

C. L. ORTH'S FILE COPY Rail Vehicle Dynamics Model Validation

Office of Research and Development

400 7th St. S.W. Washington, D.C. 20590

FRA/ORD-81/52

August 1981

Final Report

Document is available to the U.S. public through the National Technical Information Service, Springfield, Virginia 22161

Steven E. Shladover Russel L. Hull

FILE COPY

NOTICE

This document is disseminated under the sponsorship of the U.S. Department of Transportation in the interest of information exchange. The United States Government assumes no liability for the contents or use thereof.

NOTICE

The United States Government does not endorse products or manufacturers. Trade or manufacturer's names appear herein solely because they are considered essential to the object of this report.

Technical Report Documentation Page

| 1. Report No. | 2. Government Accession No. | 3. Recipient's Catalog No. |
|--|--|--|
| FRA/ORD-81/52 | | |
| 4. Title and Subtitle | | 5. Report Date |
| RATI VEHTCLE DYNAMICS MODE | Τ. ΥΑΤ.ΤΡΔΤΤΟΝ | August 1981 |
| | | 6. Performing Organization Code |
| | | 8. Performing Organization Report No. |
| 7. Author's) Steven E. Shladover and Ru | ussell L. Hull | |
| 9. Performing Organization Name and Add SYSTEMS CONTROL INC (Vt) | ress | 10. Work Unit No. (TRAIS) |
| 1801 Page Mill Road Palo Alto, CA 94304 | 11. Contract or Grant No. DOT-FR-9050 | |
| | | 13. Type of Report and Period Covered |
| 12. Sponsoring Agency Name and Address U.S. DEPARTMENT OF TRANSPO Federal Pailroad Administr | RTATION | FINAL REPORT September 1979 - January 198 |
| Office of Research and Dev Washington, D.C. 20590 | elopment | 14. Sponsoring Agency Code RRD-11 |
| 15: Supplementary Notes | | |
| 16. Abstract | | |

The validation of mathematical models of rail vehicle dynamics using test data poses a number of difficult problems, which are addressed in this report. Previous attempts to validate rail vehicle models are reviewed critically, and experience gained in validating dynamic models of aircraft and marine vehicles using system identification methods is then applied to the formulation of a general procedure for validating rail vehicle dynamic models. The procedure is outlined, step by step, for application with existing test data and for use as part of a new model validation test program. An example of the application of the initial stages of the procedure is demonstrated using data from the Perturbed Track Tests (PTT) at Pueblo to validate a simple linear model of the forced vertical dynamics of a six-axle locomotive. Recommendations are offered for the conduct of future model validation efforts.

| 17. Key Words | 18. Distribution Statement | | | | |
|---|--|--|--|--|--|
| Model validation Rail vehicle modeling Rail vehicle testing | This document is available to the U.S. public through the National Technical Information Service, Springfield, VA 22161 | | | | |
| | | | | | |
| 19. Security Classif, (of this report) 20. Security Class | if. (of this page) 21. No. of Pages 22. Price | | | | |
| UNCLASSIFIED UNCLASSIFIED | SD 41 | | | | |

Form DOT F 1700.7 (8-72)

Reproduction of completed page authorized

i

METRIC CONVERSION FACTORS

| oł ··· | When You Know | Multiply by | To Find | о Солония — 1 | · · · · · · | - 2 | 4. 4 | • • • | Sec | | enti nulli 🗋 | (| n ' |
|--------|--|--------------------|-------------------------|------------------|---------------------------------------|------------|--|-----------------------|---------------------------------------|-------------|---------------|-------------|------------|
| | When You Know | | io ring | | | | | Symbol | When You Know | Multiply by | | | sym |
| | | | | Symbol | 8 | | | | | | | Sec. 14 | |
| | · · · . | ·. · · | | | - | - | 20 | | · · · · · · · · · · · · · · · · · · · | | - | | |
| | | LENGTH | | • , • | | | 10 | | millimator | 0.04 | in abox | | |
| | * ************************************ | | ••• • • | ` | <u> </u> | | | cm | centimeters | 0.4 | inches | | |
| | inches | •2.5 | centimeters | cm | 7 | [| | m | meters | 3.3 | feet | | ÷ . |
| · · | feet | ् 30 ् | centimeters | cm | 1 | | , *• • . | m km | meters kilometers | 1.1 | yards | | • |
| | yards miles | U.9 16 | meters | î m Ivez | · · · · · · - | : : | 17 | KIII | Kironingters | 0.0 | inua? | | |
| | 1111/22 | | Kilometers , , | KM. | | | ` | • | * بر | 1001 | | | • • • |
| | - | AREA | 12 - A - 1 | | · - | | 16 | 5 · · | | AREA | _ | | |
| | · · · | | • | · · | 6 | | | | square centimeters | 0 16 | souare inches | | |
| • | square inches | 6.5 | square centimeters | cm² | | | 19 | m ² | square meters | 1.2 5 | square yards | ÷ ; ; . | |
| | square feet | 0.09 | square meters | т ² | ÷ | | ······································ | km² | square kilometers | 0.4 | square miles | | |
| • | square yards | 0.8 | square meters | m ² | | | | ha | hectares (10,000 m ²) | 2.5 | acres | | ÷. |
| | | 2.0 | square kilometers | km≁ ha | - | | | | | | | | |
| | | , 0.4 | Tipetares . | | 5 | | | | Α | ASS (weight | 1 | · · · | · |
| | , | MASS (weight) | , _. | | <u> </u> | | | · · · | | | 1 | | |
| | | | | · | · | | ` | đ | arams | 0.035 | OUDCOS | | |
| | ounces | 28 | grams | 9 | • | | 11 | kg | kilograms | 2.2 | Dounds | - | |
| . • | short tons | 0.45 | Kilograms | , KG - | | S <u>-</u> | с÷., | t - | tonnes (1000 kg) | 1.1 | short tons | * | |
| | (2000 lb) | •••• | tonnea | • | · · · · · · · · · · · · · · · · · · · | | 10 . | · . | | | | · _ | |
| | 4 | VOLIME | | | · - | | · . | | | | | • | · |
| | . - | VOLUME | | | <u> </u> | | . 9 | | ÷ — | | ■ , `} | | · |
| | teaspoons | 5 | milliliters | ml . | - | | <u> </u> | mi | milliliters | 0.03 | fluid ounces | * | |
| | tablespoons | 15 | milliliters | ml | 3 | | 0 | _ I | liters | 2.1 | pints | | |
| | fluid ounces | 30 | milliliters | ml | _ | | 7 | | liters | 1.06 | quarts . | | |
| | cups | 0.24 | liters | H F S | | | • • | √ I m ³ | riters | 0.26 | gailons | | |
| | quarts - | 0.95 | liters | | | : | 6 | m ³ | cubic meters | 1.3 | cubic vards | | - |
| | gallons | 3.8 | liters | | - | | | | | | | - | |
| | cubic feet | 0.03 | cubic meters | m ³ | 2 | | 5 | ÷., | TEMP | | exact) | | |
| | cubic yards | 0.76 | cubic meters | m ³ | · · · - | | | • | | | | | |
| | TEM | PERATURE (ex | act) | | | : : | 4 | • C | Celsius | 9/5 (then | Fahrenheit | | |
| | | | | · . · | | | | | temperature | add 32) | temperature | | |
| · . | Fahrenheit | 5/9 (after | Celsius | °C | - | | 3 | | · · · · · · · · · · · · · · · · · · · | | | | - |
| | temperature. | subtracting | temperature | | 1 | | | | 0 | • | | 1 | |
| | <i>6</i> , | 32) | | ų i – | | | Z | | | 2 | 98.6 | 160 | |
| 2.54 | 4 cm (exectly), For o | ther exact convers | ions and more detail ta | bles see | | | 1 | .' | - JILLILL | | | ĨL | 1 |

ACKNOWLEDGEMENTS

This work was performed under the technical supervision of Dr. N. Thomas Tsai, the FRA Task Monitor. His guidance and suggestions throughout the course of the project are much appreciated. The consulting assistance of Professor Larry Sweet of Princeton University during the early stages of work was very helpful.

This project was initiated at Systems Control, Inc. (Vt) by Dr. David L. Klinger and was then continued by the authors of this report. Mr. David Vanacek did the computer programming, and Ms. Clare Walker directed the preparation of the technical report.

iii

EXECUTIVE SUMMARY

The purpose of this report is to derive and demonstrate the application of a generalized methodology for validating rail vehicle dynamics models. Much effort has been devoted in the past to testing and analytical (mathematical) modeling of the dynamics of rail vehicles, but these efforts have almost invariably been exerted by different groups. The result has been that virtually none of the models has been validated by experimental data.

Previous attempts to validate rail vehicle models using a variety of approaches were reviewed, and most were found to have suffered from inadequate test data because the tests were not designed for the purpose of validating models. Based on the problems encountered in these attempts and on experience gained in successfully validating models of aircraft and marine vehicles, some general principles or guidelines for model validation were then formulated and discussed.

Two separate, but closely related, step-by-step procedures for validating rail vehicle models were derived. The first was based on development of a new test program customized for model validation, while the second was designed for use with existing test data. Particular emphasis in both cases was assigned to the comparison between model predictions and test results and the adjustment of model parameters to improve agreement with the test results.

An example application of part of the methodology was demonstrated using data collected in the Perturbed Track Tests (PTT) at Pueblo, Colorado. The tests performed on tangent track containing piecewise linear profile perturbations were used to try to validate a simple six-degree-of-freedom linear model of the vertical dynamics of a six-axle locomotive. The preliminary steps of the validation procedure were demonstrated and comparisons between the test results and model predictions were illustrated for one case. The remaining steps of model adjustment and comparison for a variety of conditions were described but were not demonstrated.

i٧

TABLE OF CONTENTS

I

I١

| | INTR | OD UCTION | Page 1 |
|--|----------------------------------|--|--------------------|
| | 1.1 | Overview of The Report | 2 |
| I. | REVI | EW OF PREVIOUS WORK | 2 |
| 1-5 ¹⁻⁴ 20-5-5-5 20-5-5-5-5 | 2.1 | Traditional Model Validation | 2 |
| | | 2.1.1Subsystem Models2.1.2Vehicle Models | 2 2 |
| | 2.2 | System Identification Applications | - 5 |
| | 2.3 | General Studies | 7 |
| II. | GENE | RAL PRINCIPLES GOVERNING MODEL VALIDATION | 7 |
| | 3.1 3.2 3.3 3.4 3.5 | Terminology Application of Statistics To Model Validation Generality of Method Importance of Model Purpose Knowledge of Inputs | 7 8 9 9 |
| | 3.6 3.7 3.8 3.9 3.10 | Parameter Adjustments Range of Operating Conditions Nesting Validation Independent Data Sets | 9 9 10 10 |
| Z • | MODE | L VALIDATION PROCEDURES | 10 |
| · · · | 4.1 | Outlines of The General Validation Methodologies | 11 |
| د. م | | 4.1.1New Data Collection4.1.2Existing Test Data | 11 14 |
| · · · · · · · · · · · · · · · · · · · | 4.2 | Application To Rail Vehicle Dynamics Models | 15 |
| | | 4.2.1Analytical Methods4.2.2Dynamic Processes4.2.3Combination of Analytical Methods and Dynamic | 15 16 |
| · · · | | Processes | 17 |
| 5 | 4.3 | System Identification | 17 |

TABLE OF CONTENTS (Continued)

| V. APPL | CATION-TO-A VERTICAL DYNAMICS MODEL | 17 |
|------------|---|----------------------|
| 5.1 | Vertical Dynamics Model | 18 |
| | 5.1.1 Model Description | 18 18 |
| 5.2 | Data For Validation | 21 |
| | 5.2.1 Parameters For Model 5.2.2 Measurement Suitability 5.2.3 Performance Regimes 5.2.4 Limitations Of Data | 22 22 22 28 |
| 5.3 | Comparison Of Model Outputs And Test Results | 28 |
| | 5.3.1 Data Reconstruction 5.3.2 Responses 5.3.3 Validation Criteria 5.3.4 Range of Validity | 28 29 29 29 |
| VI. CONC | LUSIONS | 30 |
| REFERENCES | | 31 |
| | | |

vi

I. INTRODUCTION

Mathematical models have been found to be extremely useful tools for predicting the performance of complicated systems in lieu of costly test programs. The advent of modern high-speed digital computers and the continuing sharp decline in the cost of computing have made modeling an increasingly attractive alternative to testing of dynamic systems. However, even the most sophisticated models are of no use unless they can be proven to represent reality (the performance of the physical system being modeled) adequately. Validation is the process of proving the adequacy of a model by use of appropriate test data.

It is neither feasible nor desirable to design a mathematical model to represent all possible modes of response of a vehicle for every anticipated situation. Models are tools which are best designed for specific, well-defined purposes. A model may be ideally suited for one purpose and totally inappropriate for another, so the model purpose must always be borne in mind. The design of a model always includes trade-offs among different attributes. The intended purpose should govern how these trade-offs are managed. Increasing the complexity of a model (more degrees of freedom or nonlinearities) tends to permit it to simulate more types of behavior more accurately, but imposes the penalties of adding greater cost to develop, debug, and execute the model, and making user interface more difficult (more input requirements, more difficult to understand). Similarly, the model which is designed to be as general as possible will probably be more difficult and costly to use than one which is designed for a specific purpose and specific conditions.

Mathematical models, once they are properly validated, can offer distinct advantages over full-scale testing for studying some of the dynamics of rail vehicles. Models are much less expensive to run than full-scale tests. Fullscale tests require the use of very costly vehicles, train crews, instrumentation, data processing equipment, and testing personnel (technicians), as well as track which either has to be specially constructed for testing or must be taken out of revenue-producing uses for a period. Models can also be run more rapidly than full-scale tests (even when they are slower than real time on the computer), permitting more con-ditions to be studied in the same period of time. Finally, models pose no safety hazards and do no damage when used to represent hazardous situations such as potential derailments.

Many mathematical models of rail vehicle dynamics have been developed in the past 15 years. However, in most cases these models have not been validated. The work that has been performed to validate rail vehicle models has not been completely successful. Often, it is incomplete in that some data are looked at, occasionally parameters are modified, but the final steps of comparing the model outputs to independent data sets and defining the range of validity of the models are missing. These shortcomings are not necessarily oversights, but generally are limitations of the available data and funding. In some cases, the nonlinearity of the dynamics is not well understood and causes modeling to be invalid.

The inadequacy of the test data is a major contributor to problems in validation. Some test data sets do not include important, but difficult to measure, quantities such as wheel/rail forces and wheel/rail displacements. Oversights in test planning are a common problem. In general (with the exception of models that require detailed wheel/rail measurements for validation), the problems have not been with the state of the art of testing or instrumentation technology, but with the omission of needed measurements or test conditions.

Given the difficulty of validating rail vehicle dynamics models, the potential benefits to be enjoyed from improved validation methods and test planning procedures are substantial. If validated models could be applied to predict performance with confidence, testing could be minimized and many questions about dynamic performance of rail vehicles could be answered more quickly, accurately, and inexpensively than under present circumstances.

System identification techniques offer a solution to these problems. System identification is the integrated technology for determining mathematical models of dynamic systems from measurements of the system responses to their inputs. It is used for model validation, test design, model structure determination, and parameter estimation. The technology encompasses techniques useful in all stages of the development of validated models, from test planning to model verification. System identification procedures are discussed in more detail in kef. 1, with particular application to rail vehicle dynamics.

The contents of Ref. 1 complement the work reported here. In Ref. 1, considerable attention was devoted to the data processing algorithms used for parameter estimation and model structure determination. The practical considerations involving experimental design, instrumentation, noise and statistical assumptions were reviewed in detail. Model structure determination and parameter estimation methods were applied in pilot runs to data generated by a nonlinear simulation model of locomotive forced lateral response, and then to real vehicle data gathered in the Perturbed Track Test (PTT) program. The challenges posed by the imperfections in the test data were illustrated by comparison with the use of the simulated data.

1.1 OVERVIEW OF THE REPORT

The scope of the work documented in this report is limited to the description and partial demonstration of a methodology of rail vehicle dynamics model validation. Section II is a critical review of previous attempts to validate rail vehicle dynamics models, highlighting the lessons learned from that work and the experience which can be applied to the current project. Section III describes the general principles and assumptions which governed the development of the suggested methodologies, which are themselves outlined in Section IV. Section V describes the application of one of the validation methodologies to a model of the vertical dynamics of a six-axle locomotive. Conclusions and recommendations for further work appear in Section VI.

II. REVIEW OF PREVIOUS WORK

Previous work on validation of rail vehicle dynamics models has involved a number of investigators. These separate validation efforts have been directed at a variety of purposes and have employed diverse methods. The work has produced a large body of experience which can be used to enhance the success of future validation endeavors. In many cases this experience consists of cautionary information about what types of situations to avoid.

Compared to the amount of effort which has been expended on modeling and testing, very little attention in rail vehicle dynamics has been paid to combining the two in a validation process. Generally, the modelers and testers have been different people with divergent interests. Only rarely have they been brought together. The majority of the rail vehicle testing which has been done has not been designed with model validation in mind and, therefore, the data are not well suited for use in validation (insufficient accuracy, sample sizes, and varieties of test conditions, and incomplete sets of needed variables).

This section is a review of the existing rail dynamics literature which incorporates experience applicable to model validation. The review begins with examples of the traditional model developed from physical "first principles" being compared with test data collected under the same conditions. The comparisons tend toward "eyeballing," and often lead to heuristic model adjustment in order to produce better matches to the data. In general, the difference between model and test data is not compared to a preestablished validation criterion. Applications of this approach to both vehicle subsystems and complete vehicles are discussed in sequence. The review continues with a discussion of applications of system identification techniques to rail vehicle dynamics problems. In the system identification approach the validation of the model and any modification that is necessary are achieved using a combination of suitable analyses of test data, statistical concepts and the analyst's knowledge of system structure. The review in this section concludes with a discussion of some of the more general, philosophical studies of rail dynamics model validation. The studies offer guidance about procedures and the relative importance of different factors.

2.1 TRADITIONAL MODEL VALIDATION

2.1.1 Subsystem Models

Cooperrider, et al. [2] compared a mathematical analysis used to determine the wheel/rail contact point as a function of lateral displacement with results measured using plexiglass models of wheel and rail profiles. The plexiglass models were developed for the specific purpose of validating the analysis, which was designed to be used to compute parameters needed in complete vehicle dynamic models. The validation was successful. All discrepancies between the analysis and experimental results were found to be attributable to known measurement tolerances and unintentional differences between the cases which were investigated in the analysis and the test.

Hsu and Peters [3] studied the dynamic charactoristics of a draft gear, extending the work of Ward and Leonard [4], which will be described with the other system identification work. Hsu and Peters determined the friction-velocity relationship for a class of draft gear by defining model parameters based on inspection of the empirical force-deflection curves of individual draft gear units. Validation was then claimed, without prior specification of the validation criteria, by visual comparison of the time histories of the simulated and actual tests.

2.1.2 Vehicle Models

Boocock [5] used both roller rig and field tests to validate a quasi-static curving model. Practical difficulties involving test procedures (dynamic roughness responses dominating steadystate curving responses, and profiles of test wheel different from those assumed in analysis) impaired the validation. The agreement between the model and test results (based on "eyeball" comparisons) was found to be good for low curvature and within the linear creep regime, but was degraded by high curvature (where slip of leading wheelset was hypothesized) and creep force saturation (which was not incorporated in the model).

The Office for Research and Experiments (ORE) of the International Union of Railways (UIC) sponsored a program of testing [6] aimed at improving the modeling of the interaction between vehicles and track. A two-axle freight car was

operated over a variety of track conditions in the field. The accelerations, velocities, and deflections of different parts of the car were measured along with the track cross-level, gauge, and elevation. A purely linear approach was followed in analyzing the data. The approach used the ratios of output to input PSDs to estimate vehicle frequency response. The analysts sought to identify the dynamic modes which corresponded to each response peak. They were not successful in identifying all the peaks. Also, the experimental results were consistently higher than the accelerations predicted by the models. Some arbitrary adjustments were made to the models in order to try to improve the agreement. The inherent linear assumptions in the methodology limited its fidelity, but there were also problems with important variables remaining unmeasured (especially lateral inputs). This study served as an exploratory investigation of the use of field test data to validate analytical models, but did not produce any definitive model validations.

Hutchens, Haight, and Milner from MITRE [7] reported on a similar test program using a passenger car subjected to random track inputs in a field test. This study was also founded on linear assumptions. It dealt extensively with natural frequencies and mode shapes of the vehicle response, but in this case, the results of numerous test runs were averaged together to approximate a white-noise input assumption. The modal frequencies evident in averaged test data were compared with the frequencies predicted by an analytical model which was derived from physical first principles. The comparison was used to adjust the parameter values in the model to obtain closer agreement. In this case, the test results did not serve to validate the analytical model, but were only used to adjust the parameters of the model; the model was then applied to modal analysis of the vehicle dynamics under various conditions. These adjustments to the assumed values of model spring constants, which were substantial, were based not on any causal reasoning founded on the physics of the problem, but on closer reproduction of the frequencies of the peak responses in the test data.

Abbott, Morosow, and MacPherson [8] reported on the truck-hunting-model validation testing performed by Martin-Marietta as part of the AAR/ FRA Track-Train Dynamics Program. This program was unusually interesting because of the emphasis it placed on a building-block approach to model validation, beginning with testing and validation of key subsystem elements before proceeding to the entire vehicle. Quasi-static testing of the trucks was used to help define the nonlinear properties of the truck-carbody interface (dry friction, hysteresis), and shaker tests of the car body helped identify its elastic modes. The discrepancies between the test results from the car body modal survey and the predictions from a finite element model based on the drawings of the car body were traced to differences between the drawings and the actual construction of the car

hody. The comparisons between test and analytical results wege based on mode shapes and natural frequencies, only the latter being readily susceptible to quantitative comparison. The quasi-static truck tests produced force-deflection plots showing hysteresis characteristics, which were plotted together with the forcedeflection plots produced by an analog computer model, but no criterion for evaluating the differences between these plots was apparent. Vehicle transfer function tests were performed using sinusoidal force inputs, over a range of frequencies, to a single wheel or axle. Responses to these inputs were processed through an FFT analysis which revealed the nonsinusoidal behavior induced by system nonlinearities, to identify effective vehicle transfer functions. This procedure constituted a crude linear frequencydomain system identification process, but was not a model validation.

Healy [9] compared the response forces measured in test runs of an ASF hopper car on shimmed track with predictions from a simplified nonlinear ll-degree-of-freedom model, in an attempt to validate rock-and-roll predictions. A low-speed test was run to obtain quasi-static measurements, which were used to infer the track cross-level geometry to be used as the input for model runs. The snubber characteristics in the vehicle model were adjusted to produce the best "eyeball" match to the test response time histories at a single operating speed, and the model and test results were then compared visually at other speeds. This was an exercise in model adjustment based on experimental data, rather than validation of a model.

Elkins and Gostling [10] developed a sophisticated nonlinear model for quasi-static curving behavior and then executed a test program, with instrumented wheelsets and special sensors for measuring wheel/rail displacements, to validate the model's predictions. They displayed plots of predicted and experimental values of lateral forces and yaw torques for different vehicles, speeds, and curve radii. These plots clearly demonstrated that the authors' model predicted the experimental results much more closely than preceding theories, but there was no formal, quantitative application of a validation criterion. This is a moot point, however, because the agreement between the model and data was so close and the authors' argument so convincing that there was little doubt remaining about whether the model was validated for the range of conditions tested.

Helms and Strothmann [11] concentrated on details of wheel/rail interactions in comparing lateral acceleration PSDs measured using instrumented wheelsets and simulated in a relatively simple vehicle dynamic model which included a complicated representation of track interactions. Detailed track geometry measurements were used to provide inputs (such as rolling line offset PSD) to the spectral response simulation. The simulated and experimental PSDs of car body

and wheelset lateral accelerations were plotted together, but no quantitative comparisons were made and no conclusions were drawn except that agreement was better for the body-center measurements than for the individual wheelset measurements. Despite the meticulous attention to detail in the track and interaction modeling and measurement, agreement was in general not very good and no attempt was made to claim model validation.

Illingworth [12] used roller-rig testing to validate a model of wheelset lateral dynamics based on Kalker's creep-force/ creepage theo-Within the linear creepage regime, his ries. predictions of lateral and yaw frequency response were nearly indistinguishable from the experimental results, presenting a convincing validation (regardless of quantitative criteria) of the predictions. In the nonlinear creepage regime, the experimental results were plotted against three different theories and the closest agreement was found to be with 40% of Kalker's assumed creep force. However, this result was not strong enough to encourage Illingworth to claim validation of the model by his cwn (unquantified) criteria.

Kachadourian and Tsai [13] reported on the use of vertical shaker tests at the Rail Dynamics Laboratory to validate the FRATE lumped-parameter lading response model for a TOYC configuration. They defined the performance measures of interest to be the resonant frequencies and deflection shapes and amplitudes at resonances, but did not specify tolerances on those measures. A generalized frequency domain parameter identification approach was followed with the analyst playing a crucial role in manually adjusting model parameters to improve the agreement with the test data. The remaining differences between model and test results were interpreted to indicate the need for additional model degrees of freedom (for trailer flexibility) and nonlinear representations of springs (amplitude sensitivity of results). If these conclusions had been used to modify the model, and the model outputs again compared with the test results, the "outer loop" of the validation process (to be described in Section IV) would have been closed and a convincing validation exercise could have been reported.

Cooperrider, Law, Fries, and Tsai [14] described a very thorough program of field testing aimed at validating lateral dynamics models for freight cars. The ambitious scope of this program indicated the need for a building-block approach to validation, beginning with individual truck components and then assembled trucks before proceeding to complete vehicles. Despite the wealth of data collected and the care which was used in taking measurements and processing data, this test program still did not produce any definitive model validation, although it did provide valuable experience regarding the importance of having the right type of test conditions and data (constant speed, initial condition measurements, displacement of vehicle components relative to track, rail-head and roadbed variations along track, etc.). A major contribution of this work was the recommendation of which output measures to use for validating models of dynamic behavior on tangent and curved track, even though the tolerances which should be applied to those measures were not specified. The data which were collected were not suitable for identifying creep coefficients, and recommendations were offered for remedying this shortcoming in future validation test efforts.

Garg's review of the Track-Train Dynamics Project [15] referred to the need for validation of rail vehicle dynamic models. It included some plots of model and test results on the same axes, but did not address any quantitative validation criteria.

Rinehart [16] investigated the hunting stability of a three-axle locomotive truck using a linear ll-degree-of-freedom model which he sought to validate using test data from the Transportation Test Center at Pueblo, Colorado. The natural frequencies and damping of the truck modes at a single speed were tabulated and compared with the model predictions. The agreement was considered to be close enough to constitute validation. The remainder of the study was conducted using the model alone, without benefit of test data.

Sweet, Sivak, and Putnam [17] described the testing of a one-fifth scale model instrumented wheelset on a similarly scaled track 244m long for the purpose of validating a nonlinear analytical model of whee!/rail interactions. The test program was designed to incorporate a variety of constraints on the wheelset, permitting a systematic investigation of different wheel/rail interaction phenomena. A building-block approach was followed, first to validate the scaling laws applied to the design and interpretation of the scale model tests (via testing on a scaled roller rig), and then to use the scale model track tests to validate the wheel/rail interaction theory. The model track test plan was also structured to build from validation of linear relationships to increasingly complicated nonlinear relationships (larger wheel/rail displacements). Results were presented as experimental data points plotted along with curves predicted by theory, but explicit validation criteria were not defined and the differences between theory and experiment were not explicitly quantified. In some cases, the measured data points were indistinguishable from the theoretical predictions on the authors' plots, providing convincing evidence of model validation, but in other cases the agreement was not nearly so close, and there was considerable scatter in the experimental data. In these latter cases, the authors found the data sufficient to validate important performance trends predicted by the models, but they considered the data inadequate to validate the models completely for flange contact with nonzero yaw angles. This work represented an unusually careful and conservative approach to model validation, with testing

designed specifically to support the validation, and validation being claimed only where it was strongly supported by the experimental evidence.

2.2 SYSTEM IDENTIFICATION APPLICATIONS

Several studies of rail vehicle dynamics have applied system identification techniques to determine model parameters such as creep coefficients, suspension stiffnesses, and damping coefficients. Estimation criteria have included output error, equation error, combined state and parameter estimation, and frequency response matching techniques. Only one study has reported the use of a systematic model structure determination method. On the other hand, no papers have reported the successful completion of the crucial verification step, namely the use of an identified model to predict a data set not used in the parameter estimation process. No reported result demonstrates a complete model estimation and validation sequence.

The papers referenced below have applied a wide variety of system identification techniques to the development of rail vehicle dynamics models. Techniques used include:

- frequency response (gain and phase) curve fits [18,19];
- equation error minimization [1,4,18];
- output error minimization [1,20]; and
- combined state and parameter estimation [21].

Models identified have sometimes included a limited number of nonlinearities as well as linear dynamic effects such as suspension stiffness and damping. None of the reported efforts tried to estimate, systematically, a mathematical model structure for a nonlinear function. The mathematical forms were always specified a priori. Three of the papers [1,18,20] recognized the lack of identifiability of parameters in the set to be estimated and recommended modifications to the experiment to correct the problem.

All of the papers reported the processing of very limited amounts of data. Some of the results reported were based on work with only a single data set, while others worked with two or three. This limits the possibility of model validation. Model validation requires the processing of multiple, independent data sets.

Table 2.1 summarizes the contents of six papers which have appeared since 1973 on the topic of rail vehicle system identification.

Hasselman and Johnson [18] used frequency response data to estimate suspension stiffness parameters in a four-degree-of freedom linear model of the "rock-and-roll" dynamics of a 100ton hopper car. The paper recognized the importance of assessing the statistical significance of estimation accuracies of parameters. The authors correctly concluded, for example, that five of the eight parameters in their suspension model could not be estimated from the data available to them. They suggested modifications to the experiments to produce improved identifiability for the five parameters which could not be estimated from the original data sets.

Fallon [19] used frequency response methods to estimate parameters in a linear model of the vertical dynamics of a railcar. He also used an equation error method to determine parameters in a nonlinear model of suspension dynamics. The estimation with a nonlinear model used nine different data sets, each at a different speed. The parameter values estimated (representing suspension stiffness and damping) showed a strong trend with variation of vehicle speed. This indicated that more complex nonlinear effects were present than implemented in the author's model. Perhaps a nonlinear suspension model could have been estimated from the data if a systematic model structure determination method were to have been applied. It is disappointing that this report did not present any indication of the goodness of fit of the resulting models to the data for the nonlinear model estimation.

Ward and Leonard [4] applied an equation error method to the estimation of a nonlinear model of a draft gear-coupler connecting two rail cars. The paper presented a plot of the fit of data to the identified model superimposed on a plot of the data, but did not present any estimate of the accuracy of the identified parameters. It also did not attempt to predict a set of data which was not used to estimate parameters, omitting the validation step.

Broersen [20] used an output error criterion to estimate 19 parameters in a mathematical model of the lateral motion of a rail vehicle bogie. Parameters estimated included suspension stiffness and creep coefficients. The author determined relative parameter accuracies by examining the sensitivity of mean square fit error to small (2%) changes in parameter values. "Accurate estimates may be expected," according to Broersen, if the mean square error is highly sensitive to changes in the assumed value of a parameter. "Accurate," however, was not quantitatively defined. Accuracy levels could have been determined if several data sets had been processed independently. The scatter of estimated values for the various parameters would have given quantitative information on the estimation accuracy.

The sensitivity study indicated that two parameters, modeling suspension stiffness, could not be estimated from the data. The author suggested modifications to the experiment design which would have allowed estimation of these and other parameters.

Herzog [21] reported efforts to estimate wheel/rail creep coefficients from data taken from the testing of a scale model wheelset in the laboratory. The ectimation method combined state

| | | SY. | STEN IDENTIFIE | D , | | | 1 |
|---|---|---|--------------------------------------|--|--|--|--|
| SOURCE | SYSTEM DYNAMICS HODEL | PARAMETER DESCRIPTION | NUMBER OF PARAMETERS ESTIMATED | MEASUREMENTS | IDENTIFICATION CRITERION | ESTIMATION HETHOD | REMARKS |
| HASSELMAN, JOUNSON (1979) [18] | ROCK AND ROLL OF CAR BODY | SUSPENSION STIFFNESS | 8 | RELATIVE TRACK ALIGNMENT, CAR BODY ROLL ANGLE | FREQUENCY RESPONSE MATCHING | GAUSS- Newton | DETERMINES STATISTICAL SIGNIFICANCE OF ESTIMATION ACCURACY |
| FALLON (1979) [19] | TRUCK SUSPENSION | SUSPENSION STIFFNESS, DRY FRICTION OF BOLSTER- SIDEFRAME SUSPENSION | 2 | VEHICLE ACCELERATION, RELATIVE SUSPENSION DISPLACEMENT | FREQUENCY RESPONSE MATCHING; EQUATION FRROR | LINEAR LEAST SQUARES | PARAMETER ESTIMATES VARY HIDELY WITH TEST VEHICLE FORWARD SPEED, SHOGESTING A MOHE COMPLEX MODEL STRUCTURE IS NEFOED FOR A CONSISTENT MODEL |
| WARD, LEGWARD (1974) [4] | DRAFT GEAR CONNECTION BETWEEN TWO CARS | LINEAR AND NORLINEAR SPRING AND DAMPING COEFFICIENTS | 4 | FORCE. ON ANG DISPLACEMENT OF DRAFT GEAR | EQUATION ERROR | LINEAR LEAST SQUARES | OKLY ONE DATA RECORD PROCESSED |
| BROERSOM (1973) [20] | 2 AXLE TRUCK | SUSPENSION STIFFNESS, CREEP COEFFICIENTS, WHEEL/RAIL GEGAETRY | 19 | LATERAL ACCELERATION AT ODGIE; ANGULAR ACCELERATION; NHEELSET, DISPLACEMENT | OUTPUT ERROR | GAUSS-NEWTON BY CYCLICAL ADJUSTNENT OF PARAMETERS | FIT TO DATA APPEARS POOR FROM REPORTED CORRELATION OF DATA TO MODEL PREDICTION; OCES WAT PRESENT PLOTS OF MODEL FITS TO DATA |
| IIERZOG [21] | WIEELSET LATERAL DYNAHICS | CREEP COEFFICIENT | l | MINELLSET LATERAL VELOCITY, YAM ANGLE, LATERAL DISPLACEMENT | OUTPUT ERRUP WITH STATE ESTIMATION | GAUSS-HEWTON | ONLY REPORTS RESULTS FOR PROGRAM CHECKOUT USING SIMULATED DATA. DOES NOT USE NEASURFHENTS OF TRACK ALIGNMENT IN ESTIMATION |
| HULL, TRANKLE, KLINGER [1] | HALF LOCOMOTIVE MODEL | CREEP COEFFI- CIENTS, LIMIT- ING MIEEL/RAIL FRICTION, RAIL STIFFNESS,PRI- MARY LATERAL SUSPENSION, SECORDARY YAN SUSPENSION | 8 | LATERAL WHEEL FORCES AXLE-TO-TRUCK DISPLACEMENTS TRUCK-TO-CAR-SOUY DISPLACE- MENTS, CAR BODY LATERAL AND ROLL ACCELERATIONS, DISPLACE- MENTS BETWEEN RAIL AND TRUCK | OUTPUȚ ERROR | GAUSS-NEWTON | REPORTS SUCCESSFUL RESULTS For simulated data. Results With Real data not completed. |

and parameter estimation into a maximum likelihood algorithm. This algorithm is the most sophisticated data processing method applied in any of the references reviewed here. It explicitly accounts for both errors in measuring track irregularities (inputs to the dynamic system) and errors in measuring wheelset displacement (outputs from the dynamic system). None of the other algorithms used in the papers reviewed here takes both of these errors into account in formulating a performance index for measuring the goodness of fit of the estimated model. The combined state and parameter estimation algorithm has the potential of being able to make more consistent parameter estimates if both random track variations and wheelset measurement errors are significant.

Reference 21 attempted to estimate a single parameter, creep force coefficient, from a linear model of the wheelset dynamics. The estimation assumed that track perturbations were a purely stochastic signal, but the actual track profile was not measured. No deterministic perturbations, such as a sinusoidal alignment variation as used in other tests, were included in the profile. Attempting to estimate creep coefficients from dynamic responses (system outputs) would probably be much more effective if the track alignment (system input) were to include a measured, deterministic component.

Reference 21 did not prove that it is possible to estimate creep coefficients using only stochastic information about track alignment perturbations. This work used only simulated data and did not derive any estimates from real test data. A complete demonstration of the feasibility would have required the estimation of the coefficients from several independent data sets. A small scatter of estimated values, combined with the ability to use the identified model to predict data sets not used in the parameter estimation, would have been required to validate the method.

Hull, Trankle, and Klinger [1] reviewed system identification theory and investigated its application to rail vehicle dynamics. System identification techniques were demonstrated on simulated data from an ll-degree-of-freedom half locomotive model. Field test data obtained during the PTT made at the DOT Transportation Test Center in 1978 were analyzed to determine their suitability for use with these techniques. The data were not originally taken for system identification purposes. Using simulated data, Hull, et al. successfully identified the structure of the model which represented lateral wheel-to-rail force in the simulation and also identified several parameter values. These parameters included lateral and longitudinal creep coefficients, lateral damping and stiffness between the wheelsets and truck, yaw stiffness and damping between the truck and locomotive body, and equivalent lateral stiffness and damping of the rail. They compared time history plots of simulated noisy measurement data with the identification algorithm's prediction of the data showing an extremely close fit.

Hull, et al. found system identification techniques had several applications in rail vehicle modeling. Shortcomings in the PTT data which limited their use with these techniques were pointed out. Some preliminary tests were performed with the data, but no parameters were identified or models validated in the study. Several criteria for model validation that proved useful in the past were described. These included fit error, prediction error statistics, parameter estimate scatter, residual autocorrelation, and computed parameter covariance. It was recommended that all of these criteria be applied if sufficient data were available.

2.3 GENERAL STUDIES

The paper by Hasselman and Johnson [18] included some general common-sense guidance to govern a model validation procedure, with an emphasis on devoting the same attention to matching the scope and depth of the effort to the model purpose as one would in the initial model development.

Cooperrider and Law [22] prepared a thorough survey of rail vehicle model validation testing, describing the advantages, disadvantages, and previous usage of the different types of tests (field, shakers, roller rigs, scale models, etc.). Their review paper serves as a useful distillation of the problems and unexpected outcomes which have been encountered in validation tests, with valuable guidance in particular about which measurements are most important for validating models of different dynamic processes. Cooperrider and Law concentrated on the testing portion of the validation process, rather than the comparison between results and model outputs, but they did specify three different levels of validation which could be applied to models to serve varying purposes:

- (1) qualitative (validation of trends);
- (2) single critical value (critical speed, resonant frequency, etc.) validation; and
- (3) complete quantitative correlation of results within specified limits (such as frequency or dynamic range).

The authors indicated that they thought too much attention has been devoted to the second level of validation, to the detriment of the others.

The TDOP Analytical Tool Assessment Report [23] reviewed rail vehicle dynamic model validation efforts by several previous investigators and then recommended performance indices for use within four different "performance regimes" (lateral stability, trackability, curve negotiation, and ride quality). These performance indices were directed toward the evaluation of truck performance, rather than the validation of dynamic models. The review of existing models restated the claims of the models' developers regarding validation, rather than probing the justifiability of those claims or the methods applied.

III. GENERAL PRINCIPLES GOVERNING MODEL VALIDATION

An analytical model of vehicle dynamics cannot avoid incorporating simplifying assumptions and limitations. The model is not an exact replication of the physical system whose performance it represents. Neither can any single dynamic model be designed to even approximate the behavior of a vehicle under all conditions. Each model is (or certainly should be) designed to serve a specific purpose by representing a particular dynamic process (or carefully chosen combination of processe). The purpose which the model is intended to serve determines the output quantities it should compute and the accuracy required for each.

The concept of the model as an analysis tool designed to serve a specific purpose underlies the validation methodologies in Section IV. The final purpose of the model, as embodied in the questions which the model user wants it to be able to help him answer, should remain in sight throughout the formulation of the model and its validation. The validation process deserves the same care and attention as the original model formulation. The remainder of this section highlights the principles which should be applied in the validation process to ensure that. Most of these principles are standard elements of good modeling (i.e., model derivation) practice, but have rarely been applied to rail vehicle validation efforts in the past.

3.1 TERMINOLOGY

Several of the important terms which will reappear throughout this report need to be defined at this time so that their meanings will be unambiguous. This is particularly important because of the imprecision of the meanings which have been assigned to some of these terms in the past.

Validation is of course the most significant term here. Validation of a model is the process by which one gains assurance that the model offers a valid representation of reality. What constitutes a "valid" representation, however, is a key issue which depends strongly on the purpose for which the model is to be used. A model could, for example, be considered valid for rough preliminary design but invalid for detailed final design. Similarly, a model could be a valid descriptor of one vehicle design but not another; or the model could be valid under one set of operating conditions but not another. In summary, validation is not an absolute concept. It is indeed dangerous to treat it as such. Validation of a model can only be determined with respect to stated system (vehicle) characteristics, operating conditions, and modeling purposes.

<u>Causal</u> and <u>correlative</u> models represent two different approaches to predicting the performance of a system. The <u>causal</u> model seeks to represent the physics of the system as directly as possible, using equations (differential equations for dynamic models) derived from the laws of physics which govern the interactions among system elements. The parameter values in such a model have specific physical meaning (such as masses or stiffnesses or dimensions of pieces of equipment). Wherever possible, these parameter values are obtained from direct measurements of the pieces of equipment being represented.

<u>Correlative</u> models, on the other hand, are designed to predict system performance by producing a "best fit," according to some specified criterion such as minimizing the square of errors, to an existing data set. The parameter values in a correlative model are derived from the available output performance data, and do not necessarily correspond to any physical characteristics of the system being modeled. Correlative models are generally useful only when the system being studied is too complicated to model causally or when the modeling purpose does not require higher fidelity or detail than a correlative model can provide.

System identification is an approach to model formulation and validation which combines aspects of the causal and correlative models. Using the system identification approach, the analyst represents as much of the system as he confidently can using causal arguments. However, certain portions of the system may be too complicated or poorly understood to be represented that way and values of some of the physical parameters of the system may not be readily measurable. Those problems can be addressed by making careful measurements of the system inputs and outputs in a specially designed test program and processing and interpreting those measurements appropriately. Parameter identification methods can be used to identify unknown unmeasureable parameter values, while model structure determination methods are applicable for choosing the form and level of detail (number of degrees of freedom)

the model requires to replicate, adequately, the test results for comparable input conditions.

The techniques used in system identification are founded on statistical concepts. They have been gathered into an integrated system identification procedure for model validation.

3.2 APPLICATION OF STATISTICS TO MODEL VALIDATION

As already mentioned, model validation is not an absolute or deterministic process. No model can ever be expected to replicate reality perfectly, even under the best conditions. On the other hand, a model is not very useful unless it can be applied in situations (parameter values, inputs, etc.) other than precisely those for which it was formulated. Furthermore, the environment always introduces random inputs to physical systems (particularly rail vehicles), and noise is always present in the measurements of test inputs and outputs. The combination of these factors requires that model validation be viewed statistically.

A very useful framework for thinking about model validation is statistical hypothesis testing. Hypothesis testing can be conducted according to a variety of statistical methods, but is basically concerned with quantifying the probability of drawing the "wrong" conclusion under given conditions. Because the model does not represent reality perfectly, the outputs it produces will differ from those experienced in practice. The values of an output measure produced by a model under a large number of different conditions can be considered to form a probability distribution, while the outputs experienced in practice form another, different distribution. The hypothesis to be tested is that the model represents the performance observed in practice. Typical applications of hypothesis tests are based on assumed Gaussian distributions of the output measures being compared, and involve consideration of two different types of potential errors:

- Type 1: Rejecting the hypothesis when it is true (finding the model invalid when it is actually valid).
- Type 2: Accepting the hypothesis when it is false (considering the model to be validated when it should not be).

The confidence level, x (the probability of not making either type of error) can be specified and used to derive a <u>confidence interval</u>, which is the range of values of the output measure for which one can assume the stated hypothesis to be correct 100x% of the time.

Unfortunately, the model validation problem cannot be made to fit neatly into the Gaussian hypothesis testing framework. Inspection of only the output measures eliminates the variability introduced by not being able to measure the inputs accurately enough to ensure that they are

identical for the test and the model. In addition, it is quite unlikely that the range of conditions for which one seeks to validate a model will produce a Gaussian distribution of any of the output measures. In the absence of the Gaussian distribution, standard tables cannot be used to calculate confidence intervals. The only recourse would be to execute a series of parallel model runs and tests extensive enough to produce statistically valid results, which is totally impractical. However, the conceptual framework of hypothesis testing remains applicable to the model validation issue, and provides a very useful perspective on the problem.

3.3 GENERALITY OF METHOD

The model validation process requires a substantial element of engineering judgement, based on model purpose and complexity and the nature of the available data. It is, therefore, not possible to formulate a completely general, fully detailed and algorithmic procedure for validation. In place of such an algorithmic method, some principles and guidelines for rail vehicle dynamic model validation are offered here, while Section IV includes the outlines of two general validation procedures which are recommended for the alternative cases of reliance on existing test data and specification of new testing.

3.4 IMPORTANCE OF MODEL PURPOSE

The single most important concept behind the validation approaches to be recommended is the focus on model purpose. A model of rail vehicle dynamics is a tool, and that tool should be designed to serve a specific purpose or purposes. The criteria which will be used to judge whether or not a model is valid must be defined as those which best indicate the model's suitability or unsuitability for the intended purpose. Similarly, the allowable tolerance between model predictions and test results (the tolerances in validation criteria) must be specified by the analyst on the basis of how the model's suitability for the specified purpose is affected.

The validation process does not produce the result that the model is either "validated" or "not validated" across the board. Rather, the assessment of model validity must be made in terms of model purpose. For example, the same model could be found valid for one purpose but not for another, more demanding purpose.

3.5 KNOWLEDGE OF INPUTS

Accurate knowledge of the input conditions encountered in testing is much more important for model validation (and system identification) than for other testing purposes. This is one of the principal reasons that test programs designed for other purposes often do not produce data suitable for model validation. It is essential that the test input magnitudes and timing relative to the outputs be known very accurately. The inputs include vehicle speed, external force loadings and detailed measures of track geometry and wheel/ rail contact geometry.

The emphasis on precise knowledge of the inputs arises because it is not the outputs alone which are of interest in the validation process, but the relationship between the outputs and inputs. If the differences between the input conditions for the test and model can be made small enough, the outputs can be compared directly and all differences attributed to model and measurement inaccuracies. The validation criteria can then be applied directly to those observed differences. In the absence of good input information, there is no way of knowing whether differences between test and model outputs are attributable to differences in the inputs or to model deficiencies.

3.6 PARAMETER ADJUSTMENTS

When a rail vehicle and a dynamic model designed to represent it are subjected to the same inputs, the outputs will not be identical. The discrepancies could be caused by an inappropriate model form (ill-chosen degrees of freedom or nonlinearities), by poorly chosen values for parameters within an appropriate model form, or a combination of both. It has been a common practice in previous "validation" efforts to adjust the values of model parameters in order to improve the match between test and model results. This application of correlative model adjustments to a causal model is potentially dangerous, unless it is done extremely carefully, paying close attention to the physical significance of the results. The causal basis of the model may be jeopardized, because the "best fit" model parameter values may well correspond to unreasonable values for physical constants such as masses and spring rates. In fact, unreasonability of the implied values for physical constants serves as a strong indication that the model form is inadequate. In addition, the parameter adjustment which improves agreement between the model and a sample test case could very well produce poorer agreement under other conditions because of the loss of physical causality. Parameter adjustment is applicable to model validation only when the physical reasonableness of the adjusted parameter values can be assured and the adjusted model can be checked against test data for a variety of independent operating conditions.

3.7 RANCE OF OPERATING CONDITIONS

Rail vehicle systems are highly nonlinear, and nonlinear systems behave differently when driven by inputs of different amplitudes. A model validation process which includes testing at only a single amplitude cannot generally reveal these amplitude-dependent effects. Whenever nonlinear elements are present, the validation will need to be conducted for a variety of amplitudes and combinations of inputs (superposition no longer being applicable). Extreme caution must be used in attempting to apply a model outside the range of the validation conditions. Indeed, a model of a nonlinear system cannot be considered validated for conditions outside that range unless all the nonlinearities have been isolated and the trends produced by their influences on system performance have been quantified unequivocally. This caution on use of models outside their regimes of validation is particularly important for the nested validation process to be described in Section 3.8, because failure to heed it could invalidate the validation procedure for other models.

3.8 NESTING VALIDATION

Î,

A valuable approach to model validation, but one which should be used with caution, is the nesting of models, or the "building-block" approach. In this approach, one begins by modeling subsystem elements and validating those models with component or subsystem tests. The fully validated subsystems can then be combined into larger systems and the models of those larger systems validated for appropriate representation of the interactions among the subsystems. This nesting process simplifies each validation and helps to ensure that subsystem elements which can be represented easily and accurately are not modified for the sake of improving the agreement between test results and a model of a larger part of the system. A substantial amount of engineering judgement must be applied in a nested validation to ensure that a supposedly validated subsystem element is not required to operate under input conditions for which it was not in fact validated. If the subsystem model were driven outside its range of validity, the attempt to validate the model of the larger system would produce erroneous results.

3.9 INDEPENDENT DATA SETS

A single data set is rarely sufficient to validate a dynamic model. Nonlinear system models require validation under a variety of input conditions because of their differing modes of response. Furthermore, models which have been derived or adjusted on the basis of dynamic test data must be validated using data independent from that used during model formulation. Otherwise, the results of the validation process would be analogous to comparing a curve derived as the "best fit" to a set of data points with the very data points used in the derivation. Obviously, a model cannot be considered validated until it has demonstrated its ability to predict data <u>not</u> used in its derivation.

3.10 CLOSURE

The issues which have been discussed in this section were influential in the development of the model validation procedures to be described in Section IV. They have been offered here as background and explanation of those procedures, in the hope that the procedures can now be presented as clearly and concisely as possible. Section IV will explain how to go about validating a rail vehicle dynamic model, while Section III has explained in advance some of the reasons why the approaches to be presented were chosen.

IV. MODEL VALIDATION PROCEDURES

There is no single, all-purpose, "best" method for validating rail vehicle dynamic models. The diversity of these models and their uses and the available data make it impossible to specify one algorithmic procedure to be followed under all conditions. There is a considerable variety of possible validation methods, each having advantages and disadvantages. The validation process cannot be standardized to the extent that it can be treated as a "black box," but always requires substantial engineering judgement. The validation methodologies described in Section 4.1 form a reasonable, general set of procedures for model validation, but by no means the only possible such procedures.

There are certain elements of good validation practice which all validation approaches should incorporate (based on the underlying principles which were described in Section III of this report). In terms of a step-by-step procedure, these key elements can be summarized as:

(1) statement of modeling purpose;

- (2) specification of validation criteria;
- comparison of model and test results on independent data sets for comparable operating conditions;
- (4) comparison of model and test results for several additional conditions.

The precise implementation of these elements may vary from procedure to procedure, and their interactions may vary, but they should always occur in the sequence listed above.

In discussing validation procedures, it is necessary to specify exactly what is meant by a "model." A model, for purposes of this report, is a set of dynamic equations having a specified form and number of degrees of freedom, and possibly some specific nonlinear elements. It is not necessarily a computer program or set of programs. Validation of a model according to the procedures to be specified here does not refer to the validation of any computer programs which incorporate the equations describing that model. Similarly, the validation of a particular model structure must be distinguished from valid- ation of the parameter values used to represent a particular vehicle with that model structure. The validity of a model structure for representing a class of vehicles under some range of operating conditions can be established by validating that model structure using separate sets of parameter values to represent several different vehicles of that class. The model structure cannot be validated in general until both it and the parameter values are validated for several different conditions. Thus, it is easier to validate

the model structure and parameter values for a single vehicle than it is to validate just the model structure in general.

The validation procedures which follow are general outlines, rather than being specific algorithmic procedures. The dimensions needed to categorize rail vehicle dynamics models are such that it is not practical to specify the variables which should be compared and the tolerances which should be applied for all possible model validations. Two separate methodologies are described, one for use with existing test data and the other for use when new data collection can be specified. Factors apart from the availability of data which would be expected to influence the conduct of a model validation (with the number of dimensions assumed for each in parentheses) include:

- dynamic process being modeled (8);
- analytical solution method (5);
- model complexity (nonlinearities and degrees of freedom) (n).

All possible combinations of these factors would produce the need for 40n different validation procedures, where n may be a sizeable number, considering the range of possible nonlinearities and degrees of freedom which can be incorporated into a model.

The specific breakdown of the above categorization of the models to be validated and some more specific guidance about the application of the validation procedures to specific rail vehicle dynamics models appears in Section 4.2, following the discussion of the procedures. The procedures cannot be specified in sufficient detail here to apply directly to each of the 40n possible models.

4.1 OUTLINES OF THE GENERAL VALIDATION METHODOLOGIES

Two separate model validation methodologies are described here because no single methodology can be designed both for use with existing data and for use with the new data collected specifically with validation in mind. The ideal procedure, which one should follow when designing a program of testing for model validation from scratch, appears in Section 4.1.1. Because the data available from existing test programs are likely to be much more limited, and may very well be inadequate for validating a model, the procedure to be followed with existing data is substantially different. That procedure is outlined in Section 4.1.2.

The methodologies described here can be used with current state-of-the-art techniques, and do not require exotic test procedures or instrumentation. However, they do require that the test programs be carefully planned and executed, so that all the important variables are measured and the important operating conditions are tested.

4.1.1 New Data Collection

The flow of information which occurs in this validation methodology is shown schematically in Figure 4.1. The entire procedure is predicated on prior specification of the candidate model structure (i.e. equation form, degrees of freedom, and nonlinearities).



Figure 4.1 Model Validation Schematic Using New Data Collection

<u>Step 1</u>: Specify the purpose for which the model is to be used and, based on that, determine what the output quantities of interest will be. The choice of model purpose is fundamental to all that follows because of the strong influence it must have on the choice of validation criteria. The attempt to validate the model should not proceed until the purpose is clearly defined.

<u>Step 2</u>: Formulate the validation criterion which is appropriate for the stated model purpose and the dynamic process being modeled. This is where the analyst's judgement has the most important influence and, as a result, this step is the most difficult to reduce to an algorithmic procedure. The validation criterion includes the choice of which dynamic variables to compare from among both output and intermediate variables. It also includes the choice of the most appropriate measures (or statistics) of those variables to examine, the selection of tolerances for each, and-the-choice of mathematical form.

The choice of dynamic variables is strongly influenced by the structure of the model which is being validated and the model purpose. The variables to be selected should be those which are most important for the end use of the model and those which are expected to be most revealing of model deficiencies, based on the analyst's understanding of the model and the physics of the vehicle being modeled. The model purpose and the analytical method embodied in the model influence the choice of which statistic(s) of each dynamic variable to investigate. These statistics could be mean values, variances, rms, maxima or minima, or they could be time-history traces or spectra (or even portions of spectra) as well. Once again, the choice depends on the analyst's judgement about what is most significant for the purpose which the validated model is intended to serve and what is most likely to reveal significant model deficiencies. The selection of the tolerances to apply to each performance measure should be based on model purpose, because a model fidelity adequate for one purpose could be totally inadequate for a different purpose. The form of the validation criterion should depend on both model purpose and dynamic process, but remains a highly judgemental choice. This form could be a weighted summation of errors or discrepancies in different performance measures or a product of errors, or a series of separate tolerance tests which have to be satisfied in whole or part. Each form embodies different inherent assumptions about the relative importance of each performance measure.

Obviously, the combination of factors which must be considered in formulating the validation criterion is such that no simple procedure can be offered for this purpose. The influence of the validation criterion on the results of a validation attempt is so strong that every reported validation should be accompanied by a thorough description of the criterion which was applied.

Step 3: Select the range of operating conditions which need to be tested in order to validate the model for the intended purpose. These operating conditions are characterized both by the parameter values used to describe the vehicle and by the inputs to the vehicle (speed, track perturbations, and external forces). A selection of these conditions adequate to cover all dynamic effects which are expected to be important for the eventual use of the model should be chosen. Particular attention should be paid to obtaining enough data to characterize significant nonlinearities confidently. On the other hand, it is inefficient to work with a multiplicity of cases which fall within the same easily defined linear performance regime. In order to obtain sufficient data on nonlinear response without overdoing the linear cases, it may be necessary to

experiment with the model for a variety of input amplitudes. By observing the state variables associated with the nonlinear elements, it can be determined which input amplitudes produce linear and nonlinear responses. These pilot runs of the model can thus serve to help choose which input amplitudes should be used in the test program.

Step 4: Design the test program which will be used to collect the data needed for model validation, considering the validation criteria and operating conditions previously specified. The nature of the testing which is needed must first be decided. If the resources and facilities are available, this could be full-scale field testing. On the other hand, depending upon the model to be validated and the conditions to be tested, it might be more suitable to use special facilities such as the Rail Dynamics Laboratory, or there could be good reasons for using a scale model test or even a previously validated mathematical "truth model." If a "truth model" is to be used, it should be of higher order and contain more nonlinearities than the model being validated in order to serve as a reliable benchmark against which to judge the candidate model. Moreover, the analytical method it employs to obtain solutions should produce results which are compatible with the candidate model's.

An explicit experimental design procedure should be followed to choose the conditions (inputs, operating environments, vehicle characteristics) and combinations of conditions to be tested. Each case in the test matrix should reveal some aspect of model validity, and the total set of test conditions should provide reasonable coverage of the input and parameter space expected to be encountered when the model is used for its intended purpose. This coverage must be designed into the test program at the start, because the model cannot be validated for conditions remote from those for which it was tested.

A key part of the test design is the specification of measurement requirements, including both the variables to measure and the tolerances required. All input conditions which can influence the behavior of the vehicle (track geometry, external forces, etc.) must be measured very accurately, although the precise quantification of that accuracy depends on the purpose for which the model is intended and on the sensitivity of the vehicle to each input. Any directly measurable vehicle characteristics (masses, spring rates, geometric configuration) should also be measured very carefully in order to supply the model with an accurate vehicle characterization. References 1 and 24 discuss analytical procedures for determining the variables to measure and measurement accuracy requirements. The choice of which dynamic response variables to measure during the tests depends on the form of the model and its intended purpose. A large element of analyst judgement, based on an understanding of the dynamic characteristics of the vehicle, also enters into this choice of measurements. The required measurement accuracy is once again derived from model purpose, by way of the tolerances specified in the validation criterion.

The quantity of data required for each test condition (or length of data records) should be specified during the test design step to ensure that statistically valid results can be produced. It must be adequate to permit the validation to be evaluated to within the tolerances specified in the validation criterion by using the measurements available from the contemplated test program.

<u>Step 5</u>: Execute the test program (or truth model) for the conditions specified in the test design, collecting both input and response data. Process the data to the extent needed to place them in a form suitable for direct comparison with the candidate model inputs and outputs. For example, if the model is to be validated on the basis of comparisons of time-biscory traces, plot the appropriate time histories. If the model is to be validated on the basis of response spectra, compute those spectra and plot them.

<u>Step 6</u>: Using data collected in static testing of the vehicle (or provided from the manufacturer's drawings or documentation), specify the values of the parameters needed to characterize the vehicle in the candidate model. Identify those parameters which are uncertain and specify reasonable expected distributions for their values.

<u>Step 7</u>: Execute the candidate model for the vehicle subjected to the inputs which were actually experienced in the test program. It is important that the best available characterization of those input conditions be used in the model runs, while the vehicle parameters should be those specified in Step 6. The sensitivity of the model results to the values chosen for the uncertain parameters should be tested by running several sample cases for appropriate combinations of the values defined in Step 6.

<u>Step 8</u>: This, the most complicated step in the methodology, is the comparison between the model predictions and test results, and the use of that comparison to determine whether the model should be considered validated. The comparison should treat one case at a time, not proceeding to a new vehicle or set of test conditions until the model has been found valid or invalid for the previous case.

The test results derived in Step 5 and the model outputs from Step 7 are to be compared using the criterion advanced in Step 2. If the comparison is within the allowable tolerances, the model can be considered validated for that test condition, and the next condition or vehicle can be assessed. If the discrepancies exceed the allowable tolerances, the analyst must seek an explanation.

Assuming that measurement and programming errors, and unintentional differences between the

cases tested and modeled are accounted for and eliminated, the analyst should apply his knowledge of vehicle dynamics and the existing literature to explain the discrepancies. Using that knowledge, the analyst may recognize the need to incorporate additional degrees of freedom or nonlinearities into the model, or to change some of his modeling assumptions. He may also suspect that adjustments to the values of some parameters in the model will produce closer agreement with the test data.

Once the analyst has formulated a promising model adjustment which can be justified on a causal basis, he should implement it, rerun the adjusted model, and once again compare its outputs with the test data to see if he is converging on a better model. The modifications' to model structure which are based on causal, physical, reasoning should be assigned priority, and the parameter value adjustments should only be attempted when model structure changes no longer appear to be worthwhile (i.e. when the discrepancies between the model and test cannot be explained by physical processes which can be incorporated into the dynamic model). The adjustments to parameter values must be effected cautiously, always tempered by engineering judgement about the physical reasonableness of the new implied values of masses, stiffnesses, and geometry.

If, following the model adjustments, the validation criterion is still not satisfied, that fact should be noted and the remaining cases should be dealt with. If the adjusted model does satisfy the criterion, that fact should also be noted and the adjustments which were necessary in order to achieve the validation should be recorded. If those adjustments included changes in model structure, the new model form should be treated as the baseline model for the remaining data sets.

Once all of the cases called for in the test plan have been processed, the pattern of validation successes and failures should be summarized so that the analyst can readily identify under which conditions he has a validated model. When changes in model structure were required to achieve validation, it would be advisable to pass through the procedure once again, using the adjusted model for those cases under which it was not previously tried. In that way, the adjusted model may be found valid for a wider range of conditions than those for which it was originally tested.

Note that Step 8 includes two feedback loops, which make it considerably more complicated than the earlier steps. The process of adjusting the model structure to achieve an improved representation of the vehicle is referred to as the "outer loop" of the validation process. This incorporates the two feedback loops just described, both the adjustment and rerunning of the model and the testing of the adjusted model against the data sets which did not require the use of the adjusted model initially.

The comparisons between model and test results within Step 8 should occur in a welldefined sequence, with all cases for one vehicle being completed before proceeding to the next vehicle. The cases for each vehicle should begin with the simplest, smallest amplitude inputs (presumably producing the most nearly linear response) and then proceed to the more complicated and larger amplitude conditions, which are more likely to require model adjustments. If parameter adjustments are needed to achieve validation for some conditions, the adjusted parameters should be applied to all the model runs for the same vehicle. If that invalidates some cases which were previously validated, the parameter adjustment should not be adopted and the validations which were based on its use must be reconsidered. If a systematic pattern emerges among the parameter adjustments required to achieve validation for a variety of different conditions, then that could provide the discerning analyst with the evidence needed to decide what additional degree(s) of freedom or nonlinearities to apply to the model. This use of the information generated in the validation process would not be possible if validations for several different vehicles were considered simultaneously.

4.1.2 Existing Test Data

This methodology has certain steps in common with the methodology suggested for new data collection, but also has substantial differences. Its information flow is shown schematically in Figure 4.2.

<u>Step 1</u>: Specify the purpose for which the model is to be used (same as in other methodol-ogy).

<u>Step 2</u>: Formulate the validation criterion (same as in other methodology).

<u>Step 3</u>: Review the existing test data in detail and prepare a summary which can be used to assess the feasibility of using these data for validation:

- test conditions (vehicle, operating conditions, original purposes of test);
- input and output variables measured;
- characteristics of measurements (bandwidth, dynamic range, instrumentation accuracy, any known errors or missing channels, etc.);
- known parameters of vehicle and track;
- correspondence or phasing between vehicle and track measurements.

<u>Step 4</u>: Evaluate the feasibility of using the existing data to validate the candidate model or a portion thereof for some range of operating conditions. This is an initial screening test to check whether there are enough measurements of



Figure 4.2 Model Validation Schematic Using Existing Test Data

inputs and response variables, under enough operating conditions, to make it worthwhile to continue. This step can terminate the validation attempt, but it cannot assure any successful validations. Based on the data which are available, apply engineering judgement to estimate the range of conditions for which it is worth attempting to validate the candidate model. This judgement should be based in large part on estimating the dimensions of the expected linear response regimes and the regimes influenced by isolated, simple, well-understood, nonlinear effects. The remaining effort should be focused on those conditions for which the test data already exist and those which can be understood readily by extrapolation from the data.

<u>Step 5</u>: Test the sensitivity of the candidate model to the uncertainties in the available data. Assume reasonable distributions for the values of the unmeasured inputs or parameter values to use in a series of sensitivity runs of the model. The uncertainties introduced into the model outputs can be estimated by applying covariance propagation analysis to the model equations (for linear or quasi-linear systems). In either case, the estimated distributions of the output measures can then be compared to the tolerances

specified in Step 2, and the probability of complying with those tolerances can be determined. There is not enough information available to perform any formal hypothesis testing, so heavy reliance must be placed on the analyst's judgement. If the distributions of the results produced by the model sensitivity runs are "broad" relative to the tolerance bands there is not enough information available from the test data to establish the validity of the candidate model for the intended purpose and the validation attempt should be terminated here. On the other hand, if the distributions produced by the sensitivity runs fall well within the tolerance bands, this screening test is passed and the validation can proceed to Step 6.

<u>Step 6</u>: Execute the candidate model for the conditions which were present in the test program, using the best available characterization of the input conditions which were experienced and the expected values of the uncertain parameters which were reviewed in Step 5.

<u>Step 7</u>: Compare the model predictions and the test results, following essentially the same procedures as in Step 8 of the previous methodology. However, it should be borne in mind that the additional uncertainty introduced by deficiencies in the available data limits the strength of the conclusions which can be drawn in this case. The confidence limits need to be wider than before and it will be correspondingly more difficult to validate a model using the same tolerances in the validation criterion.

4.2 APPLICATION TO RAIL VEHICLE DYNAMICS MODEL

Mathematical models have been developed to represent many of the dynamic phenomena which are experienced by rail vehicles. These models have employed a variety of analytical methods to solve for the response characteristics of interest. Because of the wide range of purposes and requirements these models have been designed to address, they are also characterized by widely varying levels of detail and complexity.

Rail vehicle dynamics models can be characterized by the dynamic processes they represent, the analytical methods they employ, and their levels of detail (portions of vehicles or number of vehicles described, degrees of freedom, nonlinearities). Although the validation procedures for all the models can be accommodated within the two frameworks described in Section 4.1, the step-by-step details of these procedures must be tailored to the characteristics of the specific models. The dimensions of this problem are such that it is totally impractical to attempt to specify a step-by-step validation procedure for every possible model. The remainder of this section explains those dimensions and provides guidance for dealing with some issues which are peculiar to attempts to validate models which employ specific analytical methods or represent specific dynamic processes.

4.2.1 Analytical Methods

The analytical methods which are typically embodied in rail vehicle dynamics models are listed in Table 4.1. Each analytical method requires the use of a different mathematical solution technique to calculate the response quantities of interest. Furthermore, the responses calculated using the different analytical methods are fundamentally different from each other, requiring different kinds of comparisons with test data in order to evaluate model validity.

| Table | 4.1 | Analytical | Methods | Applied | to | Rail |
|-------|-----|------------|-----------|---------|----|------|
| | · | Vehicle Dy | mamics Mo | odels | | |

| e | QUASI-STATIC (ALGEBRAIC) |
|---|---|
| 9 | FREQUENCY DOMAIN - LINEAR - QUASI-LINEAR - SPECTRAL ANALYSIS - MODAL ANALYSIS |
| • | TIME COMAIN - LINEAR - NONLINEAR |

Quasi-Static (Algebraic) Solutions

The quasi-static, algebraic, analytical method is applied to steady-state models which are designed to predict equilibrium values of vehicle performance measures. These models are the simplest to validate because each output quantity is a single number, which can be compared with a single number describing the same performance experienced in testing.

Frequency Domain Solutions

Several types of frequency domain analysis methods can be applied to rail vehicle dynamics models. These methods are inherently founded on linear assumptions, but can be adapted for use on nonlinear systems by employing quasi-linearization techniques such as describing functions. Eigenanalyses are used to determine natural frequencies, damping ratios, and mode shapes of vehicle response. Although eigenanalyses can be calculated very efficiently, they are difficult to compare with test data because testing cannot directly produce evidence of the natural frequency. However, eigenanalyses can be used to predict the critical speed and damping of each response mode for onset of hunting, and that critical speed can be compared with the speeds at which hunting becomes apparent in tests. More commonly applied frequency domain analyses involve the use of transfer functions to calculate

vehicle response spectra and rms values. Substantial effort has been expended in attempting to validate models which embody spectral analyses, making this an important category for further discussion.

Simple, direct comparison of measured and model-predicted spectra is not a very good test, for validation. The appearances of rail vehicle response spectra are dominated by the zeros produced by the cancellation of track inputs associated with the combination of fixed axle and truck spacings and constant train speed. These zeros can make the simulation and test output spectra look surprisingly similar at first glance even though the peaks in the spectra, which contain the majority of the information about vehicle dynamics, may differ by an order of magnitude or more. The problem is exacerbated by the logarithmic ordinate scale used for plotting spectra, although that can be compensated for by careful scrutiny of the differences in the amplitudes of the peaks in the simulated and experimental spec-" tra. It is also advisable to run the validation tests at several speeds so that the zeros are shifted to different frequencies, permitting responses which would otherwise be obscured to become observable.

The validation criteria to apply to comparisons of response spectra are not easy to define, a priori, but must be tailored to the individual model and its intended purpose. These may include frequencies and/or amplitudes of peaks, compared uniformly throughout the spectrum, or with different relative weights for different frequency ranges. This information could also be collapsed into a single figure of merit such as an rms estimate or an rms weighted by frequency range.

Although output spectra are most commonly used for validating frequency domain models, there is considerable merit to the use of cross spectra, whether they be input/output or output 1/output 2. The output 1/output 2 cross spectra can be particularly helpful in diminishing the need to rely on very accurate and simultaneous measurement of the inputs to the tested vehicle, although each output/output cross spectrum can only be used to validate portions of the model, rather than the entire model. The input/output spectral comparisons benefit greatly from simultaneous input and output measurements. If the track input information is only available from prior (or post-test) measurements, the loss of phase information can be significant, especially on flexible track.

Attempts to validate models using spectral comparisons must also contend with some serious statistical issues, since the spectrum plots derived from finite-length test data are only (noisy) estimates of the true spectra. The confidence which can be assigned to those estimates for the number and length of samples used should be quantified before comparing the test spectra with model predictions.

Time Domain Solutions

The majority of the rail vehicle dynamics models which have been developed employ timedomain solution methods. For purely linear systems, the solutions to the system state equations can be propagated using linear algebra (via the state transition matrix). For general linear or nonlinear systems, the system differential equations can be solved by a variety of numerical integration techniques. In either case, the model produces a sampled time history of each state variable and any auxiliary variables the modeler may choose. These time history outputs can be compared directly with the analogous measurements from tests, or both simulated and measured data can be Fourier transformed and their spectra compared.

Direct comparison of the test and simulated time histories can be useful for relatively simple models of low-frequency behavior in which the most important phenomena are large-amplitude transients or when the vehicle's dynamics are essentially dominated by one or two modes which include major nonlinearities. Examples of such conditions include freight car rock-and-roll responses to track cross-level inputs and locomotive responses to perturbed track test inputs.

Quantification of a validation criterion for time-history comparisons requires some subtle application of engineering judgement because of the multitude of possible ways of comparing time histories. For some applications, the <u>peak</u> value of an output measure (lateral force, L/V force ratio, wheelset displacement, etc.) could be most significant, while in other cases, integral average output measures (to estimate energy dissipation in damping elements) could be more significant.

Time history data can be used for model validation under more general conditions by applying techniques developed for system identification. Hull, Trankle and Klinger [1] described some of the criteria which have been used to compare time domain measurements and simulation outputs. These include fit error, residual autocorrelation, estimated parameter scatter, and covariance measures.

4.2.2 Dynamic Processes

Table 4.2 lists the dynamic processes which are typically represented by rail vehicle dynamic models. For each process, different variables are the significant indicators of vehicle performance and of model validation. The validation procedures and criteria must reflect this diversity. Previous studies have offered some suggestions, based on the physics which govern the dynamics of the vehicles, regarding which measurements (forces, displacements, etc.) to concentrate on for each dynamic process. For any particular model validation effort, the choice of which variables to use to establish validation



SINGLE VEHICLE:

- ROCK AND ROLL
- LATERAL STABILITY (HUNTING)
- CURVE ENTRY TRANSIENT
- STEADY-STATE CURVING
- VERTICAL FORCED RESPONSE
- LATERAL FORCED RESPONSE

MULTIPLE VEHICLES:

- LONGITUDINAL DYNAMICS
- WHOLE-TRAIN DYNAMICS

must be made on the basis of an intimate understanding of both the dynamic process being modeled and the individual candidate model.

It would not be productive to devote space here to listing the variables which should be scrutinized when validating models of each dynamic process. However, it should be noted that in general, both output and intermediate variables should receive attention. For example, in a lateral stability or hunting simulation, it is important to determine that the interbody forces and motions are represented correctly in the model, as well as having the model correctly predict the critical speed and damping of the least damped mode. If the internal workings of the model (the interactions which do not directly produce the outputs) can be shown to be valid, it is much more likely that the model can be used successfully to predict stability under new, previously untried conditions. It is not necessary to validate correct prediction of all internal variables, but engineering judgement should be applied to select those variables which should be most revealing of the important dynamic effects.

4.2.3 <u>Combination of Analytical Methods and</u> Dynamic Processes

The complete cross-categorization of the five analytical methods and eight dynamic processes which have been considered here is shown in Table 4.3. Not all of the possible combinations are reasonable. Some of those which are possible have been marked on the table. The linear time domain category is included, even though such models convey no more information than linear frequency domain models, because of the additional flexibility it offers in the choice of validation criteria.

It is obviously impractical to enumerate separate validation procedures for the 22 types of models indicated in Table 4.3. The dimensions

Table 4.3 Cross-Categorization of Rail Vehicle Dynamics Models

| | [| | ANALYTICAL METHO | DS | | |
|--|--------|--------|------------------|-------------|-----------|--|
| DYNAKIC | QUASI- | FREQ | UENCY DOMAIN | TIME DOMAIN | | |
| PROCESSES | STATIC | LINEAR | QUASI-LINEAR | LINEAR | NOMLINEAR | |
| SINGLE VEHICLES: Rock & Koli | | | x | | x | |
| Lateral Stability | | x | x | | x | |
| Curve Entry | | | | | x | |
| Steady-State Curving | x | X | x | | x | |
| Vertical Forced Response | | x | x | | x | |
| Lateral Forced Response | | X | x | | x | |
| MULTIPLE VEHICLES: Longitudinal Dynamics | | | · x | | x | |
| Whole-Train Dynamics | | 1 | x | | x | |

of this problem become still more daunting when one considers that each entry in the table can refer to many models, all having different degrees of freedom and nonlinear elements. This dimensionality problem is one of the principal obstacles to the formulation of a "black box" validation method applicable to all kinds of rail vehicle dynamic models.

4.3 SYSTEM IDENTIFICATION

The system identification approach operates in the time domain using measured, deterministic inputs to the model. It has been developed into an integrated system identification procedure for developing valid models that includes techniques for test planning, instrumentation analysis, model structure determination, parameter identification, and validation.

The techniques are especially useful in test design. Additional information about them is available in Ref. 1. They can be used to determine what measurements are required, what measurement accuracy is required, what the form of the input disturbances should be, and what test cases should be considered. Using system identification during test design to simulate the validation and testing can help guarantee the success of the validation.

V. APPLICATION TO A VERTICAL DYNAMICS MODEL

The validation methodology will be demonstrated on a six-degree-of-freedom, lumped parameter, vertical dynamics model of a six-axle locomotive. The model simulates forced response to vertical track irregularities. The model is linear, but for validation, is numerically integrated so that the response to a measured deterministic track input can be compited. The model normally would be implemented in a linear simulation (e.g., frequency response analysis). A time domain approach, as discussed in Ref. 1, has been chosen because it is compatible with the use of a deterministic, nonsinusoidal track input.

The outline of this chapter follows the flowchart given in Figure 4.2 for validating a model. The first section describes the model and the model purpose. The following sections describe validation criteria, review available data, and evaluate the performance for which it should be possible to validate the model. The actual testing of the model then takes place. Parameter values are assumed for the model. The model is integrated and its outputs compared to the measurements. Validation criteria are computed and conclusions are drawn about the range of validity of the model.

The vertical dynamics model is not validated in this chapter. As discussed in the introduction to this report, the scope of this project is limited to describing and demonstrating a validation procedure within the constraints imposed by limited project resources. Thus, at several points in the chapter, procedures are recommended but are not implemented. The procedures are discussed to show what could be accomplished in a complete validation task.

5.1 VERTICAL DYNAMICS MODEL

The locomotive vertical dynamics model is to be used to assess the effects of variations in vertical suspension damping on displacements and accelerations of the locomotive body. The locomotive is excited by irregularities in the track profile. The model is to be applicable over the full range of locomotive speeds.

The model predicts the time history of the vertical and pitch motions of the locomotive body and trucks. It is linear, except for the disturbances, so it could be solved by frequency response techniques or time domain techniques.

5.1.1 Model Description

A schematic drawing of the locomotive model is shown in Figure 5.1. This drawing describes the six degrees of freedom of the model. These include pitch and bounce of the two trucks and pitch and bounce of the locomotive body. The figure also defines the vehicle dimensions and notation used in developing the model equations of motion. All of the body motions are defined in a Newtonian reference frame moving at the constant forward velocity, V, of the locomotive.

The wheelsets are assumed to remain in constant contact with the rails. The vehicle suspension, wheelsets, and mass distribution are assumed to be symmetrical so the vertical and lateral dynamics of the vehicle are decoupled from each other. All flexibility is lumped at the suspension elements which are modeled by linear springs and viscous dampers. Angular displacements are assumed small in linearizing the equations.

Track profile (the vertical displacement, from its nominal value, of the midpoint of the line connecting the tops of the two rails) is taken to be the only track input exciting the vertical dynamics. Track cross-level and alignment are assumed to have no effect because the vehicle is symmetrical. Gage variation and rolling line offset are assumed small so that the effects on vertical dynamics are also small.

The resulting equations of motion are listed in Table 5.1. The notation used in the equations is defined in Table 5.2 and Figure 5.1. The derivation of these equations is straightforward and is not shown. The equations can be developed directly from Newton's laws of motion. Similar models have been described in prior publications [15,36,37].

5.1.2 Model Purpose and Validation Criteria

The vertical dynamics model is intended for use as a tool to assess the effect of variations in the vehicle suspension design on the displacement and acceleration of the vehicle body in response to track disturbances. The criteria for validation should measure how well the model predicts the trends in the displacements and accelerations with changes in the vehicle suspension design over the range of track disturbances and operating speeds. The model must also be phenomenologically accurate. There must be a direct correlation between component characteristics of the vehicle and parameters of the model so that design changes and suspension property changes can be implemented easily in the model.

In this particular case, an important aspect of the validation process is ascertaining whether a simple, linear, rigid-body model is adequate to represent the trends in locomotive vertical dynamics.

The vehicle response variables which are important for evaluating the validity of the locomotive vertical dynamics model are: body vertical and pitch accelerations, primary (wheelset) and secondary suspension vertical deflections, and truck frame vertical and pitch accelerations. The body accelerations are the primary outputs of the model, that is, the outputs which it is designed to predict. The other response variables are needed to ensure the phenomenological accuracy of the model, providing assurance that the primary outputs will be properly represented under conditions other than those tested. In particular, if the model does not agree with the test results for suspension response, the chances are not good that it will be able to represent the effects on body accelerations of changes in suspension characteristics.



Figure 5.1 Vertical Dynamics Model of Locomotive

Table 5.1 Six-DOF Vertical Locomotive Model CAR BODY

 ${}^{m}_{B}\ddot{y}_{B} = -2k_{yT} y_{B} -2c_{yT} \dot{y}_{B} + (l_{1} - l_{2})k_{yT} \dot{\psi}_{B} + (l_{1} - l_{2})c_{yT} \dot{\psi}_{B}$ ${}^{k}k_{yT} y_{T1} + c_{yT} \dot{y}_{T1} + k_{yT} y_{T2} + c_{yT} \dot{y}_{T2}$ ${}^{+(a_{3} + a_{4})k_{yT}\psi_{T2}} + (a_{3} + a_{4}) c_{yT} \dot{\psi}_{T2}$ ${}^{-(a_{3} + a_{4})k_{yT}\psi_{T1}} - (a_{3} + a_{4})c_{yT} \dot{\psi}_{T1}$

$$\begin{split} \mathbf{I} \psi_{\mathbf{B}} \ \ddot{\psi}_{\mathbf{B}} &= - \left[2k\psi_{\mathbf{T}} + (z_{1}^{2} + z_{2}^{2})k_{y\mathbf{T}} \right] \psi_{\mathbf{B}} - \left[2c_{\psi\mathbf{T}} + (z_{1}^{2} + z_{2}^{2})c_{y\mathbf{T}} \right] \dot{\psi}_{\mathbf{B}} \\ &+ (z_{1} - z_{2})k_{y\mathbf{T}} \ y_{\mathbf{B}} + (z_{1} - z_{2})c_{y\mathbf{T}} \ \dot{y}_{\mathbf{B}} \\ &+ z_{2}k_{y\mathbf{T}} \ y_{\mathbf{T}2} + z_{2}c_{y\mathbf{T}} \ \dot{y}_{\mathbf{T}2} - z_{1}k_{y\mathbf{T}} \ y_{\mathbf{T}1} - z_{1}c_{y\mathbf{T}} \ \dot{y}_{\mathbf{T}1} \\ &+ [k_{\psi\mathbf{T}} + (a_{3} + a_{4})z_{1}k_{y\mathbf{T}}]\psi_{11} + [c_{\psi\mathbf{T}} + (a_{3} + a_{4})z_{1}c_{y\mathbf{T}}]\dot{\psi}_{\mathbf{T}1} \\ &+ [k_{\psi\mathbf{T}} + (a_{3} + a_{4})z_{2}k_{y\mathbf{T}}]\psi_{\mathbf{T}2} + [c_{\psi\mathbf{T}} + (a_{3} + a_{4})z_{2}c_{y\mathbf{T}}]\dot{\psi}_{\mathbf{T}2} \end{split}$$

FRONT TRUCK

$$\begin{split} & \mathfrak{m}_{T} \ \ddot{y}_{T1}^{*} = -(k_{yT} + 3k_{y}) \ y_{T1} \ -(c_{yT} + 3c_{y} + c_{y1}) \ \dot{y}_{T1} \\ & +k_{yT}y_{B} + c_{yT} \ \dot{y}_{B} \ -\hat{z}_{1}k_{yT}\psi_{B} \ -\hat{z}_{1}c_{yT}\psi_{B} \\ & +[(a_{3} + a_{4})k_{yT}^{*} \ (a_{2} - a_{4})k_{y} \ +a_{4}k_{y} + \ (a_{1} + a_{4})k_{y}]\psi_{T1} \\ & +[(a_{3} + a_{4})c_{yT} \ -(a_{2} - a_{4})c_{y} \ +a_{4}(c_{y} + c_{y1}) + (a_{1} + a_{4})c_{y}]\psi_{T1} \\ & +k_{y}(v_{1} + v_{2} + v_{3}) \ +c_{y}(\dot{v}_{1} \ +\dot{v}_{2} + \dot{v}_{3}) \ +c_{y1}\dot{v}_{2} \\ I_{\psi T}\ddot{\psi}_{T1} = -[k_{\psi T} + (a_{3} + a_{4})^{2}k_{yT} + (a_{2} - a_{4})^{2}k_{y} + a_{4}^{2}k_{y} + (a_{1} + a_{4})^{2}k_{y}]\psi_{T1} \\ & -[c_{\psi T} + (a_{3} + a_{4})^{2}c_{yT} + (a_{2} - a_{4})^{2}c_{y} + a_{4}^{2}(c_{y} + c_{y1}) + (a_{1} + a_{4})^{2}c_{y}]\dot{\psi}_{T1} \end{split}$$

+ $[k_{\psi T} + \hat{z}_1(a_3 + a_4)k_{\gamma T}]\psi_B + [c_{\psi T} + \hat{z}_1(a_3 + a_4)c_{\gamma T}]\psi_B$ - $(a_3 + a_4)k_{\gamma T} y_B - (a_3 + a_4)c_{\gamma T} \dot{y}_B$

 $+ \{ (a_{3} + a_{4})k_{yT} - (a_{2} - a_{4})k_{y} + a_{4}k_{y} + (a_{1} + a_{4})k_{y} | y_{T1} + [(a_{3} + a_{4})c_{yT} - (a_{2} - a_{4})c_{y} + a_{4}(c_{y} + c_{y1}) + (a_{1} + a_{4})c_{y}] \dot{y}_{T1} + k_{y} [-(a_{1} + a_{4})v_{1} - a_{4}v_{2} + (a_{2} - a_{4})v_{3}] + c_{y} [-(a_{1} + a_{4})\dot{v}_{1} - a_{4}\dot{v}_{2} + (a_{2} - a_{4})\dot{v}_{3}] - a_{4}c_{y1}\dot{v}_{2}$

REAR TRUCK

- $$\begin{split} {}^{m}_{T} \ddot{y}_{T2} &= -(k_{yT} + 3k_{y})y_{T2} (c_{yT} + 3c_{y} + c_{y1})\dot{y}_{T2} \\ &+ k_{yT} y_{B} + c_{yT} \dot{y}_{B} + \frac{a_{2}k_{yT}\psi_{B}}{2} + \frac{a_{2}c_{yT}}{2} \dot{\psi}_{B} \\ &- [(a_{3} + a_{4})k_{yT} + (a_{1} + a_{4})k_{y} + a_{4}k_{y} (a_{2} a_{4})k_{y}]\psi_{T2} \\ &- [(a_{3} + a_{4})c_{yT} + (a_{1} + a_{4})z_{y} + a_{4}(c_{y} + c_{y1}) (a_{2} a_{4})c_{y}]\psi_{T2} \\ &+ k_{y}(v_{4} + v_{5} + v_{6}) + c_{y}(\dot{v}_{4} + \dot{v}_{5} + \dot{v}_{6}) + c_{y1}\dot{v}_{5} \\ I_{\psi T} \ddot{\psi}_{T2} &= -[k_{\psi T} + (a_{3} + a_{4})^{2}k_{yT} + (a_{1} + a_{4})^{2}k_{y} + a_{4}^{2}k_{y} + (a_{2} a_{4})^{2}k_{y}]\psi_{T2} \end{split}$$
 - $-[c_{\psi T} + (a_3 + a_4)^2 c_{y T} + (a_1 + a_4)^2 c_y + a_4^2 (c_y + c_{y1}) + (a_2 a_4)^2 c_y]_{T2}^{\downarrow}$
 - $+(a_{3}+a_{4})k_{yT}y_{B} + (a_{3}+a_{4})c_{yT}\dot{y}_{B}$
 - $+[k_{\psi T} + {}^{\varrho}_{2}(a_{3} + a_{4})k_{yT}]^{\psi}_{B} + [c_{\psi T} + {}^{\varrho}_{2}(a_{3} + a_{4})c_{yT}]^{\psi}_{B}$
 - $-[(a_{3}+a_{4})k_{yT} + (a_{1}+a_{4})k_{y} + a_{4}k_{y} (a_{2}-a_{4})k_{y}]y_{T2}$
 - $-[(a_{3}^{+} a_{4})c_{yT}^{+} + (a_{1}^{+} a_{4})c_{y}^{+} a_{4}(c_{y}^{+} c_{y1}^{+}) (a_{2}^{-} a_{4}^{+})c_{y}^{-}]\dot{y}_{TZ}^{+}$
 - + $k_{y}[-(a_{2}-a_{4})v_{4} + a_{4}v_{5} + (a_{1}+a_{4})v_{6}]$ + $c_{y}[-(a_{2}-a_{4})v_{4} + a_{4}v_{5} + (a_{2}+a_{4})v_{6}] + a_{4}c_{y1}v_{5}$

Table 5.2 Notation for Vertical Locomotive Model

| | MOTIONS | • |
|---|---|--|
| | УВ | Vertical displacement of carbody cg, positive up |
| | ΨB | Pitch angle of carbody, positive when front end down |
| | y _{T1} ,y _{T2} | Vertical displacement of front and rear trucks at the truck cg's positive up |
| | ^ψ τ1• ^ψ τ2 | Pitch angle of front and rear trucks, positive when front end of trucks are down |
| | PARAMETER | <u>s</u> |
| | a1,a2,a3, *1,*2, | a _g , Truck and carbody dimensions, (see Figure 5.1) |
| 1 | ^m 8 | Mass of carbody plus bolsters |
| | I _{\$\$B} | Pitch inertia of carbody plus bolsters |
| | m r | Truck mass (does not includ? wheelsets, drive motors, or bolsters) |
| | ^к ут• ^с ут | Vertical stiffness and equivalent viscous damping, truck to carbody suspension, per truck |
| | ĸ _{ŸŢ} , CŸŢ | Pitch stiffness and equivalent viscous damping, truck to carbody suspension, per truck |
| | к _у , с _у | Vertical stiffness and equivalent viscous damping, wheelset to truck, per alle. |
| | c _{yl} | Vertical viscous damping coefficient of two shock absorbers on middle axle of truck, per axle. |
| | DISTURBAN | ICES |
| | `1 ` 2'`3' | 4,5,6 Vertical irregularity of track at axle location, positive up. Axles numbered from front of locomotive to rear. |
| | [•] 1• [•] 2• [•] 3• | 4,5,6 Time rate of change of vertical irregularity of track at axle location |
| | | |

In addition to the choice of which response variables to consider, the selection of validation criteria includes the choice of which statistical measures to apply to each variable. This choice of statistics must be made carefully because of the implicit assumptions associated with the use of each statistical measure. Steady-state (or quiescent) values are useful for quantifying and then eliminating measurement system biases before comparing model predictions and test results, but they are not incorporated in the validation criteria for this model. Maximum value (peak) responses are significant for evaluating the validity of the model, and particularly for identifying worst-case conditions. It is especially important that the peak responses predicted by the model follow the same trend as those measured in the tests. However, the peak values are the hardest to model accurately because they are strongly dependent on random conditions and damping effects, which can be strongly nonlinear. Comparisons of frequency domain characteristics of the model and test results (natural frequencies and damping ratios) can be very revealing of model validity. Time histories produced in the tests and in simulations using the model can be compared quantitatively using the fit error (mean square difference between measured and predicted time histories). The fit error is easy to observe visually if the predicted and measured response are plotted on the same graph. Numerical values of the fit error should only be computed when there is already a fairly good visual fit because slight phase errors can cause the computed fit error to become very large and therefore meaningless. The fit error is not very useful as an absolute measure to compare with a specified validation criterion, but should be used to compare alternate model structures and parameter values.

The validation criteria chosen for the primary output variables of the locomotive vertical dynamics model, the body vertical and pitch accelerations, are:

| Maximum values | +25% each case |
|--------------------------|----------------|
| × | +10% on trends |
| Damped natural frequency | <u>+</u> 10% |
| Equivalent damping ratio | +50% |

As previously mentioned, the steady-state quiescent values are used to eliminate biases and the time history fit errors are used to compare alternate models once proper phasing is ensured.

The maximum value criteria were chosen to reflect both the importance of peak responses and the difficulty of modeling them. The representation of trends is more important than the prediction of absolute magnitude for the purposes this model is intended to address, and the trends should be less susceptible to random perturbations as well, leading to the tighter tolerances on trends. The tolerance on natural frequency is tight because of its fundamental importance in describing system dynamics and because it should also be relatively easy to predict on the basis of simple estimates of vehicle parameters (mass and stiffness properties). Although damping is also an important description of the vehicle's dynamics, the most common measure, damping ratio, is difficult to identify from test data, as well as not being applicable to nonlinear responses such as those produced by dry friction in suspensions. The exponential decay envelope assumed for linear damping cannot be matched directly to the triangular envelope produced by dry friction. Consequently, the tolerance on damping ratio was chosen to be much looser than the tolerance on natural frequency.

Other frequency domain measure such as rms and spectra were not chosen for use here because the PTT conditions did not produce the stationary responses which those measures assume.

5.2 DATA FOR VALIDATION

The best validation data available on the performance of a locomotive were gathered in the Perturbed Track Test (PTT) at the Transportation Test Center in Pueblo, Colorado, during November and December 1978. These tests were sponsored by the Office of Research and Development, Federal Railroad Administration, U.S. Department of Transportation.

In the PTT tests [25], E-8 and SDP-40F locomotive consists were operated at speeds between 35 and 80 mph over two PTT zones: a tangent zone on the Railroad Test Track and a 1.5 degree, 3" superelevation curved zone on the Train Dynamics Track at the Pueblo Test Center. The PTT zones were designed to excite significant dynamic responses under controlled conditions satisfying Class 4 track standards. The test zones included sections of alignment, cross-level, and profile perturbations, as well as a section of combined alignment and cross-level perturbations.

The principal test variables, besides the perturbed track sections, were speed, vehicle type and loading, locomotive position and orientation, rail surface condition, primary suspension damping, and restricted vertical coupler freedom.

The response of the test vehicles to the perturbed track was measured extensively using both on-board and wayside instrumentation. On the SDP-40F, the trailing truck was instrumented the most, with very little instrumentation on the front truck. The rear truck was instrumented for wheel/rail forces, accelerations, and relative displacements of its primary and secondary suspension components. Additionally, the coupler between the SDP-40F and the following baggage car was instrumented to measure coupler forces and angles. The lateral displacements between the wheels and rails were not measured. The track geometry was measured several times during the tests and the data used to generate a complete. track geometry data base.

Documentation on the PTT tests is readily Available [25-29].

Many test runs of the SDP-40F were made on the tangent track. A representative set of the available data which applies to the vertical dymanics of the SDP-40F on tangent track is deberibed in Table 5.3. The table lists the test tonditions, parameters, and inputs of the representative runs. Table 5.4 lists the response measurements taken during the runs that are applicable to validation of the vertical dynamics model. Some data specifying the valid range and frequency of the measurement system for each measurement are also given.

The track geometry was measured three days before the locomotive test runs described in Table 5.3 were performed. Table 5.5 lists the measurements made on the track.

5.2.1 Parameters for Model Validation

The parameters used in model validation should come from measurements of the vehicle(s) used in the testing. Parameters measured for a vehicle class are acceptable, though, if they are representative of the vehicle tested. Errors in the parameter values will show up as errors between the predicted and measured responses.

Parameter values for the SDP-40F locomotive are available in the PTT literature [25,27,30,31] and in the work of Garg, et al. [32]. Little of these data were measured on the vehicle used in the test. As shown in Table 5.6, data are available for all of the parameters but one. This parameter, truck pitch inertia, had to be estimated.

The data for some of the structural stiffnesses and damping are nonlinear. Since the model is linear, some procedure had to be used to linearize these data. The procedure followed needed to be well defined so that it could be repeated with consistent results.

In this analysis, equivalent linearized values for the parameters were obtained by estimating the amplitude and frequency of the deflections across the suspension elements, then computing the describing function quasi-linearizations [33,34]. This procedure could be applied in an iterative manner, as in Ref. 34, to correct the quasi-linearized values for each speed and input, but such an approach was beyond the scope of this preliminary analysis. Here a single linearized value for each parameter was used.

The parameter values used in the model are not exact. The sensitivity of the predicted responses to errors in the parameters could be computed using the system identification software, if necessary, to determine how accurate the parameters needed to be or, conversely, how much error to expect in the predicted responses due to errors in the parameters.

5.2.2 Measurement Suitability

Measurements of both the track inputs and the vehicle responses are required for validation. The track measurement required was track profile. Vehicle response measurements that would be useful are car body pitch and vertical acceleration; front and rear truck pitch and vertical acceleration; vertical displacement between wheelsets and truck sideframes; vertical displacement between the ends of the bolsters and the truck frames; and the displacement between the car body and bolster.

The accelerations of the car body are the outputs of the model. Truck motions and relative motions between the trucks, bolsters, and car body are of interest because they can be used to validate the suspension element models used in the vehicle model, and thus validate the phenomenological accuracy of the model. Not all of these measurements are required for validation. The track input (profile) and model outputs (car body vertical and pitch accelerations) are required, though some measurements of truck, truckversus-car, and truck-versus-wheelset motion must also be available to ensure phenomenological accuracy of the model.

The measurements which are available are listed in Tables 5.4 and 5.5. The track measurements required for validation were taken. Only some of the vehicle response measurements were included. There appear to be sufficient measurements to validate the vertical dynamics model.

The measurements were taken at a sampling rate of 256 Hz. This is well above the response frequencies of interest; thus, it is more than adequate.

The measurement accuracy, number of measurements, instrumentation range, instrumentation frequency range, and instrumentation phase shift should be considered for the validation task prior to testing. This was not possible here because the PTT tests were not performed specifically with this task in mind. These measurement requirements can be defined prior to testing by using some of the system identification techniques discussed in Refs. 1 and 35. The techniques involve testing the validation procedure using simulated data.

5.2.3 Performance Regimes

The portions of the PTT tests used here were run on tangent track with perturbations built into the track profile to excite the vertical dynamics modeled oy the locomotive model. The built-in perturbations were five cycles long, with smooth track following. The designed shape of the profile disturbance is shown in Figure 5.2. The number of cycles was chosen to allow the response of the vehicle to build to its maximum. The smooth section of track following the perturbation allows the rate at which the vehicle response disturbance damps out to be observed.

Table 5.3 PTT Test Runs for Validation of Vertical Dynamics Model

| | 1 | TRACK P | ERTURBATIONS | | · · | | |
|---------|---------|---------|--------------|--------------------------|----------------|-----------------------|----------------------|
| RUN NO. | PROFILE | X-LEVEL | ALIGNMENT | X-LEVEL AND ALIGNMENT | SPEED (MPH) | SUSPENSION DAMPING | COUPLER Alignment |
| 121001 | x | × | x | x | 40 | NO SHOCKS* | NOMINAL |
| 121002 | × | , × , | x | x | 50 | | u |
| 121004 | x | x | x | x | 60 | u | u |
| 121006 | x | x | x | x | 70 | u | ä |
| 121012 | x | x | x | x | 40 | | SHIMMED |
| 120903 | x | × | x | x | 40 | NOMANAL | NOMINAL |
| 120907 | x | × | x | × . | 60 | a | . B |

No shock absorbers in primary vertical suspension. Nominally, there is one 1800/1800 vertical shock on either end of the middle axle of each truck.

| | TRANSDUCER | | SIGNAL CONDITIONING AND RECORDING | |
|--|------------|---------|--------------------------------------|--------------------|
| MEASUREMENT | RANGE | FREQ | RANGE | FILTER CORNER FREQ |
| VERTICAL DISPLACEMENT, AXLE TO TRUCK AT JOURNALS OF REAR TRUCK | 10 IN. | 0-9 HZ | 5 IN. | 20 HZ (3DB) |
| VERTICAL ACCELERATIONS ON CAR BOCY TO COMPUTE BOUNCE, PITCH AND BENDING | 0-2 G'S | 0-90 HZ | 16 | 10 HZ (3DB) |

Table 5.4 Vertical Dynamics Measurements from PTT

Table 5.5 PTT Track Geometry Measurements

| GAGE |
|--------------------------------------|
| PROFILE LEFT RAIL RIGHT RAIL |
| ALIGNMENT LEFT RAIL RIGHT RAIL |
| CROSSLEVEL |
| DISTANCE ALONG TRACK |

*Taken every 6" along the track

| PARAMETER | VALUE | REFERENCE | COMMENTS |
|------------------|---|-------------|--|
| al | 79.65 in | 25,p.122 | |
| az | 83.75 in ' | м | |
| ag | 1.25 in | • | |
| aą | 8.5 in | н. | |
| £1 | 276 in | • | |
| r5 | 276 in | n | |
| mβ | 766. <u>1b_sec²</u> 1n | 25,p.128 | |
| Ι _φ Β | 40.x10 ⁶ 1b sec ² in | • | |
| πτ | 40 <u>1b sec²</u> 1n | 27,p.B-6 | |
| ι _ψ τ | .178x10 ⁶ 1b sec ² in | | NO DATA AVAILABLE. ASSUMED EQUAL TO TRUCK YAW INERTIA, REFERENCE 27, P. B-6 |
| ky⊺ | .501x10 ⁶ 1b in | 27, op.8-11 | |
| с _у т | 615 <u>lbsec</u> | · | |
| kψŢ | 633.x106 <u>in 1b</u> Rad | " | THE DATA GIVEN IN THE REFERENCES FOR THESE PARAMETERS WAS NONLINEAR. EQUIVALENT LINEAR VALUES WERE ESTIMATED FROM THIS DATA. |
| Сψт | .895x106 <u>in 1b sec</u> Rad | | |
| ky . | 11,300 <u>lb</u> in | 27,pp.3-14 | |
| су | 92 <u>1b sec</u> in | . / | |
| c _{y1} | 200 <u>1b sec</u> | 31,p.5-54 | |

Table 5.6 Vehicle Data for Vertical Dynamics Model



Figure 5.2 Planned Rail Profile Perturbation



1.00 2.00 3.00 4.00 5.00 6.00 7.00 8.00 9.00 SECONDS

11.0

10.0

Figure 5.4 Track Profile Rate

-3.09 -2.00

-1.00

0.0













The track perturbation should excite all the vertical resonances of the suspensions because it has discontinuities and is not a perfect sine.

The vehicle speed was varied from 30 to 80 mph in different runs over the disturbance. This allows the model to be validated over a range of speeds.

There is only one parameter variation that is applicable to the current validation task. This variation is the deletion of shock absorbers from the primary suspension. Thus it will not be possible to demonstrate the validity of the model for variations in primary suspension stiffness or secondary suspension design, as would be desirable.

5.2.4 Limitations of Data

The main limitation of the PTT data for validating the vertical dynamics model is that only one of the vehicle parameters used in the model (primary suspension vertical damping) was varied during the test. This limits the range of suspension variations for which the validity of the model can be demonstrated.

Not all the response measurements that were desired were taken. There are sufficient measurements for validation of the simple model described in Section 5.1, but not enough to study more detailed models of the suspension.

One of the parameters for the model (truck pitch inertia) was not known, but was approximated. In a comprehensive validation, the sensitivity of the model outputs to changes in this parameter should be computed.

5.3 COMPARISON OF MODEL OUTPUTS AND TEST RESULTS

Before the model outputs could be compared with the response measurements, the effect of the measurement system on the data had to be considered. Correcting for these effects is called data reconstruction. This is discussed below. Following this section, the comparison of results is made, and recommendations and conclusions drawn.

5.3.1 Data Reconstruction

5.3.1.1 Correlation of Track and Response Data

The track data and vehicle response data had to be reconstructed to correlate the track profile measurements with the vehicle response measurements. Additional reconstruction was necessary to create a time history of track profile rate-of-change data. Only the track profile as a function of position was given. The rate of change of track profile enters into the equations of motion for the vehicle through the primary suspension damping terms. The wheelsets are assumed to follow the track exactly. Realignment of the track records with the vehicle response records was not straightforward because the two sets of data were taken at two different, non-constant speeds. Signals from automatic location detectors (ALDs) were available on each data tape to use in realignment.

The vehicle response is both time (frequency) and position (track input) dependent. The model assumes the vehicle is moving at a constant speed. Thus, it is important that time and position be represented accurately on the data tapes.

The procedure used to align the data on the track data tape with the data on the vehicle response data tapes was to assume the distance record associated with the response measurements was correct and to "stretch" and "squeeze" the track measurement data so that the key locations, given by ALD signals on each tape lined up. The response data were then treated as coming from a constant-speed vehicle. Because the speed was not really constant, the track inputs appear to the model at times associated with a varying vehicle speed. Time correctness is kept by using the response data tape as is. Position relevance is kept by interpolating data off the track tape to line it up with the corresponding responses on the response tapes. The model gets data for a non-constant vehicle speed, but the small speed variation will not affect the creep forces, which are the only speed-dependent terms in the model. If the speed were too jittery, there might be an effect on the vehicle response, but such a speed variation was not observed.

5.3.1.2. Derivative of Track Inputs

Track profile (vertical irregularity) rates of change are required inputs to the vertical dynamics model. The inputs must be computed by differentiating the track profile data. Because of the "scatter" of the data, simple differencing methods produce extremely noisy, unusable data. Fourier transform techniques were used to smooth and differentiate the data.

5.3.1.3 Measurement Models

In system identification, the response measurements are compared with model predictions of the measurements, rather than trying to compare the model outputs with some functions of the measured responses derived to be the same as the outputs that are modeled.

The measurement transducers commonly do not measure the exact states used in the model. Also, the measurement system may lose some of the available information when the measurements are filtered or digitized. It is not always possible to recreate the state without using the model. Therefore, the measurement system is modeled as part of the vehicle model. The model of the measurement system processes (i.e., filters and combines) the model outputs in the same manner as the measurement system processes the responses. Vertical displacement measurements and vertical acceleration measurements, as described in Table 5.4, are to be used to validate the vertical dynamics model. The measurement system ranges and cut-off frequencies are shown in Table 5.4. These ranges and frequencies were chosen to encompass the expected range and frequency of the dynamic response. If it turns out that this is true (it should be checked during the validation) then it will not be necessary to filter and limit the outputs of the model to represent the measurement system limitations.

5.3.2 Responses

Initial results of the comparison of the model outputs with measurements from one test run (Number 120903, as described in Table 5.3) are shown in Figures 5.3 to 5.8. These plots show the measured value of the actual as-built track profile disturbance, the computed rates of change of the disturbance, plus plots comparing the measured responses with the responses predicted with the model. Note in Figure 5.3 that the as-built track profile resembles the profile that was planned, Figure 5.2, but that there are significant differences as well. Note the smoothing of the "corners" (transitions) of the piecewise linear profile, the amplitude variations from one perturbation to the next and the significant profile variations on the "unperturbed" segment of track. This shows the importance of measuring the actual track profile for validation.

Figure 5.4 shows the rate of change of the track profile computed from the data shown in Figure 5.3 using Fourier techniques. These techniques were applied successfully to computing a usable derivative by digitally filtering out high-frequency noise.

Figure 5.5 shows the measured and estimated car body bounce acceleration. Figure 5.6 shows the measured and estimated car body pitch acceleration. Figures 5.7 and 5.8 show the vertical displacement between the front axle of the rear truck and the truck frame. The measurement bias apparent in Figure 5.7 was removed, using the analyst's judgement, to produce Figure 5.8.

5.3.3 Validation Criteria

The validation criteria to be considered are fit error, peak acceleration, frequency and damping of the responses. The peak values, frequency and damping of the responses will be reviewed first.

Figure 5.5 shows measured and predicted car body bounce acceleration for the transient track perturbation shown in Figure 5.3. This figure shows that the predicted frequency is slightly lower than the measured frequency (1.50 Hz compared to 1.57 Hz, or 96%), but well within the limit of the validation criterion. The predicted peak bounce acceleration is 0.10g. The measured peak acceleration is 0.08g. The ratio of predicted peak acceleration to measured is 1.25, just within the limit for validation. The predicted damping for this response appears to be low compared to the measured damping because the measured response damps out more quickly than the predicted response and the peak measured accelerations are lower than predicted. Numerical values for the damping could be estimated roughly from the decay rates but, because of the dry friction in the suspension, this damping will vary with speed and response amplitude. Also, the decay envelope for viscous damping is different from that due to dry friction (exponential for viscous, linear for friction), complicating the comparison.

Fit error [1] is the other criterion that was suggested to validate the model. Fit error is the mean square difference between the predicted and measured responses. Because of the slight error in predicted frequency (shown in Figure 5.5), the numerical fit error for this response would be extremely large and thus would not be useful. This index would not be able to indicate that a small correction in frequency would produce an extremely good fit. However, the purely visual measure of fit error is very useful here as it gives the user of the model a quick, clear indication of how good the fit is.

The agreement between the predicted and measured values of pitch acceleration, as shown in Figure 5.6, is not as close as the agreement for bounce acceleration. Although there is no apparent discrepancy between predicted and measured frequencies in that figure, the predictions of peak pitch accelerations exceed the measured peaks by amounts ranging from about 10% to about 200% (outside the limit for validation). In particular, the measured response was characterized by sharply alternating peaks of large and small amplitudes, while the estimated peaks did not vary by nearly as much from one to the next. This discrepancy was systematic enough to indicate that the model was not fully characterizing the body pitch acceleration response. More detailed evaluation of the model and test results would be needed to establish the cause of the discrepancy (such as an inadequate suspension model or neglect of car body flexibility, for example). This would occur within Step 8 of the validation procedure, in the box labelled "Seek Explanation of Differences," in Figures 4.1 and 4.2.

The substantial bias separating the predicted and measured axle displacements in Figure 5.7 demonstrates the need for careful visual inspection of all responses before quantitative comparisons are attempted. Once the bias is computed and then removed, the comparisons can proceed (Figure 5.8). The absence of frequency or phasing discrepancies is expected, because the timing of the axle displacement response is fixed by the track perturbation input (under the assumption that each wheel always remains in contact with the rail). The degree of agreement in the magnitudes and shapes of the peaks is a reflection of the validity of the modeling of the locomotive's suspension. The higher amplitude and sharpness of the predicted relative to the measured peaks (especially the lower amplitude peaks) is a reflection of lower effective damping from the equivalent linear viscous dampers in the model, an inherent difficulty when using a linear model of nonlinear friction. Although the axle displacement is not one of the principal outputs needed to satisfy the model purpose, the comparison between predicted and measured axle displacements is needed to help ensure the phenomenological validity of the model (comparison of responses other than principal model outputs).

In the one case illustrated, the damping ratios could not be readily quantified for the test results, but were probably within the +50% tolerance of the model. The model produced a valid prediction of bounce acceleration, but not pitch acceleration, for this case. Nothing is yet known about its ability to predict trends or accommodate other performance regimes.

5.3.4 Range of Validity

Test data from the PTT are available on the vehicle modeled for a range of speeds and two suspension variations. Because this report is only a preliminary demonstration of the validation approach, only one test case was analyzed. This is not sufficient to establish model validity; however, much experience was gained from this application. In the initial work (not shown in the plots) the model outputs were found to be extremely sensitive to variations in the suspension parameters. This makes the procedure for approximating linear damping and stiffness coefficients from lab test data very critical. The parameters used for the results shown in Figures 5.4 through 5.8 were linearized from lab test data using sinusoidal input describing functions and assumed values of the response amplitudes and frequencies. This application was incomplete because no iteration was performed to correct the describing functions calculated using assumed amplitudes and frequencies by recomputing them using the frequencies and amplitudes observed in the simulations. The responses were also not purely sinusoidal. Also, quasi-linearization of the friction forces requires knowledge of the tractive effort forces. These forces were only estimated during this study.

Following the adjustments indicated above, the next step in the validation procedure should be comparing the model predictions and test results for several different speeds. The consistency with which the validation criteria are satisfied for the different speeds provides an indication of the validity of the assumed model structure. If the agreement between model predictions and test results suffers as the speed changes, the model structure is likely to be deficient. Adjustments which are made to the model parameters to improve agreement at any individual speed must be tested at the other speeds to ensure that the model remains valid over the entire speed range (or to identify the speed range for which it remains valid).

Once the validity of the model is established for the baseline loccmotive, the suspension parameters should be changed to represent the alternative configuration (without primary suspension shock absorbers) and the predictions of this version of the model compared with the test data gathered for the alternative suspension. The validation criteria should then be applied to these cases in order to determine whether the model can represent the effects of suspension changes.

These steps require multiple executions of the simulation and comparisons of the simulation results with the test data for the two vehicle configurations at the various test speeds. The processing of the lest data tapes requires substantial computational effort (and expense) for each test case because of the quantity of data involved and complications introduced by tape labels and headers. The analyst must spend substantial time as well with each test case to detect and correct anomalies, such as instrumentation biases, and to ensure that track and vehicle measurements are properly synchronized. He must also decide what adjustments to the model parameters are needed to improve agreement with the test results, often on the basis of limited available evidence. The validation process cannot be considered complete until these steps have been successfully accomplished.

There was insufficient time to validate many of the assumptions used in deriving the model, such as (1) decoupling of lateral and vertical dynamics due to vehicle symmetry, (2) insignificant locomotive body bending, (3) insignificant train interaction effects, and (4) reasonably linear dynamics. In a comprehensive study, these assumptions could be validated because the PTT data contain sufficient measurements. For example, the measurements of vertical acceleration at the center and ends of the locomotive body could be used to determine the significance of the omitted first body bending mode. Test results for different coupler configurations could be used to assess the significance of train interactions.

VI, CONCLUSIONS

This report has documented a systematic methodology for validating rail vehicle dynamics models and has demonstrated the initial steps of that methodology for a relatively simple model. The key issues which must be considered when working on model validation have been highlighted in this report and explicitly incorporated into the methodology. The approach described here incorporates the use of system identification methods for careful advance planning of test cases and selection of instrumentation. The technology for conducting the testing is within the current state of the art, but the care with which this technology must be applied exceeds that for most other types of testing.

The validation procedures developed here cannot be applied in a purely algorithmic way because of the great diversity among the types of models and test data involved. The judgement of a highly skilled analyst remains central to the validation process. This report provides that analyst with some guidance and a framework for structuring his particular validation effort, but it cannot be used as a "cookbook" for unsophisticated analysts.

The example application of the validation procedures in Chapter V illustrated some of the difficulties involved in validating a model using data which were not collected for purposes of model validation. This example only proceeded as far as the initial comparison between one set of test data and the model prediction for comparable conditions. The procedure would need to continue with model adjustments and use of data gathered at other vehicle speeds in order to complete the validation for the nominal vehicle configuration. Comparable test cases for other configurations would then need to be compared with the model predictions to validate the model struc-These steps, needed to complete the model ture. validation, were beyond the scope of the current study.

The data collected on locomotives in the PTT program at Pueblo in 1978 were found to be the most suitable available in the public domain for model validation, even though the test program was not designed with validation as a principal The track perturbations, selection of goal. operating speeds, variations in vehicle characteristics, and number of test runs were suitable for use in model validation. The measurements which were taken were usable for validation of the vertical forced response model considered here, although they would not have been entirely adequate for some other models (particularly those involving lateral wheel/rail interactions). Additional information about measurement system accuracies, noise, bandwidths, biases, and dynamic range (truncations) would have been very useful.

In general, a test program must be designed specifically with model validation in mind if its results are to be useful for validation work. The tests must be planned to provide inputs to excite all important modes of response, measurements of inputs, vehicle parameters, and all relevant outputs (vehicle responses), and sufficient test cases to cover all performance regimes of interest. The test cases under consideration must be simulated by the candidate model prior to testing in order to ensure that all the necessary conditions have been satisfied (performance regimes, measurement ranges, etc.). The tests must be designed to ensure consistency of inputs from one run to the next, and the test procedures should facilitate the synchronization of vehicle and track measurements when the data are processed.

The data processing phase of the model validation process should not be short-changed. Significant and potential costly data processing is required for reformatting of data tapes (particularly if supplied by outside organizations) and aligning track and vehicle data. If the computers used to read and write the data tapes are not fully compatible, some extensive processing, with much analyst intervention, may be required. The process of matching time-dependent vehicle data with space-dependent track data and interpolating is very laborious unless provision is made for this at the time of data collection. This process should be mechanized to the extent possible to minimize wasted effort by separate users of test data collected for model validation.

In conclusion, no fully successful validations of rail vehicle dynamics models have come to light in the course of this study. Few test programs have been designed for purposes of validating models, thereby almost guaranteeing the absence of successful validations. The technology required for model validation testing is inherently no more exotic than any other testing technology, but the tests must be very carefully planned and executed if they are to be successful. Significant effort must be devoted to processing the test data and comparing it with model predictions. This activity, occurring at the end of the validation process, is the most likely to be chort-changed when resources run low, as well as being the activity for which the costs are most difficult to estimate in advance.

Model validation testing is likely to be more costly than standard performance testing because of its greater data processing requirements and the need to perform a substantial number of separate tests. The economic advantage of investing in validation tests comes from the ability to use the validated model to predict performance for vehicle configurations and operating conditions which need not be tested. The operation of the model for these new cases will be orders of magnitude less costly than additional tests, particularly if those tests would involve potentially hazardous conditions or the fabrication of new vehicle designs or equipment.

REFERENCES

- Hull, R.L., T.L. Trankle, and D.L. Klinger, <u>Application and Evaluation of System Identification Techniques to Rail Vehicle Dynamics, Systems Control, Inc. (Vt) Report No.</u> TR-5307-100, November 1979, for Transportation Systems Center, Cambridge, MA.
- Cooperrider, N.X., E.H. Law, R.L. Hull, P.S. Kadala, and J.M. Tuten, "Analytical and Experimental Determination of Nonlinear Wheel/ Rail Geometric Constraints," U.S. DOT Report No. FRA-OR&D 76-244 (PB 25290), Dec. 1975.

- Hsu, T.K. and D.A. Peters, "A Simple Dynamic Model for Simulating Draft-Gear Behavior in Rail-Car Impacts," ASME Faper No. 78-RT-2.
- 4. Ward, E.D. and R.G. Leonard, "Automatic Parameter Identification Applied to a Railroad Car Dynamic Draft Gear Model," <u>Journal</u> of Dynamic Systems, Measurement, and Control, December 1974, pp. 460-465.
- Boocock, D., "Steady-State Motion of Rail Vehicles on Curved Track," <u>Journal of Mechanical Engineering Science</u>, Vol. 11, No. 6 1969, pp. 556-566.
- Office for Research and Experiments, International Union of Railways, <u>Question Cl16 -</u> <u>Interaction Between Vehicles and Track</u>, Report No. 2, October 1972.
- Hutchens, W.A., E.C. Haight, and J.L. Milner, "Analysis of the Dynamics of a Rail Car from Its Response to Random Inputs," <u>High</u> <u>Speed Ground Transportation Journal</u>, Vol. 9, No. 1, 1975, pp. 449-457.
- Abbott, P.W., G. Morosow, and J. MacPherson, "Track-Train Dynamics," SAE Paper 751058, 1975.
- Healy, M.J., "A Computer Method for Calculating Dynamic Responses of Nonlinear Flexible Rail Vehicles," ASME Paper 76-RT-5.
- Elkins, J.A. and R.J. Gostling, "A General Quasi-Static Curving Theory for Railway Vehicles," <u>The Dynamics of Vehicles on Roads</u> and on Tracks (Proceedings of 5th VSD -2nd IUTAM Symposium at Technical University, Vienna, Austria, September 1977), Swets and Zeitlinger, B.V., Amsterdam, pp. 388-406.
- Helms, H. and W. Strothmann, "Lateral Rail Irregularities: Measurement and Application," <u>The Dynamics of Vehicles on Roads and on Tracks</u> (Proceedings of 5th VSD - 2nd IUTAM Symposium at Technical University, Vienna, Austria, September 1977), Swets and Zeitlinger, B.V., Amsterdam, pp. 430-449.
- 12. Illingworth, R., "Railway Wheelset Lateral Excitation by Track Irregularities," <u>The Dynamics of Vehicles on Roads and on Tracks</u> (Proceedings of 5th VSD - 2nd IUTAM Symposium at Technical University, Vienna, Austria, September 1977), Swets and Zeitlinger, B.V., Amsterdam, pp. 450-458.
- Kachadourian, G. and N.T. Tsai, "Freightcar Vibration and Analysis Comparison - Validation of FRATE," SAE Paper 781049.
- 14. Cooperrider, N.K., E.H. Law, R.H. Fries, and N.T. Tsai, "Theoretical and Experimental Research on Freight Car Lateral Dynamics," Paper presented at Heavy Haul Railways Conference, Perth, Australia, September 1978.

- Garg, V.K., "Computer Models for Railway Vehicle Operation," <u>Rail International</u>, June 1978, pp. 381-396.
- Rinehart, R.E., "Hunting Stability of the Three-Axle Locomotive Truck," ASME Paper 78-RT-6.
- 17. Sweet, L.M., J.A. Sivak, and W.F. Putnam, "Nonlinear Wheelset Forces in Flange Contact. Part 2: Measurement Using Dynamically Scaled Models," <u>Journal of Dynamic Systems, Measurement and Control</u>, Vol. 101, No. 3, September 1979, pp. 247-255.
- Hasselman, T.K. and L. Johnson, "Validation and Verification of Rail Vehicle Models," ASME Paper 79-WA/DSC-8.
- Fallon, Jr., W.J., N.K. Cooperrider and E.H. Law, "An Investigation of Techniques for Validation of Railcar Dynamic Analyses," Report No. FRA/ORD-78/19, March 1978 (PB2799-96).
- Broersen, P.M.T., "Estimation of Multivariable Railway Vehicle Dynamics from Normal Operating Records," <u>Proceedings 3rd IFAC</u> <u>Symposium on Identification and System</u> <u>Parameter Estimation</u>, the Hague/Delft, 1973.
- Herzog, W.N., "Identification of Wheelset/ Rail Crrep Coefficients from Dynamic Response Data Using the Maximum Likelihood Parameter Identification Technique," Masters Thesis, School of Engineering and Applied Science, Princeton University, Princeton, N.J., January 1979.
- 22. Cooperrider, N.K. and E.H. Law, "A Survey of Rail Vehicle Testing for Validation of Theoretical Dynamic Analyses," <u>Journal of Dynamic Systems, Measurement and Control</u>, Vol. 100, No. 4, December 1978, pp. 238-251.
- Johnson, L., A. Gilchrist, M. Healy, C. Bush, and G. Sheldon, <u>Truck Design Optimiza-</u> <u>tion Project Phase II - Analytical Tool</u> <u>Assessment Report</u>, Report No. FRA/ORD-79-36, <u>August 1979.</u>
- Gupta, N.K. and W.E. Hall, "Design of Sensor Systems for State and Parameter Estimation," Journal of Guidance and Control, Vol. 1, No. 6, Nov-Dec 1978, pp. 397-403.
- 25. Coltman, M., R. Brantman, and P. Tong, "A Description of the Tests Conducted and Data Obtained During the Perturbed Track Test," Report No. FRA/ORD-80/15, January 1980.
- 26. Palmer, D.W., M.E. Hanson, and I. Sheikh, "Perturbed Track Test Onboard Vehicle Response Data Base: User's Manual," Draft Report from Arthur D. Little to U.S. DOT TSC, ADL 82919, June 30, 1980.

- 27. Abbott, P.W., "Track/Irain Dynamics, Test Results, New HTC Truck Static Test," Contract NAS8-29882, Martin Marietta Corp., Denver, Colorado, 1 March 1979.
- 28. Vanstone, R.A., <u>Perturbed Track Test Data</u> <u>Acquisition Instrumentation Systems Handbook</u>, Prepared for DOT/Transportation Systems Center under Order No. TSC-15941, December 28, 1978.
- 29. Cohen, Martin L., <u>Preliminary Error Analysis</u> of the Perturbed Track Test Measurement System. Draft report to USDOT, Transportation Systems Center, Arthur D. Little, Inc., Report No. ADL C-82919, August 28, 1979.
- Letter: Nayak, P.R., Arthur D. Little, to John Mirabella, FRA, April 28, 1978.
- 31. Tong, P., R. Brantman, R. Greif, and J. Mirabella, "Tests of the AMTRAK SDP-40F Train Consist Conducted on Chessie System Track," Report No. FRA-OR&D-79/19, US DOT FRA, May 1979.
- 32. Garg, V.K. and K.D. Mels, "Lateral Stability cf a Six-Axle Locomotive," ASME Paper No. 75-RT-7, presented at the IEEE-ASME Joint Railroad Conference, San Francisco, CA, April 15-17, 1975.
- Gelb, A., and W. VanderVelde, <u>Multiple-Input</u> Describing Functions and Nonlinear Design, McGraw-Hill, New York, New York, 1968.
- 34. Hull, R., and N.K. Cooperrider, "Influence of Nonlinear Wheel/Rail Contact Geometry on Stability of Rail Vehicles," <u>Transactions of</u> <u>the ASME, Journal of Engineering for Indus-</u> <u>try</u>, February 1977, pp. 172-185.
- Gupta, N.K., and W.E. Hall, Jr., "Input Design for Indentification of Aircraft Stability and Control Derivatives," NASA CR-2493, February 1975.
- 36. Dimasi, F.P. and A.B. Perlman, "Frequency Domain Computer Programs for Prediction and Analysis of Rail Vehicle Dynamics," Reports No. FRA/ORD-76/135.I and .II, December 1975 (PB259287 and PB259288).
- 37. Chang, E.H. and V.K. Garg, "Technical Documentation of the 6-Axle Locomotive Response Model," AAR/TTD Report.

Rail Vehicle Dynamics Model Validation, 1981 US DOT, FRA, Steven E Shladover, Russel L Hull

LIEVALA A HONELT LIEVALA A HONVELL LIEVALA A HONVELL