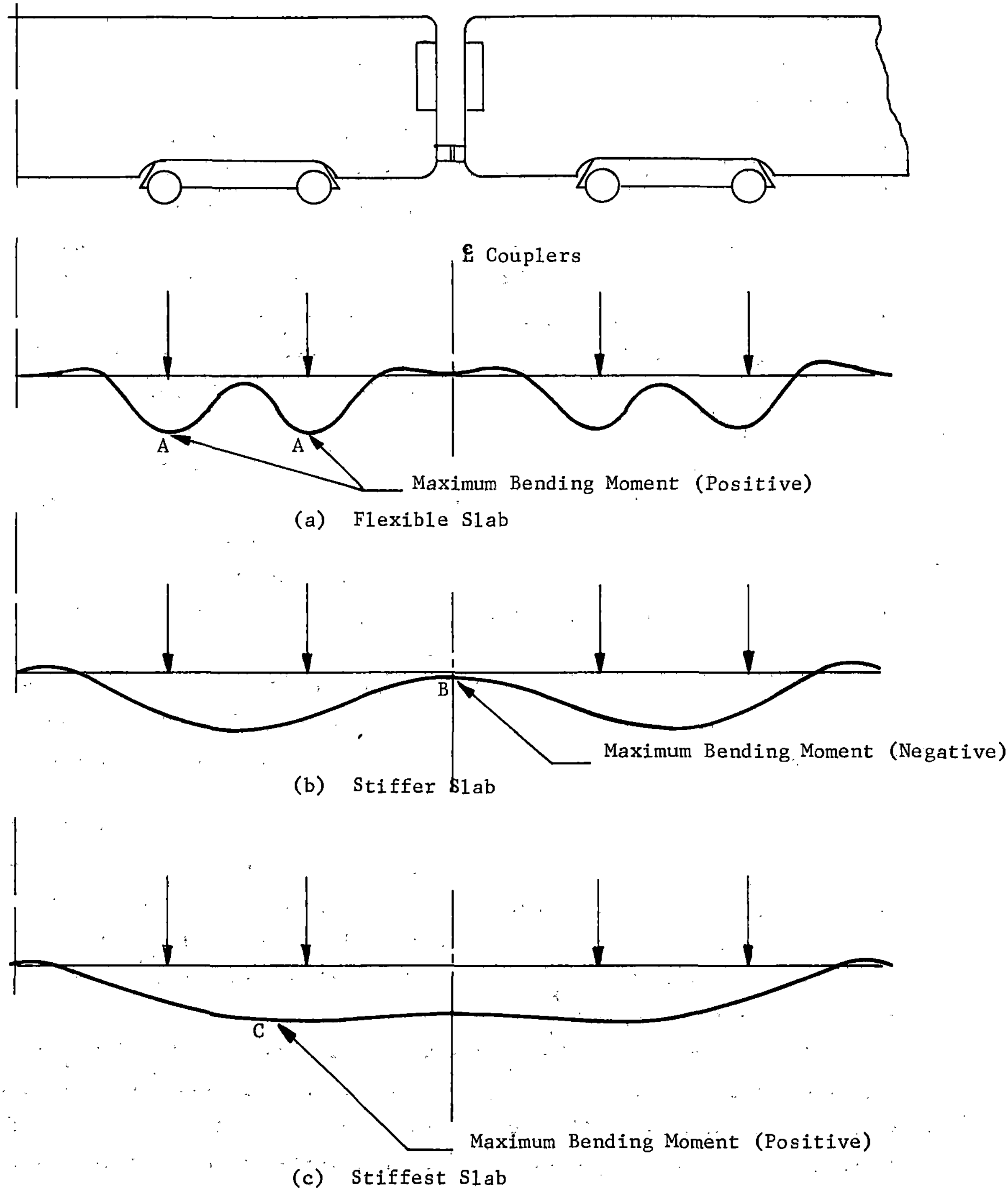


FIGURE 35. VARIATION OF SLAB DEFLECTION WITH INCREASING SLAB STIFFNESS

design may not be economically acceptable, however, and was not used in the Kansas Test Track design.

Figures 36 and 37 show that varying the modulus of the fastener affects the deflection of the rail and has almost no effect on the slab. The variation of the moment in the rail is unimportant because the stress in the rail is less than one third the allowable for all rail sizes considered. The important effects of reducing dynamic wheel-rail loads by reducing fastener stiffness are, of course, not shown by this static analysis.

Figures 38 and 39 show that varying the rail size has little effect on the deflections and negative bending moments for the slab size considered. The positive bending moment is reduced by increasing rail size, but this decrease will probably not be important since the negative bending moment in the slab will be the limiting design consideration for most slab type track.

Using the results of the parameter study, a number of design parameters were chosen. No attempt was made to evaluate initial construction costs in this preliminary design phase, but rather the basis of comparison was to equal or exceed the Metro track performance.

119-pound rail was selected because of the relatively thicker head which should provide a longer wearing life. The 119-pound rail will also help to bridge any discontinuities caused by cracks or joints. It was assumed that continuously welded rail (CWR) will be used.

As discussed in the section on rail fasteners, the fastener should have a low modulus. The critical design values will be based on dynamic considerations - particularly allowable gauge spread, and attenuation of wheel-rail dynamic loads. Although rail stress increases with a softer fastener, the rail stress due to car loads is only about one-third allowable, and therefore the effect is negligible.

The slab stiffness should be kept high in order to spread the load on the subgrade, thereby reducing soil pressure. The soil pressure is also inversely proportional to the slab width. For initial design calculations, a value of 9 feet was assumed. This width helps to prevent a center bound condition, and also gives a margin of safety to protect the subgrade from excessive loads if part of the slab loses some subgrade support. Moreover, the wide slab moves the edges of the slab further from the loaded areas, and therefore reduces edge stresses in the slab.

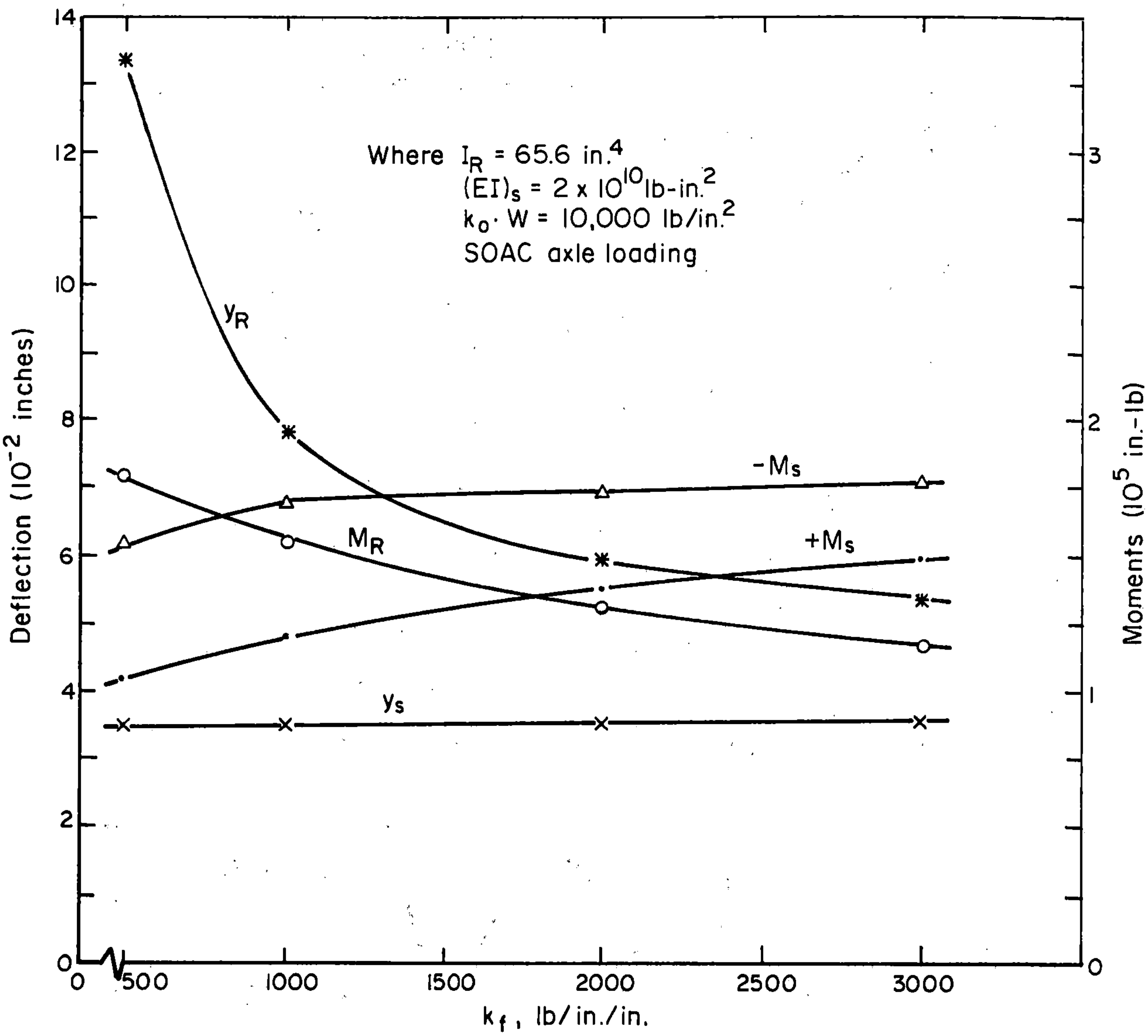


FIGURE 36. RELATIONSHIP OF TRACK STRUCTURE BENDING MOMENTS AND DEFLECTIONS TO RAIL FASTENER STIFFNESS

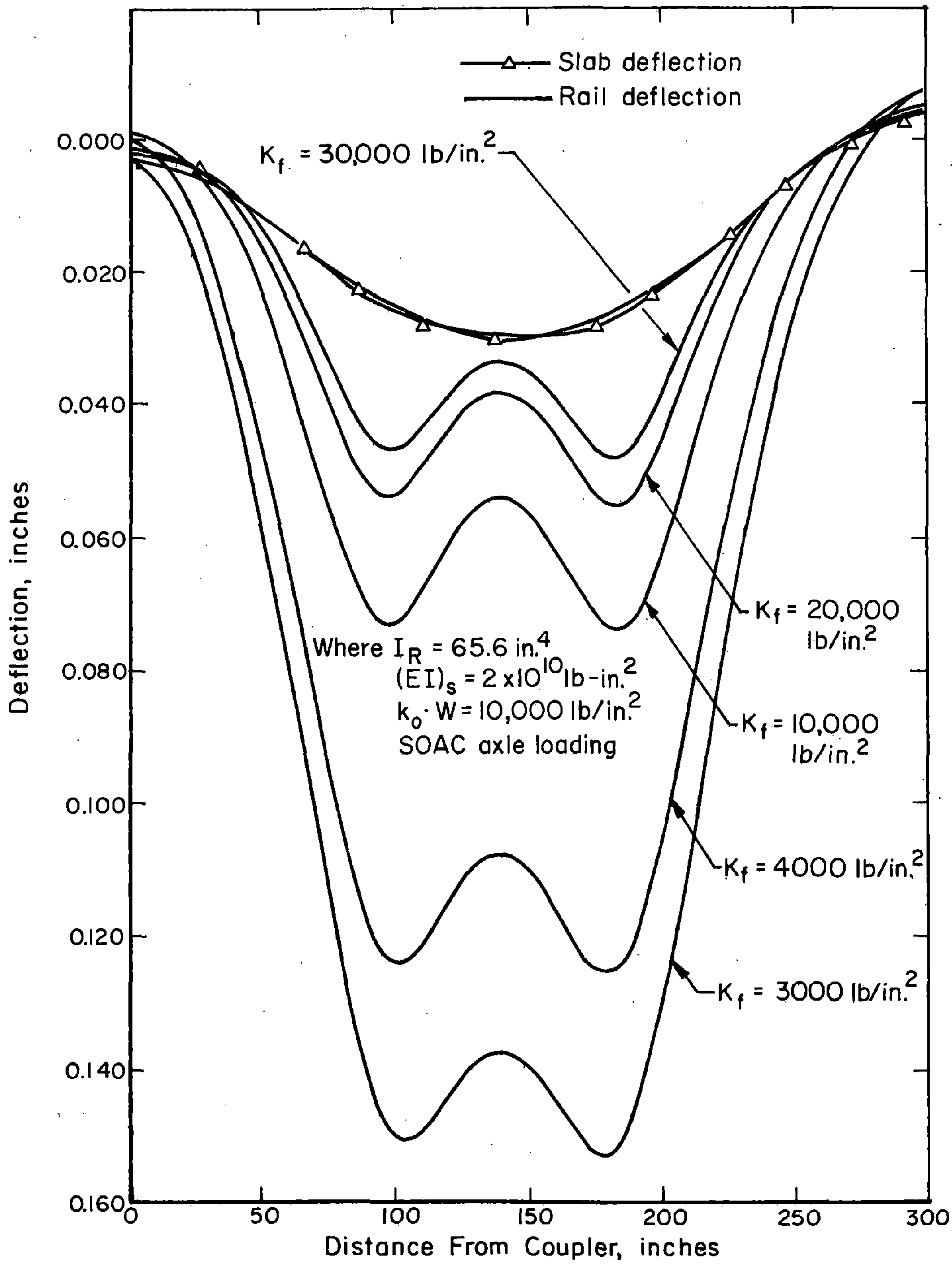


FIGURE 37. RELATIONSHIP OF RAIL AND SLAB DEFLECTIONS TO RAIL FASTENER STIFFNESS

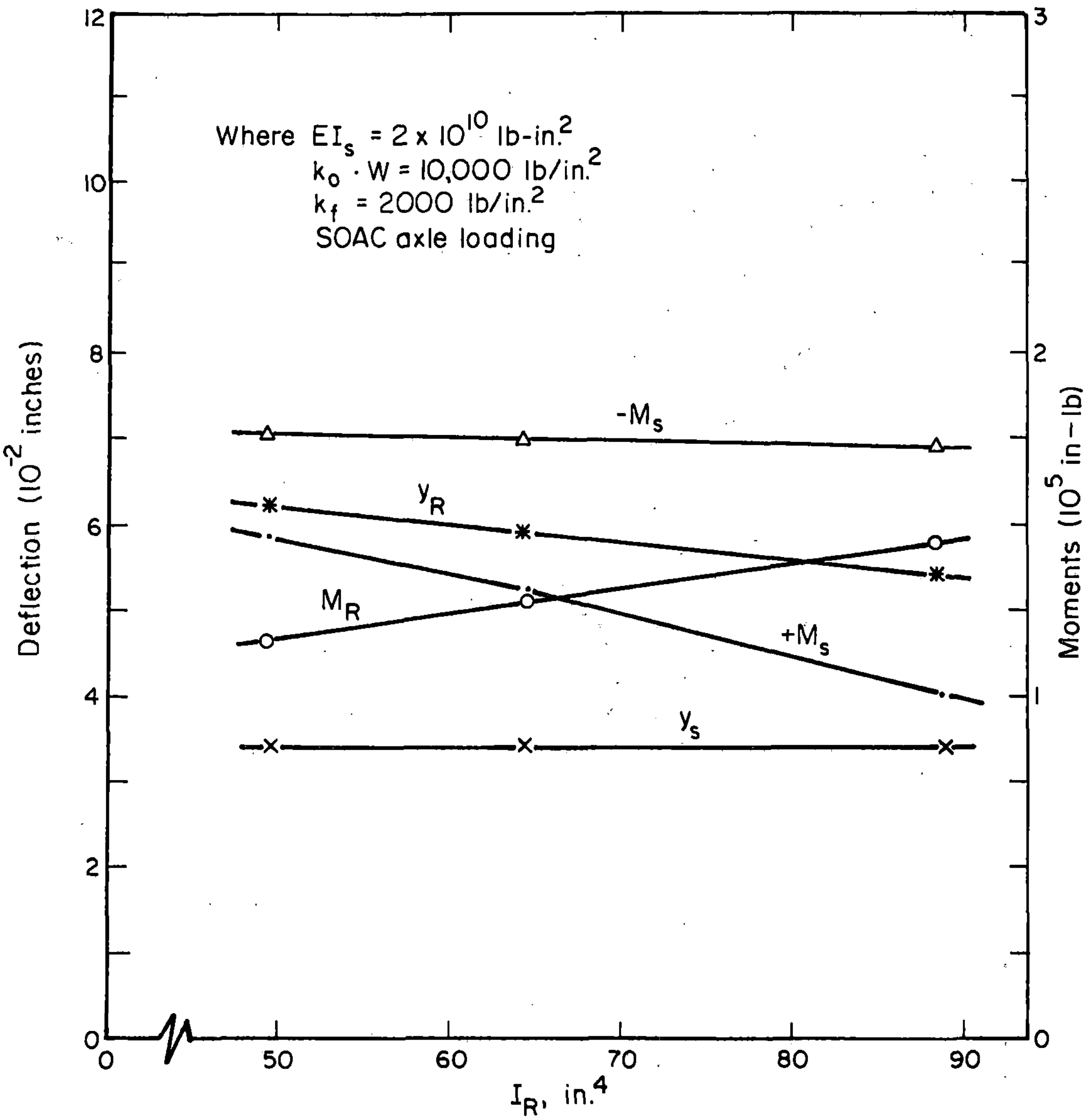


FIGURE 38. RELATIONSHIP OF TRACK STRUCTURE BENDING MOMENTS AND DEFLECTIONS TO RAIL BENDING STIFFNESS

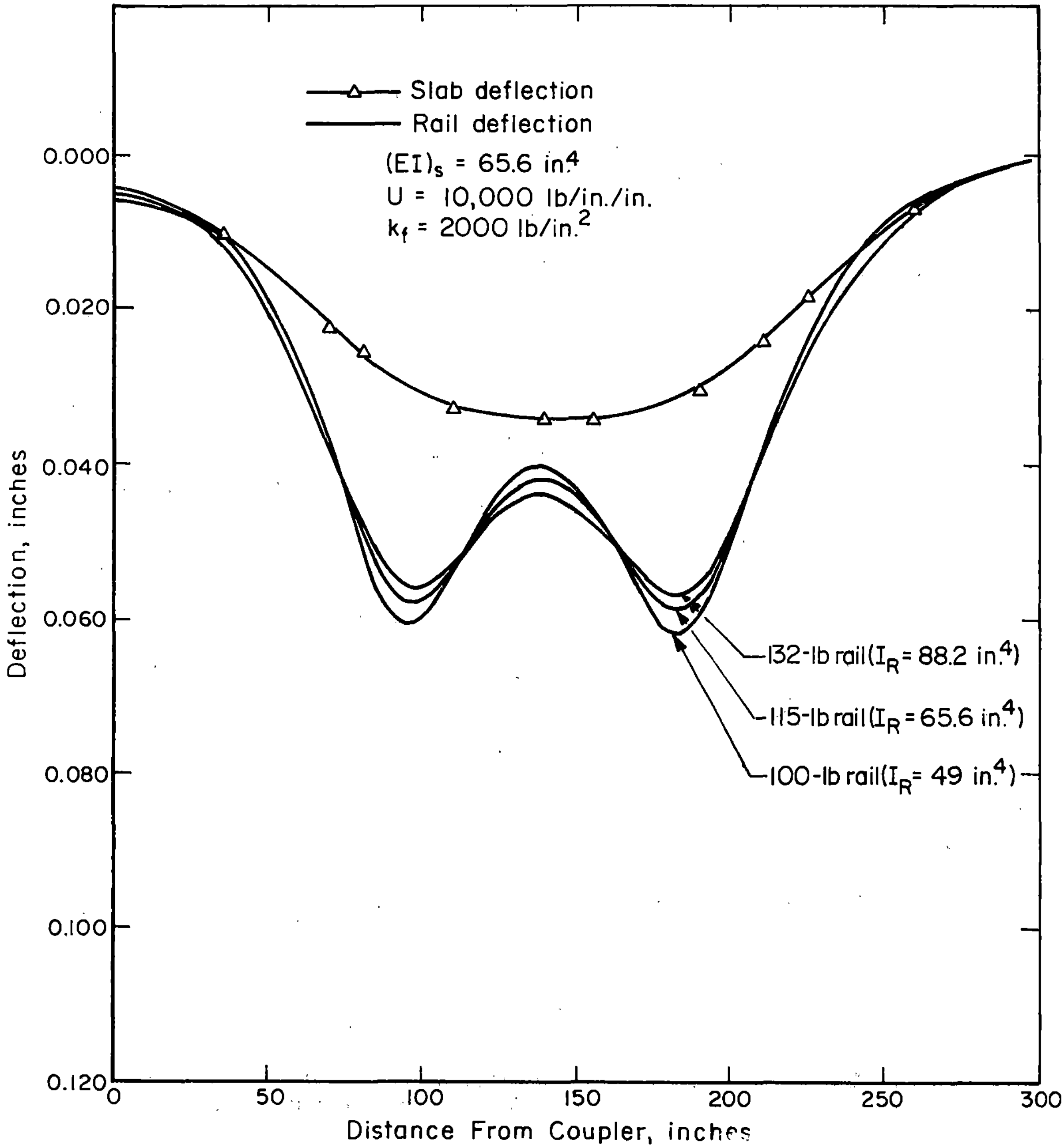


FIGURE 39. RELATIONSHIP OF RAIL AND SLAB DEFLECTIONS TO RAIL BENDING STIFFNESS

Runs were made using a range of slab-bending stiffness, and based on the criterion of reducing the pressure profile to one pulse per truck, the required bending stiffness of the slab was determined to be about 2×10^{10} lb-in.^{2} (see Figure 40). This does not eliminate all of the higher frequency components, but it gives a profile in which there is no slab bearing pressure reduction between the two axles on a SOAC truck.

If possible, the subgrade should be prepared so that a nominal k of 200 lb/in^2/in. is obtained. A peak subgrade pressure of 10 psi was chosen as the design allowable, and correspondingly the subgrade must have a bearing strength of at least 10 psi. For installation at the HSGTC at Pueblo, Colorado, concrete or bituminous stabilization of the subgrade similar to the UTACV stabilization should be seriously considered (see reference 34).

The following preliminary calculations were made for a concrete slab 40 feet long, with only temperature reinforcing. An uncracked section was assumed for design purposes. Note that this is different than the Kansas Test Track design, in which a cracked section (concrete takes no tension) was assumed. (This and many other aspects of the final reinforced concrete slab design are well beyond the scope of the present study.) In order to calculate the thickness needed to obtain a bending stiffness of 2×10^{10} lb-in.^{2}, assume

(1) The effect of the reinforcing can be ignored.

(2) The slab is rectangular in cross section.

(3) The slab is 9 feet wide

Then

$$I = \frac{bh^3}{12}$$

where b = width of slab = 108 in.

h = depth of slab (in.)

I = moment of inertia of slab (in.^{4})

Young's modulus E of concrete is assumed to be 3×10^6 psi. For an EI of 2×10^{10} lb/in.^{2}, the required depth of the slab (h) must be 9.05 inches. The rounded-off value of 9.0 inches will be used.

The maximum moment for the chosen value of EI (run shown in Figure 40) was calculated to be 110,000 in-lb. The maximum tensile stress was then calculated from the following equation:

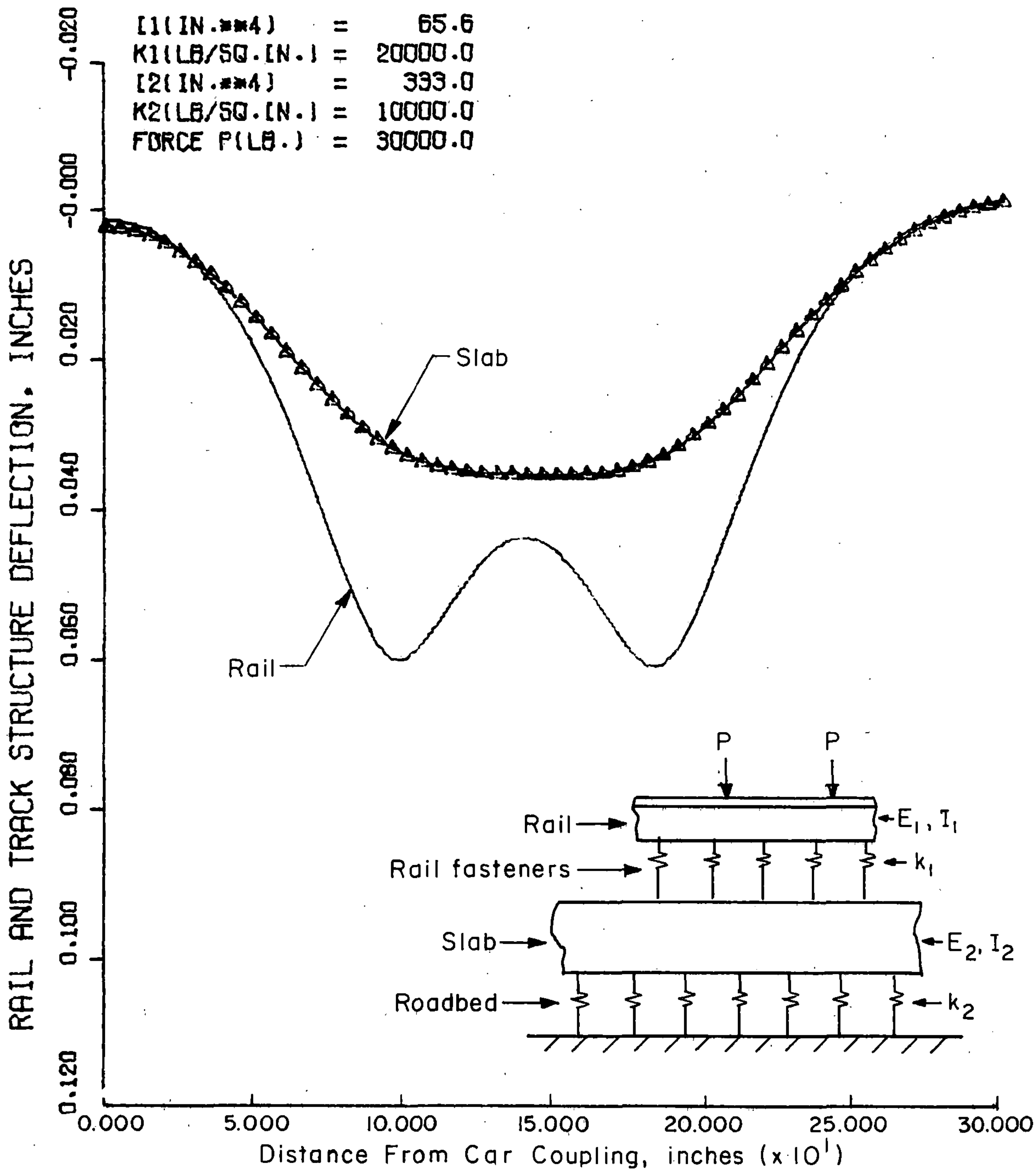


FIGURE 40. TRACK DEFLECTIONS

$$f_s = \frac{Mc}{I}$$

where

f_s = tensile stress in concrete

M = maximum bending moment in slab = .110 x 10^6 in-lb.

c = distance to extreme fiber = 4.5 in.

I = 6667 $in.^4$.

The stress due to bending is therefore 74.5 psi.

As discussed earlier, Westergaard's equations for the stresses due to warping make use of an influence length

$$\ell = \sqrt[4]{\frac{Eh^3}{12\ (1-\mu^2)\ k}} \tag{19}$$

where

μ = Poisson's ratio of concrete (assumed to be .15)

k = subgrade modulus (assumed to be 200 $lb/in.^3$)

The calculated value of ℓ is equal to 31.1 inches.

The interior stresses in the longitudinal direction are

$$\sigma_x = \frac{E\ \epsilon_t\ \Delta_t}{2} \left(\frac{C_x + \mu C_y}{1 - \mu^2}\right) = 228 \text{ psi} \tag{21}$$

where

σ_x = longitudinal interior tensile stress

ϵ_t = volume coefficient of expansion (assumed to be 5 x 10^{-6} in/in/$^\circ$F)

Δ_t = temperature differential (assumed to be 3°F per inch)

C_x and C_y are influence coefficients from Figure 25.

The longitudinal edge stress is

$$\sigma_{xe} = \frac{C_x E\ \epsilon_t\ \Delta_t}{2} = 212 \text{ psi} \tag{20}$$

The stress due to friction between the expanding slab and the subgrade is

$$\sigma_c = \frac{W\ L\ f}{24\ h} = 22.9 \text{ psi} \tag{23}$$

where

W = weight of slab (psf) 110 $lbs/ft.^3$ (3/4 ft.)

L = length of slab (ft.) = 40 ft.

h = depth of slab (in.) = 9 inches

f = average coefficient of subgrade resistance (assume 1.5).

The expansion and contraction of the slab is also restrained by the continuous welded rail and fasteners. The rail will not deflect longitudinally except near the joints. Therefore, the stress induced in the slab can be calculated by assuming the slab to deflect unrestrained by friction. The force on the slab is a maximum in the center of the slab.

The force on the center of the slab is

$$F = \sum_{1}^{i} 2\, \Delta_i\, k_\ell \approx 2\,(1/4\ N)\, k_\ell\, \epsilon_t\, (\Delta_t)$$

where

Δ_i = longitudinal deflection of i^{th} pair of fasteners

k_ℓ = longitudinal spring rate of fasteners (assumed to be 1×10^4 lb/in.)

L = length of slab = 40 ft.

N = number of fasteners on a slab = 34

ϵ_t = linear coefficient of expansion

Δ_t = temperature change from installation temp. (assumed $65^{o}F$)

The calculated value of F is 673 pounds, which, if assumed to act over the entire cross section, gives a longitudinal tensile (or compressive) stress of 0.7 psi.

These stresses are summarized in Table 3.

TABLE 3. TENSILE STRESSES IN A CONCRETE SLAB 9" x 9' x 40'

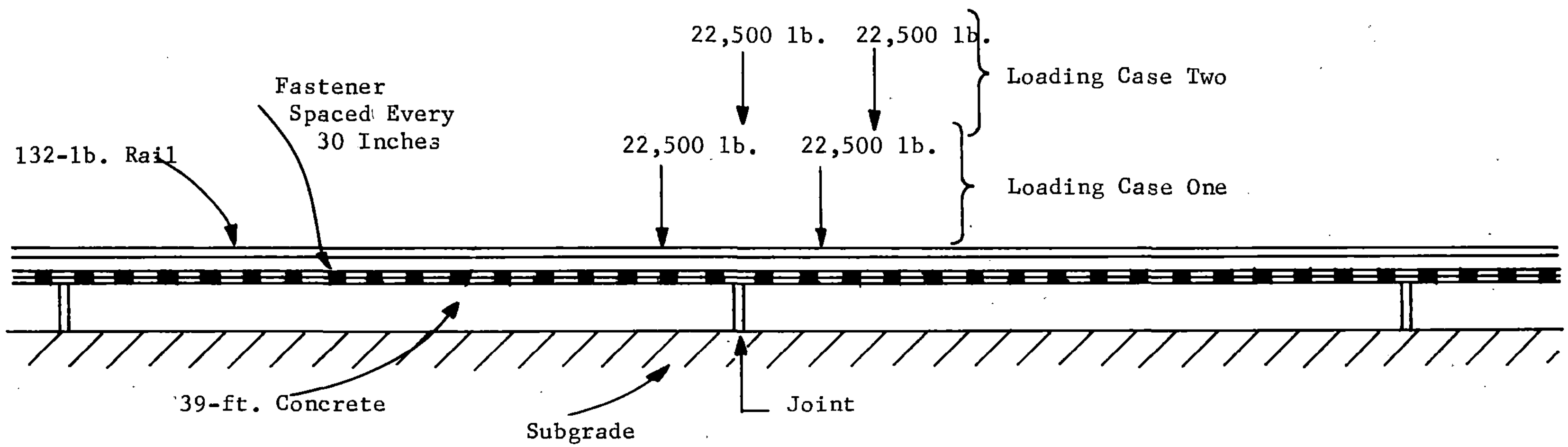
Origin of Stress	Magnitude (psi)
Vehicle Loading	74.3
Warping	228.0
Friction	22.9
Rail Fastener	0.7
Total Stress	326 psi

All of the previous analyses using the beam-beam program assumed the slab to be continuous. The track will have joints which disrupt the continuity, however. A computer analysis of the effect of joints, previously developed by Battelle [11], is described in the following paragraphs. Although the parameters are all within the range of interest considered, the figure is presented to give a qualitative rather than a quantitative picture of the joint. Additional analyses using models of this type would be necessary to obtain quantitative results of the deflections and moments in the proposed track structure.

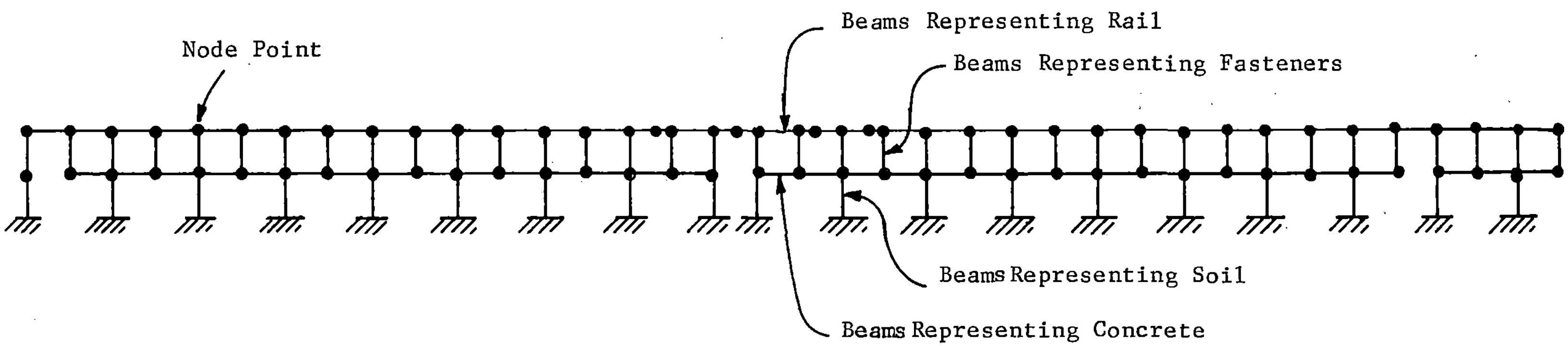
The model used to represent the earlier track structure is shown in Figure 41. A total of 97 node points and 129 beam elements were used in order to obtain a representation of more than two 39-foot slab sections and the three corresponding joints. The fasteners have a vertical spring rate of 750,000 lb/in. and are spaced on 30-inch centers. With the wide bearing area of the slab assumed to be about 100 inches, the foundation stiffnesses of 2400 and 12,000 lb/in/in. correspond to moduli of subgrade reaction of 24 lb/in.^{3} and 120 lb/in.^{3}, respectively. The model is of 132-pound rail and a slab with an EI of 4.1×10^{10} lb/in.^{2}.

The deflection curves are shown in Figure 42. The stiffness of the track structure is only slightly lower at the joint, as indicated by the deflection of the rail. Figure 42c is the deflection curve of a continuous track for comparison.

Shear ties at the joint would eliminate the relative deflections of the slabs and help alleviate the problems of infiltration into the joints and subgrade pumping.



(a) Slab-Type Track Structure



(b) Digital Computer Model of Slab-Type Track Structure

FIGURE 41. REPRESENTATION OF SLAB-TYPE TRACK STRUCTURE ON DIGITAL COMPUTER (From Reference 11, page 77)

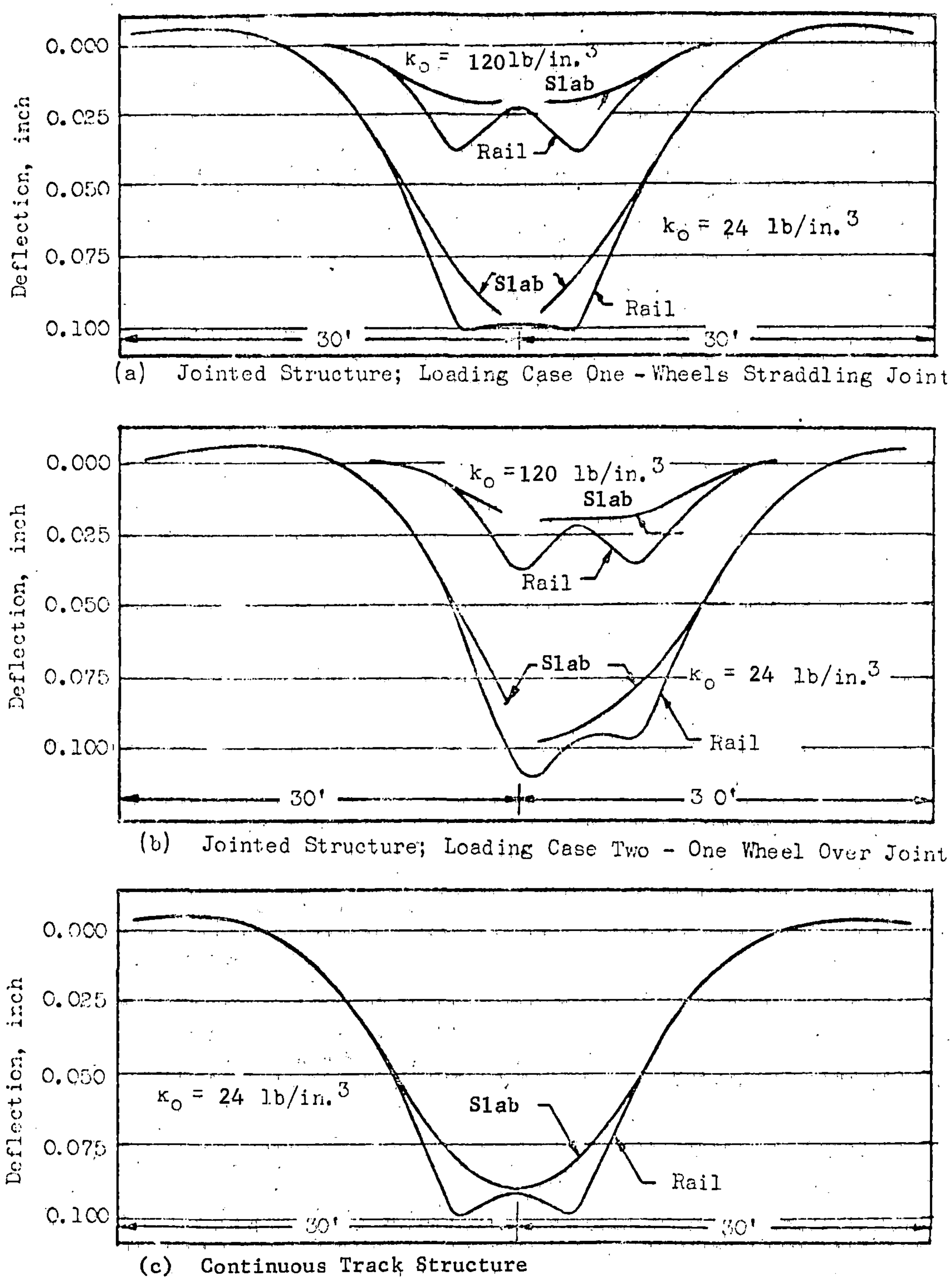


FIGURE 42. RAIL AND SLAB DEFLECTION FOR SLAB-TYPE TRACK STRUCTURES WITH AND WITHOUT JOINTS

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APPENDIX A

BEAM-ON-BEAM MODEL OF SLAB TRACK

Model

The slab supported track can be modeled by a continuous beam (rail) resting on an elastic foundation (rail fasteners) which is in turn supported by another beam (slab) on an elastic foundation (roadbed). The model is shown in Figure A-1.

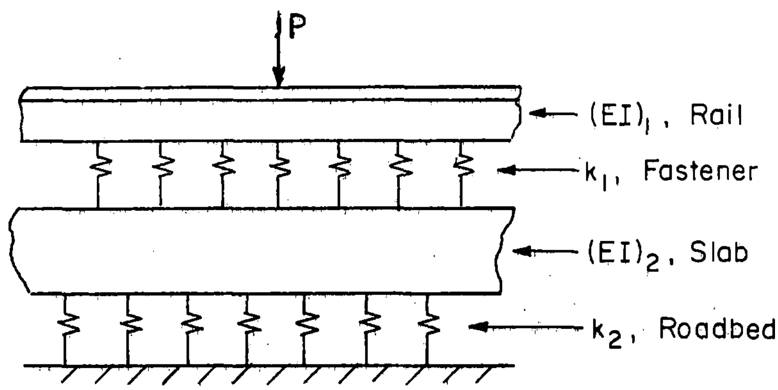

FIGURE A-1. MODEL USED TO REPRESENT SLAB-TYPE TRACK STRUCTURE

It is assumed that the upper and lower levels have the respective support stiffnesses k_1 and k_2 (lbs/in^2), that the entire system is resting on a rigid base, and that the bending rigidities of the rail and of the slab are $(EI)_1$ and $(EI)_2$, respectively.

The deflections of the rail and slab are denoted by y_1 and y_2, respectively. The wheel loads, P, are assumed as point loads. SOAC axle spacings were used for all runs; the leading axle is 95" from the coupler centerline, and the truck wheelbase is 90".

The governing differential equations are*

$$(EI)_1 \frac{d^4 y_1}{dx^4} = -k_1 (y_1 - y_2) = p_1 \qquad \text{(A-1)}$$

$$(EI)_2 \frac{d^4 y_2}{dx^4} = -(k_1 + k_2) y_2 + k_1 y_1 = (p_2 - p_1) \qquad \text{(A-2)}$$

where p_1 is the distributed pressure under the rail and p_2 the distributed pressure under the slab. The solutions to these differential equations can be obtained, and the general solution then applied to the particular case where both the rail and slab are of unlimited length and the rail is subjected to a concentrated load at x = o. For this case the condition at x = ±∞ is that the deflections and all their derivatives in both beams must vanish. The solution for vertical deflections y and flexural bending moments M are of the following form:

Upper beam:

$$y_1 = \frac{P}{16(EI)_1 \beta} \left[D_1 \frac{e^{-\lambda_1 x}}{\lambda_1^3} (\cos\lambda_1 x + \sin\lambda_1 x) - D_2 \frac{e^{-\lambda_2 x}}{\lambda_2^3} (\cos\lambda_2 x + \sin\lambda_2 x) \right], \qquad \text{(A-3)}$$

$$M_1 = \frac{P}{8\beta} \left[D_1 \frac{e^{-\lambda_1 x}}{\lambda_1} (\cos\lambda_1 x - \sin\lambda_1 x) - D_2 \frac{e^{-\lambda_2 x}}{\lambda_2} (\cos\lambda_2 x - \sin\lambda_2 x) \right], \qquad \text{(A-4)}$$

(*) The derivation of solutions may be found in Hetenyi, M., "Beams on Elastic Foundation", U. of Michigan Press, 1946, Chapter 10.

Lower beam:

$$y_2 = - \frac{P}{16(EI)_2\beta} \frac{k_1}{EI_1} \left[\frac{e^{-\lambda_1 x}}{\lambda_1^3} (\cos\lambda_1 x + \sin\lambda_1 x) - \frac{e^{-\lambda_2 x}}{\lambda_2^3} (\cos\lambda_2 x + \sin\lambda_2 x) \right], \tag{A-5}$$

$$M_2 = - \frac{P}{8\beta} \frac{k_1}{(EI)_1} \left[\frac{e^{-\lambda_1 x}}{\lambda_1} (\cos\lambda_1 x - \sin\lambda_2 x) - \frac{e^{-\lambda_2 x}}{\lambda_2} (\cos\lambda_2 x - \sin\lambda_2 x) \right], \tag{A-6}$$

where

$$D_1 = \frac{k_1}{(EI)_1} - (\alpha - \beta), \qquad D_2 = \frac{k_1}{(EI)_1} - (\alpha + \beta);$$

$$\alpha = \frac{A}{2}, \qquad \beta = \sqrt{\frac{A^2}{4} - B};$$

$$A = \frac{k_1(I_1 + I_2) + k_2 I_1}{(EI)_1 I_2}, \qquad B = \frac{k_1}{(EI)_1} \frac{k_2}{(EI)_2};$$

and

$$\lambda_1 = \sqrt[4]{\frac{\alpha + \beta}{4}}, \qquad \lambda_2 = \sqrt[4]{\frac{\alpha - \beta}{4}}.$$

The formulas give the values of the unknown quantities along the beam to the right of the point of the application of the load ($x > 0$). The curves for y and M are even functions of x. From the solution above for a single concentrated load it is possible to derive, by superposition, any combination of concentrated or distributed loadings on the infinitely long upper beam.

Computer Program

Equations (A-3) through (A-6) have been programmed in FORTRAN for the CDC 6400 computer and the CALCOMP plotter. The following sections describe the use of the program.

Input

The input parameters, which are outlined below, are placed into the deck in the order indicated. Figure A-2 identifies some of the input parameters (see next page).

FIRST DATA CARD, FORMAT (8E 10.0)

Name of Parameter	Description	Units
AJ(1)	Distance from coupler to first axle load	Inches
AJ(2)	Distance from coupler to second axle load	Inches
AJ(3)	Distance from coupler to third axle load	Inches
AJ(4)	Distance from coupler to fourth axle load	Inches
P	Wheel load	Pounds

SECOND DATA CARD, FORMAT (4E 10.0, I3)

SI1	Moment of inertia of rail	In^4
SK1	Modulus of Fastener: $\frac{K_f}{\ell_f}$	Lbs/in of deflection/inch of rail
SI2	EI of Beam/3 x 10^7	In^4
SK2	Track Modulus	Lbs/inch of deflection/inch of rail
NUM	Run Number	None

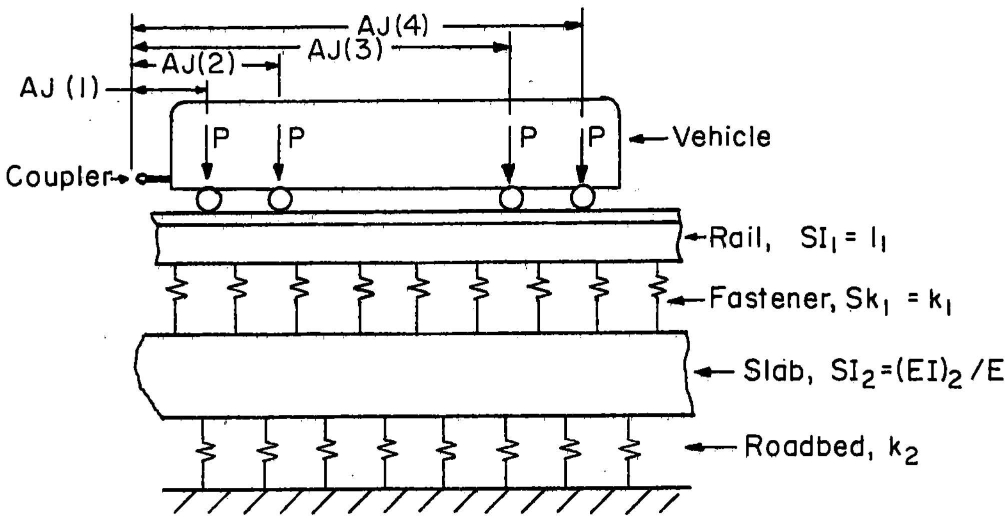

FIGURE A-2. COMPUTER PROGRAM INPUT VARIABLES

Output

The program outputs the input data, the deflection of slab and rail, and the bending moments in the slab and rail. The deflections and moments are printed for distances from 0 to 300 inches away from the coupler in 10 inch increments.

The plot routine outputs the deflection curves of the rail and slab for 0 - 300 inches from the coupler.

Sample Output

The following two pages contain sample printed and plotted outputs for Run Number 34.

RUN NUMBER = 34

AJ(1) = -95.0 AJ(2) = -185.0 AJ(3) = 95.0 AJ(4) = 185.0

P(LB.)= 30003.0 I1(IN.**4)= 65.6 K1(LB./SQ.IN.)= 20000.0 I2(IN.**4)= 667.0 K2(LB./SQ.IN.)= 10000.0

DISTANCE(IN.)		DEFLECTION(IN.)		BENDING MOMENT(IN.LB.)		SHEAR(LB.)	
X		Y1	Y2	M1	M2	Q1	Q2
	AZ						
	95.0	0.355E-02	0.416E-02	-0.824E+04	-0.780E+05		
	185.0	-0.121E-02	-0.117E-02	-0.102E+04	-0.926E+04		
	-95.0	0.355E-02	0.416E-02	-0.824E+04	-0.780E+05		
	-185.0	-0.121E-02	-0.117E-02	-0.102E+04	-0.926E+04		
0.0		0.467E-02	0.597E-02	-0.185E+05	-0.174E+06		
	105.0	0.213E-02	0.251E-02	-0.630E+04	-0.735E+05		
	195.0	-0.111E-02	-0.107E-02	-0.602E+03	-0.516E+04		
	-85.0	0.540E-02	0.620E-02	-0.114E+05	-0.770E+05		
	-175.0	-0.126E-02	-0.124E-02	-0.151E+04	-0.145E+05		
10.0		0.515E-02	0.641E-02	-0.198E+05	-0.170E+06		
	115.0	0.103E-02	0.122E-02	-0.513E+04	-0.658E+05		
	205.0	-0.972E-03	-0.931E-03	-0.268E+03	-0.205E+04		
	-75.0	0.793E-02	0.862E-02	-0.161E+05	-0.682E+05		
	-165.0	-0.124E-02	-0.123E-02	-0.205E+04	-0.208E+05		
20.0		0.664E-02	0.769E-02	-0.236E+05	-0.157E+06		
	125.0	0.190E-03	0.266E-03	-0.436E+04	-0.564E+05		
	215.0	-0.323E-03	-0.785E-03	-0.161E+02	0.220E+03		
	-65.0	0.111E-01	0.114E-01	-0.224E+05	-0.493E+05		
	-155.0	-0.111E-02	-0.111E-02	-0.262E+04	-0.284E+05		
30.0		0.934E-02	0.975E-02	-0.294E+05	-0.134E+06		
	135.0	-0.425E-03	-0.410E-03	-0.376E+04	-0.465E+05		
	225.0	-0.672E-03	-0.640E-03	0.161E+03	0.178E+04		
	-55.0	0.155E-01	0.144E-01	-0.291E+05	-0.185E+05		
	-145.0	-0.849E-03	-0.853E-03	-0.319E+04	-0.370E+05		
40.0		0.135E-01	0.125E-01	-0.359E+05	-0.100E+06		
	145.0	-0.849E-03	-0.853E-03	-0.319E+04	-0.370E+05		
	235.0	-0.539E-03	-0.504E-03	0.274E+03	0.276E+04		
	-45.0	0.214E-01	0.175E-01	-0.333E+05	0.242E+05		
	-135.0	-0.425E-03	-0.410E-03	-0.376E+04	-0.465E+05		
50.0		0.195E-01	0.157E-01	-0.400E+05	-0.566E+05		

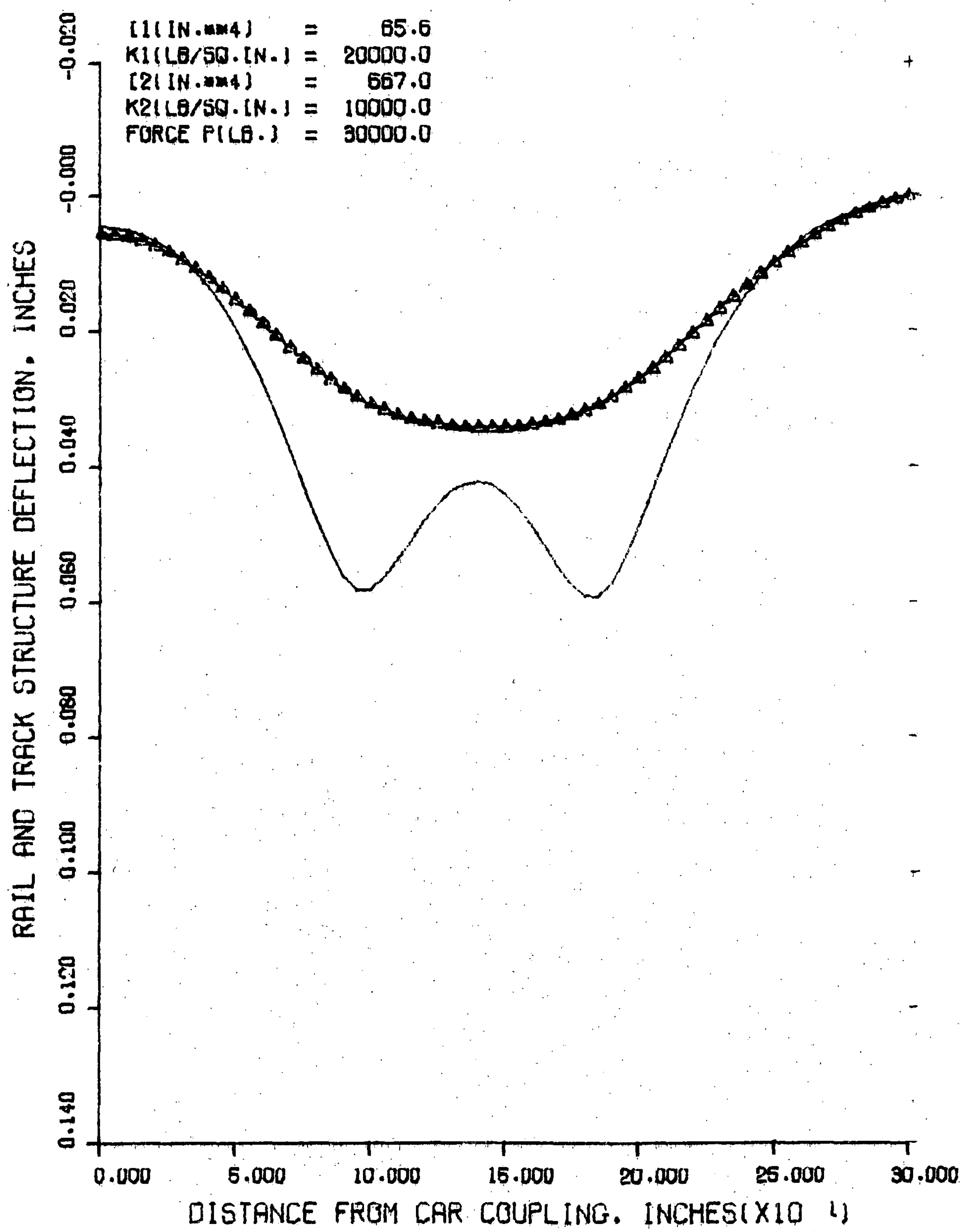

FIGURE A-3.TRACK DEFLECTIONS FOR RUN NO.34

PROGRAM LISTING

```
      PROGRAM TRACK(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PLOT,TAPE99=
     $PLOT)
      DIMENSION X(100),AJ(4),Y1S(100),Y2S(100)
      READ 100,AJ(1),AJ(2),AJ(3),AJ(4),P
  100 FORMAT(8E10.0)
C  CALCULATE EVENLY SPACED X-COORDINATES
      X(1)=0.
      DELTX=5.
      DO 10 I=2,61
      X(I)=X(I-1)+DELTX
   10 CONTINUE
      E=30.E6
    1 READ 200,SI1,SK1,SI2,SK2,NUM
  200 FORMAT(4E10.0,I2)
      IF (EOF,5) 99, 101
  101 CONTINUE
      PRINT 201.NUM
  201 FORMAT(1H1,3X,12HRUN NUMBER =I3//)
      PRINT 301,AJ(1),AJ(2),AJ(3),AJ(4)
  301 FORMAT(2X,7HAJ(1) =F7.1,5X,7HAJ(2) =F7.1,5X,7HAJ(3) =F7.1.5X,7HAJ(
     $4) =F7.1/)
      PRINT 300,P,SI1,SK1,SI2,SK2
  300 FORMAT(1H0,7HP(LB.)=F9.1,4X,11HI1(IN.**4)=F9.1,4X,15HK1(LB./SQ.IN.
     $)=F9.1,4X,11HI2(IN.**4)=F9.1,4X,15HK2(LB./SQ.IN.)=F9.1//)
      PRINT 400
  400 FORMAT (1X,13HDISTANCE(IN.),20X,15HDEFLECTION(IN.),20X,22HBENDING
     $MOMENT(IN.LB.),20X,10HSHEAR(LB.)//)
      PRINT 500
  500 FORMAT (5X,1HX,26X,2HY1,13X,2HY2,19X,2HM119X,2HM2,19X,2HQ1,10X
     $,2HQ2//)
      PRINT 800
  800 FORMAT(20X,2HAZ)
      A=(SK1*(SI1+SI2)+SK2*SI1)/(E*SI1*SI2)
      B=SK1*SK2/(SI1*SI2*E*E)
      ALPHA=A/2.
      BETA=SQRT((A*A/4.)-B)
      D1=(SK1/(E*SI1))-(ALPHA-BETA)
      D2=(SK1/(E*SI1))-(ALPHA+BETA)
      AMDA1=((ALPHA+BETA)/4.)**.25
      AMDA2=((ALPHA-BETA)/4.)**.25
  600 FORMAT(2X,F6.1,20X,E10.3,6X,E10.3,13X,E10.3,11X,E10.3//)
  700 FORMAT(17X,F6.1,10X,E10.3,6X,E10.3,13X,E10.3,11X,E10.3)
C BEGIN DO LOOP FOR X STATIONS ALONG TRACK
      K=1
      DO 2 I=1,61
      Y1SUM=0
      Y2SUM=0
      BM1SUM=0
      BM2SUM=0
C BEGIN DO LOOP FOR INFLUENCE AT X STATION FROM POINT LOADS
      DO 3 J=1,4
      AZ=X(I)-AJ(J)
      Z=ABS(AZ)
      EXAMX1=EXP(-AMDA1*Z)
      EXAMX2=EXP(-AMDA2*Z)
```

```
      CSAMX1=COS(AMDA1*Z)
      CSAMX2=COS(AMDA2*Z)
      SNAMX1=SIN(AMDA1*Z)
      SNAMX2=SIN(AMDA2*Z)
      COMP1=(EXAMX1/AMDA1**3)*(CSAMX1+SNAMX1)
      COMP2=(EXAMX2/AMDA2**3)*(CSAMX2+SNAMX2)
      COMP3=(EXAMX1/AMDA1)*(CSAMX1-SNAMX1)
      COMP4=(EXAMX2/AMDA2)*(CSAMX2-SNAMX2)
      Y1=(P/(16.*E*SI1*BETA))*(D1*COMP1-D2*COMP2)
      Y2=-(P*SK1/(16.*E*SI2*BETA*E*SI1))*(COMP1-COMP2)
      BM1=(P/(8.*BETA))*(D1*COMP3-D2*COMP4)
      BM2=-(P*SK1/(8.*BETA*E*SI1))*(COMP3-COMP4)
    8 Y1SUM=Y1SUM+Y1
      Y2SUM=Y2SUM+Y2
      BM1SUM=BM1SUM+BM1
      BM2SUM=BM2SUM+BM2
      Y1S(I)=Y1SUM
      Y2S(I)=Y2SUM
C   PRINT ANSWERS FOR 10 INCH INCREMENTS
      IF(I-K)3,13,3
   13 PRINT 700,AZ,Y1,Y2,BM1,BM2
    3 CONTINUE
      IF(I-K)2,14,2
   14 PRINT 600,X(I),Y1SUM,Y2SUM,BM1SUM,BM2SUM
      K=I+2
    2 CONTINUE
      CALL QIKSET(6.,0.,50.,8.,.14,-.02)
      CALL QIKPLT(X,Y1S,61,36H*DISTANCE FROM CAR COUPLING, INCHES*,45H*R
     $AIL AND TRACK STRUCTURE DEFLECTION, INCHES*,40H*FIGURE   TRACK DEF
     $LECTIONS FOR RUN NO.*)
      CALL PLOT(-7.0,1.0,-3)
      CALL QLINE(X,Y2S,-61,-2)
      CALL NUMBER(6.5,-1.,.22,NUM,0.,2HI2)
      CALL SYMBOL(0.2,8.2,.11,15HI1(IN.**4)     =,0.,15)
      CALL NUMBER(999.0,999.0,.11,SI1,0.,4HF9.1)
      CALL SYMBOL(0.2,8.0,.11,15HK1(LB/SQ.IN.) =,0.,15)
      CALL NUMBER(999.0,999.0,.11,SK1,0.,4HF9.1)
      CALL SYMBOL(0.2,7.8,.11,15HI2(IN.**4)     =,0.,15)
      CALL NUMBER(999.0,999.0,.11,SI2,0.,4HF9.1)
      CALL SYMBOL(0.2,7.6,.11,15HK2(LB/SQ.IN.) =,0.,15)
      CALL NUMBER(999.0,999.0,.11,SK2,0.,4HF9.1)
      CALL SYMBOL(0.2,7.4,.11,15HFORCE P(LB.)   =,0.,15)
      CALL NUMBER(999.0,999.0,.11,P,0.,4HF9.1)
      CALL PLOT(7.0,0.,-3)
      GO TO 1
C   MOVE PEN FROM PLOT AREA AND SIGN OFF
   99 CALL ENDPLT
      CALL EXIT
      END
```

APPENDIX B

REPORT OF INVENTIONS

This report contains a comprehensive review of reported work on rail track structure design, in particular, at-grade slab track. Suggestions are made for the preliminary design of an at-grade slab for possible installation at the Pueblo High Speed Ground Test Center. After a diligent review of the work performed under this contract, it was found that no new inventions or discoveries were made.