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# Modeling and Simulation of a Superconducting EMS-Type Maglev Vehicle/Guideway System

**Authors:**
Mark L. Nagurka* and Ssu-Kuei Wang*

**Performing Organization Name(s) and Address(es):**
*Carnegie Mellon Research Institute
700 Technology Drive, P.O. Box 2950
Pittsburgh, PA 15230-2950

**Sponsoring/Monitoring Agency Name(s) and Address(es):**
Volpe National Transportation Systems Center
55 Broadway, Kendall Square
Cambridge, MA 02142

**Abstract:**
Computer simulation models are developed for predicting the dynamic performance of a magnetically levitated (maglev) vehicle with a superconducting (SC) electromagnetic suspension (EMS) traversing a flexible, multiple span, elevated guideway. A single Degree Of Freedom (DOF) vehicle model incorporating model SC magnet dynamics as well as a five DOF vehicle model are developed. The SC magnets are controlled via Linear Quadratic (LQ) optimal control augmented with integral action. Vehicle disturbances include crosswind gusts as well as guideway deflections due to span compliance, step, ramp, camber, and versine irregularities, and random roughness.

A first simulation study tests LQ optimal preview control on the single DOF maglev vehicle operating on a rigid guideway. The results show that the controller regulates the air gap to a nominal value, achieves zero steady-state gap error due to a step disturbance, and offers improved performance relative to a standard controller without preview. A second simulation study applies LQ optimal control to the five DOF maglev vehicle model negotiating a multi-span flexible guideway. The results suggest that requirements on the air gap safety margin, control voltage limit, and ride quality can be satisfied for the cases studied.

**Subject Terms:**
Magnetically levitated (maglev) vehicle, vehicle/guideway interaction, vehicle dynamics, maglev control, preview control

**Security Classification:**
Unclassified

**Number of Pages:**
110

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* Disclaimer: This document is a report of work conducted by Carnegie Mellon Research Institute as part of Statement of Work Pursuant to Task Order No. 53476 Vehicle, Guideway and Terminal Systems Contract No. DTR5-57-99-D-00027, TTD No. VA-3204 "Dynamic Analysis Support for Evaluation of HSGT Systems."
### METRIC/ENGLISH CONVERSION FACTORS

#### ENGLISH TO METRIC

**LENGTH (APPROXIMATE)**
- 1 inch (in) = 2.5 centimeters (cm)
- 1 foot (ft) = 30 centimeters (cm)
- 1 yard (yd) = 0.9 meter (m)
- 1 mile (mi) = 1.6 kilometers (km)

**AREA (APPROXIMATE)**
- 1 square inch (sq in, in²) = 6.5 square centimeters (cm²)
- 1 square foot (sq ft, ft²) = 0.09 square meter (m²)
- 1 square yard (sq yd, yd²) = 0.8 square meter (m²)
- 1 square mile (sq mi, mi²) = 2.6 square kilometers (km²)
- 1 acre = 0.4 hectare (ha) = 4,000 square meters (m²)

**MASS - WEIGHT (APPROXIMATE)**
- 1 ounce (oz) = 28 grams (gm)
- 1 pound (lb) = 0.45 kilogram (kg)
- 1 short ton = 2,000 pounds (lb) = 0.9 tonne (t)

**VOLUME (APPROXIMATE)**
- 1 teaspoon (tsp) = 5 milliliters (ml)
- 1 tablespoon (tbsp) = 15 milliliters (ml)
- 1 fluid ounce (fl oz) = 30 milliliters (ml)
- 1 cup (c) = 0.24 liter (l)
- 1 pint (pt) = 0.47 liter (l)
- 1 quart (qt) = 0.99 liter (l)
- 1 gallon (gal) = 3.8 liters (l)
- 1 cubic foot (cu ft, ft³) = 0.3 cubic meter (m³)
- 1 cubic yard (cu yd, yd³) = 0.76 cubic meter (m³)

**TEMPERATURE (EXACT)**
- \[(x - 32)(5/9)]°F = y°C
- \[(9/5)(y + 32)]°C = x°F

#### METRIC TO ENGLISH

**LENGTH (APPROXIMATE)**
- 1 millimeter (mm) = 0.04 inch (in)
- 1 centimeter (cm) = 0.4 inch (in)
- 1 meter (m) = 3.3 feet (ft)
- 1 meter (m) = 1.1 yards (yd)
- 1 kilometer (km) = 0.6 mile (mi)

**AREA (APPROXIMATE)**
- 1 square centimeter (cm²) = 0.16 square inch (sq in, in²)
- 1 square meter (m²) = 1.2 square yards (sq yd, yd²)
- 1 square kilometer (km²) = 0.4 square mile (sq mi, mi²)
- 10,000 square meters (m²) = 1 hectare (ha) = 2.5 acres

**MASS - WEIGHT (APPROXIMATE)**
- 1 gram (gm) = 0.036 ounce (oz)
- 1 kilogram (kg) = 2.2 pounds (lb)
- 1 tonne (t) = 1,000 kilograms (kg) = 1.1 short tons

**VOLUME (APPROXIMATE)**
- 1 milliliter (ml) = 0.03 fluid ounce (fl oz)
- 1 liter (l) = 2.1 pints (pt)
- 1 liter (l) = 1.06 quarts (qt)
- 1 liter (l) = 0.26 gallon (gal)
- 1 cubic meter (m³) = 36 cubic feet (cu ft, ft³)
- 1 cubic meter (m³) = 1.3 cubic yards (cu yd, yd³)

**TEMPERATURE (EXACT)**
- \[(x - 32)(5/9)]°F = y°C
- \[(9/5)(y + 32)]°C = x°F

#### QUICK INCH-CENTIMETER LENGTH CONVERSION

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#### QUICK FAHRENHEIT-CELSIUS TEMPERATURE CONVERSION

| °F | -40 | -30 | -20 | -10 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|----|-----|-----|-----|-----|---|----|----|----|----|----|----|----|----|----|----|----|
| °C | -40 | -30 | -20 | -10 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

For more exact and or other conversion factors, see NIST Miscellaneous Publication 286, Units of Weights and Measures. Price $2.50. SD Catalog No. C13 10286.

Updated 9/29/95
EXECUTIVE SUMMARY

This study investigates the dynamic interactions between magnetically levitated (maglev) vehicles employing electromagnetic suspension (EMS) systems and elevated flexible guideways. In EMS or "attractive" designs, vehicle levitation and guidance is achieved by attraction between vehicle-borne magnets and iron rails mounted to a guideway. EMS maglev systems rely on feedback control to actively position the vehicle on the guideway to achieve a nominal air gap and ensure overall safe performance. Furthermore, for configurations such as the Grumman concept, which involves a superconducting (SC) EMS system with no secondary suspension, the control system plays a critical role in providing acceptable passenger ride comfort.

The objective of this work is to develop an analytic tool and computer simulation model for predicting the safety- and comfort-related dynamic performance of a SC EMS-type maglev vehicle operating over a flexible, multiple span, elevated guideway. A crucial feature in evaluating vehicle performance is the effectiveness of the suspension control system in accommodating guideway geometry disturbances and wind force excitations.

A sequence of dynamic models is developed in this work. We first derive a magnet model which characterizes the behavior of the on-board SC magnets. A single Degree of Freedom (DOF) linear vehicle model incorporating the magnet model is constructed to test different optimal control methods. A five DOF nonlinear vehicle model, representing lateral, vertical, roll, pitch, and yaw motions, is then developed for more realistic simulation studies. The vehicle's magnet modules are controlled using Linear Quadratic (LQ) optimal control. The LQ optimal control is augmented with integral action to avoid steady-state gap errors which might otherwise arise from guideway offsets or constant crosswind gusts.

A simply supported, multi-spanned, tangent guideway model is proposed to evaluate vehicle dynamic response for a range of guideway geometry and wind force inputs. In addition to the crosswind gust, the disturbances imposed on the maglev system include guideway deflection...
due to inherent span compliance and guideway irregularities such as random roughness and offsets (steps, ramps, camber, and versine irregularities). The design criterion is to minimize the gap errors and passenger accelerations without exceeding limits on the controlled voltage of the magnet modules.

Two main simulation studies are conducted. The first study applies the LQ optimal preview controller with integral action to the single DOF maglev vehicle operating on a rigid guideway. The purpose is to evaluate the effectiveness of an optimal controller with and without preview. In the case of preview, it is assumed the controller has access to guideway position information before arrival of the maglev vehicle. In the simulation, the vehicle is traveling at 100 m/s over a step guideway offset of 10 mm. The results show that the controller (i) stabilizes and regulates the air gap to a nominal value, (ii) achieves zero steady-state gap error due to the step disturbance, and (iii) offers improved performance relative to an optimal controller without preview by reducing the input voltages, gap errors, and vehicle accelerations.

The second simulation study applies LQ optimal control with integral action (but without preview) to the five DOF maglev vehicle model carrying multiple magnet modules. The maglev vehicle traverses a multi-span flexible guideway subject to different guideway irregularities as well as crosswind gust. The vehicle performance is evaluated via (i) limits on the gap error for safety, (ii) carbody accelerations relative to the ISO ride quality criteria for ride comfort, and (iii) constraints on the controlled applied voltage for practical implementation. The study investigates the individual effects of guideway flexibility, step guideway irregularities, and crosswind gust alone, and then considers general excitation consisting of combined guideway flexibility, step, ramp, camber, and random roughness disturbance inputs, and crosswind gust. The results suggest that the requirements on the air gap safety margin, control voltage limit, and ride quality can be satisfied for the simulation cases studied.

The computer simulation work, although limited, offers insights into the nature of the dynamic interaction that can be expected in high-speed EMS maglev operation. The investigation shows that to achieve acceptable safety and performance, maglev designs require detailed
dynamic analyses that account for the governing behavior of their magnet modules, vehicle and guideway DOFs, and controller structure as well as the interaction coupling the vehicle, guideway, and control subsystems.
LIST OF SYMBOLS

\(a_j\)   modal amplitude of \(j\)-th mode for Span I

\(A\) system matrix

\(A_a\) augmented system matrix

\(A_c\) closed-loop system matrix

\(A_m\) face area of each magnet pole

\(A_r\) roughness parameter

\(A_s\) carbody side area

\(b_j\) modal amplitude of \(j\)-th mode for Span II

\(B\) control influence matrix

\(B_a\) augmented control influence matrix

\(B_c\) coefficient matrix used in equation (76)

\(B_m\) magnetic flux density in the air gap

\(c\) viscous damping coefficient

\(C\) coefficient matrix in output equation

\(C_d\) drag coefficient

\(D\) coefficient matrix in output equation

\(E_a\) coefficient matrix in augmented state equation (85)

\(EI\) bending rigidity of the guideway span

\(f\) magnet force

\(f_o\) nominal magnet force

\(F_d\) drag force
LIST OF SYMBOLS (cont.)

$F_x$ total magnet force component in the $X_r$ direction

$F_y$ total magnet force component in the $Y_r$ direction

$F_z$ total magnet force component in the $Z_r$ direction

$F_w$ aerodynamic force due to crosswind gust

$g$ acceleration due to gravity

$H$ constant matrix such that $H^T H = Q$

$h$ air gap

$h_o$ nominal air gap

$h_c$ height between the magnet module and the carbody CG

$h_v$ vehicle height

$i$ trim current in the normal coil

$I$ current in the superconducting coil

$I_n$ identity matrix

$I_x$ roll moment of inertia of the five DOF vehicle

$I_y$ pitch moment of inertia of the five DOF vehicle

$I_z$ yaw moment of inertia of the five DOF vehicle

$J$ performance index

$k_h$ gap coefficient in equation (6)

$k_i$ current coefficient in equation (6)

$K$ optimal feedback gain matrix

$K_c$ optimal feedback gain matrix of the augmented system
LIST OF SYMBOLS (cont.)

\( l_m \) magnet module length

\( L \) guideway span length

\( L_i \) self-inductance of the normal coil

\( L_v \) vehicle length

\( m \) vehicle mass

\( M_x \) total moment component in the \( X_C \) direction due to magnet forces

\( M_y \) total moment component in the \( Y_C \) direction due to magnet forces

\( M_z \) total moment component in the \( Z_C \) direction due to magnet forces

\( M_w \) aerodynamic moment due to crosswind gust

\( n \) number of turns in the normal coil

\( n_s \) number of mode shapes

\( N \) number of turns in the superconducting coil

\( N_m \) number of magnet module on each side

\( P_{rms} \) RMS value of the time-varying function, \( p(t) \)

\( P \) solution of algebraic Riccati equation

\( Q \) weighting matrix

\( r \) reference signal of the feedforward controller

\( R \) weighting matrix

\( R_c \) total resistance in the normal coils

\( S_p(\omega) \) power spectral density of the time-varying function, \( p(t) \)

\( t_p \) preview time
LIST OF SYMBOLS (cont.)

$T_c$ roll-pitch-yaw transformation matrix

$U(s)$ Laplace transform of $u$

$u$ magnet input voltage (control vector)

$u_s$ unit step function

$v$ disturbance vector

$V$ Vehicle speed

$V(s)$ Laplace transform of $v$

$V_a$ average air flow velocity

$w$ guideway deviation vector

$w_c$ width between the magnet module and the carbody CG

$w_d$ guideway disturbance vector

$w_h$ white noise

$w_s$ guideway deflection vector

$W(s)$ Laplace transform of $w$

$x$ state vector

$x_0$ initial state vector

$x_a$ augmented state vector

$x_s$ axial coordinate of the guideway span

$x_v$ vehicle state vector

$X_C$ longitudinal axis of the carbody frame

$X_I$ longitudinal axis of the inertial frame
LIST OF SYMBOLS (cont.)

\( y \)  
output vector

\( y_b \)  
camber irregularity

\( y_c \)  
lateral displacement of the CG of five DOF vehicle

\( y_{\xi_j} \)  
guideway lateral deviation at module \( j \)

\( y_p \)  
ramp irregularity

\( y_r \)  
rail deviation due to guideway roughness

\( y_s \)  
step irregularity

\( y_v \)  
versine irregularity

\( y_t \)  
integrator state vector

\( Y(s) \)  
Laplace transform of \( y \)

\( Y_C \)  
lateral axis of the carbody frame

\( Y_I \)  
lateral axis of the inertial frame

\( z \)  
vertical displacement of the single DOF vehicle

\( z_c \)  
vertical displacement of the CG of five DOF vehicle

\( z_{\xi_j} \)  
guideway vertical deviation at module \( j \)

\( Z_C \)  
vertical axis of the carbody frame

\( Z_I \)  
vertical axis of the inertial frame

\( \beta \)  
magnet cant angle

\( \gamma \)  
mass per unit length of the guideway span

\( \mu_0 \)  
permeability of air

\( \phi_e \)  
roll displacement of the five DOF vehicle
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<th>Description</th>
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<td>$\theta_c$</td>
<td>pitch displacement of the five DOF vehicle</td>
</tr>
<tr>
<td>$\psi_c$</td>
<td>yaw displacement of the five DOF vehicle</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>center frequency</td>
</tr>
<tr>
<td>$\omega_j$</td>
<td>circular frequency of the $j$-th mode</td>
</tr>
<tr>
<td>$\omega_l$</td>
<td>lower bound for the one-third octave band</td>
</tr>
<tr>
<td>$\omega_u$</td>
<td>upper bound for the one-third octave band</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>roll angular velocity</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>pitch angular velocity</td>
</tr>
<tr>
<td>$\omega_y$</td>
<td>yaw angular velocity</td>
</tr>
<tr>
<td>$\zeta_j$</td>
<td>modal damping ratio of the $j$-th mode</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>density of air</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>power spectral density of guideway roughness</td>
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1. INTRODUCTION

Magnetically-levitated, or maglev, vehicles are one class of High-Speed Guided Ground Transportation (HSGGT) vehicles being considered for deployment in the U.S. A primary safety requirement for the dynamic performance of maglev systems is their ability to operate over a wide range of conditions without leaving or contacting the guideway (in analogy to derailment of rail vehicles).

For maglev vehicles with electromagnetic suspension (EMS) systems that achieve levitation and guidance by attractive forces, feedback control systems play two critical roles. First, they actively position the vehicle relative to the guideway to maintain a nominal vehicle-guideway air gap and ensure overall safe performance. Second, they are an integral part of the suspension system for attaining acceptable ride quality, especially in designs, such as the proposed Grumman concept, which has no secondary suspension.

1.1 BACKGROUND: GRUMMAN SYSTEM CONCEPT

The Grumman maglev design (Proise, et al., 1993) employs an EMS system using superconducting (SC) magnets. The vehicle wraps around a Y-shaped guideway and carries SC magnet modules cantled along the vehicle to provide simultaneous guidance and levitation. The SC magnets with trim current control coils provide an air gap clearance of 40 mm. No secondary suspension is used in this design, although an active carbody tilting mechanism to effectively increase the bank angle in curves is proposed. Propulsion is by a conventional linear synchronous motor.

The Grumman concept requires a single set of magnets to provide both lift and guidance. These are two separate functions that generally have different control response characteristics. For example, the vertical and lateral suspension designs of passenger vehicles, such as rail, automobile, and general ground vehicles, are typically not identical. Since there is no secondary suspension in the proposed Grumman arrangement, a concern is the force-capability of the
suspension. The suspension travel must be adequate for the range of guideway perturbations that the vehicle may encounter.

1.2 OBJECTIVES

An objective of this research is to develop a computer simulation modeling tool for predicting the safety-related and comfort-related dynamic performance of an EMS-type maglev vehicle that is similar to the Grumman system concept. Key features of the model are that it accounts for nonlinear magnet module characteristics, vehicle dynamics (such as rigid body Degrees Of freedom (DOFs)), guideway dynamics (such as vertical modal flexibility), and controller logic (feedback and feedforward schemes). The custom computer model is modular, and builds on Battelle’s earlier work (Daniels, et al., 1992).

Another objective of this task is to conduct limited computer simulation studies to explore vehicle operating performance. Performance-related issues of interest are safety, ride quality, and input power. Safety is measured by the air gap error, i.e., deviation from a nominal gap, ride comfort is given in terms of accelerations at points in the carbody, and applied power demand is represented by the magnitude of the input voltage.

1.3 SCOPE

This report is organized as follows. Section 2 develops models that characterize the dynamic behavior of key components of the maglev system. These include models for the SC magnet, vehicle, guideway, and controller. Then the complete (i.e., integrated) EMS maglev model is described. Section 3 describes model inputs. These include external disturbances such as crosswind gusts and guideway irregularities. Section 4 presents the results of two different simulation studies. The first study focuses on a single DOF vehicle model, and is used to verify the magnet model and test different controllers (including optimal feedback and feedforward controllers). The second study involves a five DOF nonlinear vehicle model, a multiple span vertically flexible guideway model, and an optimal feedback controller. It is designed to simulate more realistic behavior. Section 4 also introduces the performance measures. Finally,
Section 5 summarizes features of both models and suggests extensions and additional future developments.
2. SYSTEM COMPONENTS

The EMS maglev system represented in this report is comprised of four main model components: SC magnet model; vehicle model; guideway model; and controller model. The magnet model is an EMS-type magnet system that incorporates SC and trim current coils. Two vehicle models are developed. A simplified single DOF vehicle model is used to test the SC magnet model and design the controller. A five DOF nonlinear vehicle model is also proposed for full vehicle simulation. The guideway model represents an elevated multiple-span guideway with vertical flexibility that is excited by the vehicle magnet forces. The controller adjusts the magnet forces generated by the SC magnets to maintain an air gap of 40 mm and meet ride comfort specifications. Modeling of the system components is addressed in the following subsections.

2.1 SC MAGNET MODEL

The maglev vehicle is levitated by a set of on-board magnet modules. Each module consists of several magnets controlled by a single power supply. The number of magnets in each module is a design variable and can be specified based on the cost of the vehicle and required performance. The proposed Grumman design uses twenty-four magnet modules with two SC magnets in each module. In this subsection, a magnet model is derived to characterize the behavior of the SC magnet.

2.1.1 Nonlinear Magnet Model

The EMS magnet system, shown schematically in Figure 1, consists of a guideway iron rail, an iron-core magnet, a SC coil wrapped on the back leg of the iron core, and a set of normal coils, which are serially connected and attached to both pole ends of the iron core. It is assumed that the current in the SC coil is constant, and the resulting magnet force provides the lifting capability to balance the total weight in equilibrium. The trim current in the normal coils is driven by a controlled voltage to maintain the air gap at its nominal value.
To find the relationship linking the magnet force to the SC current and the trim current, the magnetic flux density in the air gap is first derived. By applying Ampere's law along path C in Figure 1, the magnetic flux density, $B_m$, in the air gap can be approximated by

$$B_m = \frac{\mu_0 (NI + ni)}{2h}$$  \hspace{1cm} (1)

where $\mu_0$ is the permeability of air; $h$ is the air gap; $N$ and $n$ are the number of turns in the SC coil and normal coils, respectively; and $I$ and $i$ are the SC current and the trim current, respectively. In equation (1), it is assumed that the leakage flux in the iron rail and iron core is negligible and the air gap is sufficiently narrow that the total flux in the iron core flows across the gap without loss. Using the law of conservation of energy for the magnetic energy stored in the air gap, the attractive magnet force, $f$, set up by the magnetic flux density can be shown to be

$$f = \frac{B_m^2 A_m}{\mu_0}$$  \hspace{1cm} (2)
where $A_m$ is the face area of each pole. By substituting equation (1) in equation (2), the attractive magnet force can be written as a function of air gap, $h$, and trim current, $i$,

$$f = \frac{\mu_0 A_m}{4h^2} (NI + ni)^2$$  \hspace{1cm} (3)

where the SC current, $I$, is assumed constant.

The trim current, $i$, in equation (3) is achieved by a power supply with the controlled voltage, $u$. From Kirchhoff's voltage law, the equation which relates the trim current with the controlled voltage, $u$, can be expressed as

$$u - R_c i - n \frac{d(B_m A_m)}{dt} = 0$$  \hspace{1cm} (4)

where $R_c$ is the total resistance in the normal coils. In equation (4) the last term on the left-hand side is the emf induced by the normal coil. (The emf induced by the SC coil is not shown in equation (4) since a constant SC current is assumed.) Substituting equation (1) in equation (4), and rearranging gives

$$u = R_c i + \left[ \frac{\mu_0 A_m n^2}{2h} \frac{di}{dt} - \left[ \frac{\mu_0 A_m n(NI + ni)}{2h^2} \right] \frac{dh}{dt} \right]$$  \hspace{1cm} (5)

where the trim current, $i$, can be obtained by solving a first order differential equation involving the controlled voltage, $u$, the air gap, $h$, and the air gap rate, $dh/dt$.

The dynamics of the nonlinear SC magnet model, described by equations (3) and (5), are summarized simply in the block diagram of Figure 2. The model assumes that the controlled voltage, $u$, the air gap, $h$, and its time derivative, $dh/dt$, are available to determine the magnet force, $f$, and trim current, $i$. As will be shown later, the magnet force is an input to the vehicle model and the trim current is used by the controller. Note that Figure 2 can also represent a SC magnet model composed of several modules in which $u$, $h$, $f$, and $i$ are vectors whose dimensions are the number of magnet modules.
2.1.2 Linearized Magnet Model

In control system design, the control law is generally derived from linear models. To develop a linear model, the magnet force of equation (3) can be linearized (via a Taylor series expansion) in the neighborhood of the nominal operating point, i.e., \( i = 0 \) and \( h = h_0 \) (the nominal gap) as

\[
f = f_0 + k_i i - k_h (h - h_0)
\]

(6)

where

\[
f_0 = \frac{\mu_0 A_m N^2 I^2}{4h_0^2}, \quad k_i = \frac{\mu_0 A_m nNI}{2h_0^2}, \quad k_h = \frac{\mu_0 A_m N^2 f^2}{2h_0^3}
\]

(7)-(9)

Similarly, from equation (5) the linearized voltage equation at \( i = 0 \) and \( h = h_0 \) can be shown to be

\[
u = R_c i + L_i \frac{di}{dt} - k_i \frac{dh}{dt}
\]

(10)

where

\[
L_i = \frac{\mu_0 A_m n^2}{2h_0}
\]

(11)

Equations (6) and (10) represent a linearized magnet model which will be combined with a single DOF vehicle model to create a linear plant model. The linear plant model can be used to test different control strategies and select controller parameters.
Table 1. SC Magnet Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability of Air</td>
<td>$\mu_0$</td>
<td>$4\pi \times 10^{-7}$</td>
<td>Weber/A-m</td>
</tr>
<tr>
<td>Face Area of Each Magnetic Pole</td>
<td>$A_m$</td>
<td>0.04</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Number of Turns in SC Coil</td>
<td>$N$</td>
<td>1020</td>
<td>None</td>
</tr>
<tr>
<td>Number of Turns in Normal Coil</td>
<td>$n$</td>
<td>96</td>
<td>None</td>
</tr>
<tr>
<td>Resistance of the Normal Coil</td>
<td>$R_c$</td>
<td>1.04</td>
<td>$\Omega$</td>
</tr>
</tbody>
</table>

2.1.3 SC Magnet Parameter Study

To test the nonlinear magnet model, parameter values suggested by Grumman (Kalsi, et al., 1993) were adopted. These are summarized in Table 1. With the SC current set to a constant $I = 53$ A, Figure 3 shows the magnet force generated by a single SC magnet as a function of the trim current for different air gaps. The solid lines represent the magnet model of equation (3). The indicated data points, obtained from (Kalsi, et al., 1993), were generated

Figure 3. SC Magnet Force Characteristic
via 3-D magnetic analyses (TOSCA and ANSYS). The magnet model shows consistency with the data in (Kalsi, et al., 1993) except for the smallest gap of 20 mm. The large deviations for the smallest gap may be the result of the leakage flux which is neglected in the proposed model. The Grumman data exhibit magnet force saturation at large values of trim current that is not observed in the proposed nonlinear model.

Since the trim current is driven by the controlled voltage, the performance of the SC magnet depends strongly on the time delay between the controlled voltage and the trim current. A smaller time delay (smaller time constant) implies a faster response and hence better performance in terms of controllability of the magnet force. To reduce the time delay, a resistor is connected serially to the normal coil. Although a larger resistance yields a smaller time constant (faster response), a tradeoff is a smaller output magnitude, meaning more required power. In this study, the total resistance in the normal coils is chosen as $R_c = 1.04 \ \Omega$.
(The resistance of the normal coils is 0.04 Ω in the Grumman baseline design; a 1 Ω resistor is serially connected.) The target is to achieve a time constant which is at least 5 times faster than the vehicle time constant (which is about 0.04 s, as discussed in Section 4.1) and unity DC gain (which is the ratio of the steady-state current to the input voltage). Using the linearized voltage equation (10) and the selected SC magnet parameters, summarized in Table 1, the response of the trim current to a 1 V step input and constant air gap ($\frac{dh}{dt} = 0$) is shown in Figure 4. The trim current has a time constant of 0.0056 s and a DC gain of 0.962; this meets the target specification for time constant and is within 4 percent of the target for gain.

2.2 VEHICLE MODELS

A single DOF vehicle model was developed to (i) select parameters in the SC magnet system and the controller, and (ii) evaluate the effectiveness of different control strategies. (This simplified vehicle model in conjunction with the linearized magnet model and a controller is referred to subsequently as Model I.) In addition, a five DOF vehicle model was constructed to represent the Grumman system concept and simulate more realistic behaviors. (This five DOF vehicle model with the nonlinear SC magnet model, flexible guideway, and controller is later called Model II.)

2.2.1 Simplified Maglev Vehicle Model

This subsection describes the system dynamics for a simple maglev model, shown in Figure 5, consisting of a single mass carrying the linearized SC magnet. At the nominal position, the air gap is $h_0$, and both the guideway displacement, $w$, and the vehicle displacement, $z$, are zero.

The magnet is attracted to the guideway iron rail by an attractive magnet force given by equation (6). The equation of motion for the vehicle can be expressed as

$$ m\ddot{z} = mg - f_0 - k_i i + k_h (h - h_0) $$

(12)

where $m$ is the mass of the vehicle, $z$ is the vehicle displacement, $h$ is the actual gap, $h_0$ is the nominal gap, and $f_0$, $k_i$, $k_h$ are determined from equations (7)-(9), respectively. Choosing the SC current such that $mg = f_0$, the equation of motion (12) becomes
where \( w \) is the guideway displacement with gap error being defined by

\[
h - h_0 = z - w
\]  
(14)

Combining the equation of motion (13) and the linearized voltage equation (10), the simple maglev model can be represented as the state-space equation

\[
\dot{x} = Ax + Bu + Ev, \quad x(0) = x_0
\]  
(15)

where

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ k_h/m & 0 & -k_i/m \\ 0 & k_i/L_i & -R_c/L_i \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/L_i \end{bmatrix}
\]  
(16)-(17)

\[
E = \begin{bmatrix} -k_h/m & 0 \\ 0 & 0 \\ 0 & -k_i/L_i \end{bmatrix}, \quad x = \begin{bmatrix} z \\ \dot{z} \\ i \end{bmatrix}, \quad v = \begin{bmatrix} w \\ \dot{w} \end{bmatrix}
\]  
(18)-(20)

Note that three state variables are needed to characterize Model I. If the system outputs are selected to be the gap error, the output equation can be expressed as

\[
y = Cx + Dv
\]  
(21)
Table 2. Parameters of Simplified Maglev Vehicle Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Mass</td>
<td>m</td>
<td>2340</td>
<td>kg</td>
</tr>
<tr>
<td>Nominal Gap</td>
<td>( h_0 )</td>
<td>0.04</td>
<td>m</td>
</tr>
<tr>
<td>Resistance of the Normal Coil</td>
<td>( R_c )</td>
<td>1.04</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>Self-inductance of the Normal Coil</td>
<td>( L_I )</td>
<td>( 5.79 \times 10^{-3} )</td>
<td>H</td>
</tr>
<tr>
<td>Current Coefficient in Eq. (6)</td>
<td>( k_f )</td>
<td>81.5</td>
<td>N/A</td>
</tr>
<tr>
<td>Gap Coefficient in Eq. (6)</td>
<td>( k_h )</td>
<td>( 1.14 \times 10^6 )</td>
<td>N/m</td>
</tr>
</tbody>
</table>

where

\[ y = h - h_0, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 \end{bmatrix} \]  

(22)-(24)

Equations (15) and (21) represent the system dynamics of the simplified maglev model in the neighborhood of the nominal gap, \( h_0 \), and zero trim current. In summary, the simplified maglev model consists of a single DOF vehicle model and a linear SC magnet model. The input is the controlled voltage and the output is the gap error.

The parameters of the third-order maglev vehicle model are listed in Table 2. A SC current of \( I = 53 \) A is used to balance the weight of the vehicle. The SC magnet parameters in Table 1 are employed to compute the self-inductance of the normal coil, \( L_h \), the current coefficient, \( k_f \), and the gap coefficient, \( k_h \), in the linearized magnet model.

### 2.2.2 Realistic Maglev Vehicle Model

An EMS-type maglev vehicle model was developed to represent the Grumman system concept (Proise, et al., 1993). The model is considered to be rigid with its vehicle center of gravity identified as point CG in Figure 6. There are \( N_m \) magnet modules on each side (\( N_m \) is an even number), inclined at angle \( \beta \) from vertical. The magnet force \( f_j \) at each module is assumed to be distributed uniformly along the module length, \( l_m \). Figure 6 shows the vehicle in the nominal position in which the magnet forces pass through the longitudinal axis of the vehicle.
Figure 6. EMS Maglev Vehicle Configuration

Figure 7. Cross Section Showing Nominal Vehicle/Guideway Position
In this analysis, two coordinate systems are established: an inertial frame and a carbody frame. It is assumed that the inertial frame, \(X_jY_jZ_j\), moves along the guideway \(X_j\) direction at constant speed. As shown in Figure 7, the origin of the inertial frame coincides with the vehicle CG when the vehicle and guideway are in their nominal positions. The nominal air gap, \(h_0\), is defined as the shortest distance between the center of the magnet module surface and the guideway iron rail at the nominal position. Also, the height and the width between the module and the carbody CG are denoted as \(h_c\) and \(w_c\), respectively. Table 3 lists the vehicle model parameters selected to be representative of a Grumman-type vehicle.

Figure 8 shows the carbody frame which is fixed in the vehicle carbody with principal axes \(X_c, Y_c, Z_c\) and origin located at CG. The vehicle motion is characterized by the displacements of the carbody CG in the lateral \((y_c)\), vertical \((z_c)\), roll \((\phi_c)\), pitch \((\theta_c)\), and yaw \((\psi_c)\) directions. The roll, pitch, and yaw angles are assumed small and represent the rotational angles about \(X_j\), \(Y_j\), and \(Z_j\), respectively. All displacements are measured in the inertial frame. Since the vehicle travels at constant speed, longitudinal dynamics are not considered and the vehicle model developed here has five DOFs.

### Table 3. Parameters of Realistic Maglev Vehicle Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of Vehicle</td>
<td>(m)</td>
<td>60,000</td>
<td>kg</td>
</tr>
<tr>
<td>Vehicle Height</td>
<td>(h_o)</td>
<td>3.9</td>
<td>m</td>
</tr>
<tr>
<td>Vehicle Length</td>
<td>(L_v)</td>
<td>18</td>
<td>m</td>
</tr>
<tr>
<td>Height, Magnet Centroid to Vehicle CG</td>
<td>(h_c)</td>
<td>1.09</td>
<td>m</td>
</tr>
<tr>
<td>Width, Magnet Centroid to Vehicle CG</td>
<td>(w_c)</td>
<td>0.763</td>
<td>m</td>
</tr>
<tr>
<td>Nominal Gap</td>
<td>(h_0)</td>
<td>0.04</td>
<td>m</td>
</tr>
<tr>
<td>Roll Moment of Inertia</td>
<td>(I_x)</td>
<td>(0.148 \times 10^6)</td>
<td>kg-m(^2)</td>
</tr>
<tr>
<td>Pitch Moment of Inertia</td>
<td>(I_y)</td>
<td>(1.73 \times 10^6)</td>
<td>kg-m(^2)</td>
</tr>
<tr>
<td>Yaw Moment of Inertia</td>
<td>(I_z)</td>
<td>(1.91 \times 10^6)</td>
<td>kg-m(^2)</td>
</tr>
<tr>
<td>Magnet Cant Angle</td>
<td>(\beta)</td>
<td>35</td>
<td>deg</td>
</tr>
</tbody>
</table>
Figure 8. Carbody Frame of Reference

Figure 9. Cross Section Showing General Vehicle/Guideway Position
The direction and the magnitude of the air gap at each module can be calculated from vector analyses. Figure 9 shows the vehicle relative to the guideway in a general position, at the cross section of module $j$, where both the vehicle and guideway are displaced from their nominal positions. (The nominal guideway position is indicated by the dotted lines.) The guideway displacement is restricted to the nominal lateral-vertical cross-sectional plane. The displacement vector $\bar{r}_b$ from the center of the magnet module face to the center of the iron rail is needed to determine the gap. It may have lateral, vertical, as well as longitudinal components. (The longitudinal component, if present, is due solely to vehicle motion. In particular, it is attributed to vehicle pitch and yaw angular deviations, since the guideway cannot displace longitudinally.)

From Figure 9, the vector $\bar{r}_b$ can be expressed as

$$\bar{r}_b = -T_c \bar{r}_{aj} - \bar{r}_c + \bar{r}_{bj} + \bar{r}_{gi} \tag{25}$$

The first term on the right-hand side of equation (25) is the negative of the vector displacement from the carbody CG to module $j$ expressed in the inertial coordinate frame. Displacement vector $\bar{r}_{aj}$ is the location of module $j$ from the CG in the carbody frame:

$$\bar{r}_{aj} = \begin{bmatrix} l_m[(N_m + 1) / 2 - j] \\ -w_c \\ -h_c \end{bmatrix}, \quad \bar{r}_{aj(N_m + j)} = \begin{bmatrix} l_m[(N_m + 1) / 2 - j] \\ w_c \\ -h_c \end{bmatrix}, \quad j = 1, \ldots, N_m \tag{26}$$

where $N_m$ is the number of modules on each side, $l_m$ is the module length as shown in Figure 6, $h_c$ and $w_c$ are the height and width distances from the module to the carbody CG, respectively, as shown in Figure 7, and $T_c$ is a roll-pitch-yaw transformation matrix which converts the vector $\bar{r}_{aj}$ from the carbody reference frame to the inertial reference frame. Using the small angle assumption:

$$T_c = \begin{bmatrix} 1 & -\psi_c & \theta_c \\ \psi_c & 1 & -\phi_c \\ -\theta_c & \phi_c & 1 \end{bmatrix} \tag{27}$$
The second term on the right-hand side of equation (25) is the negative of the vector displacement of the carbody CG. This displacement can be expressed as

\[ \vec{r}_c = \begin{bmatrix} 0 \\ y_c \\ z_c \end{bmatrix} \] (28)

Since it is assumed that there are no forces acting in the longitudinal direction, the x component in equation (28) is zero. The third term on the right-hand side of equation (25) is the vector from the origin of the inertial frame to the nominal position of the center of the adjacent iron rail at the cross section of module \( j \).

\[
\vec{r}_{b(j)} = \begin{bmatrix} I_m[(N_m + 1)/2 - j] \\ -w_c + h_0 \sin \beta \\ -h_c + h_0 \cos \beta \end{bmatrix}, \quad \vec{r}_{b(N_m+j)} = \begin{bmatrix} I_m[(N_m + 1)/2 - j] \\ w_c - h_0 \sin \beta \\ -h_c + h_0 \cos \beta \end{bmatrix}, \quad j = 1, \ldots, N_m \] (29)

The last term on the right-hand side of equation (25) is the guideway deviation vector

\[ \vec{r}_{gj} = \begin{bmatrix} 0 \\ y_{gj} \\ z_{gj} \end{bmatrix}^T \] (30)

where \( y_{gj} \) and \( z_{gj} \) are the guideway deviations in the lateral and vertical directions, respectively. (As noted previously, it is assumed that the guideway does not deviate longitudinally.)

Since in this analysis the magnet force provides only guidance and levitation with no propulsion or braking, the air gap vector is confined within the \( Y_jZ_j \) plane and is represented by

\[ \vec{r}_{gj} = \begin{bmatrix} 0 \\ \vec{r}_{gj} \cdot \vec{j} \\ \vec{r}_{gj} \cdot \vec{k} \end{bmatrix}^T, \quad j = 1, \ldots, 2N_m \] (31)

where \( \vec{j} \) and \( \vec{k} \) are the unit vectors in the \( Y_j \) and \( Z_j \) directions, respectively. It follows that the air gap and air gap rate can be determined by

\[ h_j = |\vec{r}_{gj}| \] (32)

\[ \dot{h}_j = (\vec{r}_{gj} \cdot \dot{\vec{r}}_{gj})/|\vec{r}_{gj}| \] (33)
With the direction of the air gap known, the magnet force vector can be represented as

$$
\vec{F}_j = f_j \frac{\vec{r}_n}{|\vec{r}_n|}
$$

(34)

where $f_j$ is the magnitude of the magnet force. (This force will be specified by the control law described in Section 2.4.) Note that the magnet force is not necessarily normal to the magnet face in the non-equilibrium condition. In this situation, the magnet force produces a moment about the carbody CG as

$$
\vec{M}_j = T_c \vec{r}_j \times \vec{F}_j
$$

(35)

The total magnet force applied to the vehicle can be written as

$$
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}
= \begin{bmatrix}
2N_m \\
\sum_{j=1}^{2N_m} \vec{F}_j
\end{bmatrix}
$$

(36)

where $F_x$, $F_y$, and $F_z$ are the total force components in the $X_i$, $Y_i$, and $Z_i$ directions, respectively. Since there is no force component in the $X_i$ direction for each individual magnet force, $F_x = 0$. Similarly, the total moment about the CG due to the magnet forces can be expressed by

$$
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}
= T_c^T \begin{bmatrix}
2N_m \\
\sum_{j=1}^{2N_m} \vec{M}_j
\end{bmatrix}
$$

(37)

where $M_x$, $M_y$, and $M_z$ are the moment components in the $X_c$, $Y_c$, and $Z_c$ directions, respectively. The transpose of the roll-pitch-yaw transformation matrix, $T_c^T$, is used to convert the moment vector from the inertial frame to the carbody frame.

The vehicle may be subjected to crosswind gusts that apply an equivalent lateral force, $F_w$, and a yaw moment, $M_{\psi}$, upon the carbody CG. With knowledge of the magnet force and associated moment, the vehicle model equations of motion can be written as

$$
\begin{align*}
F_y + F_w &= m\ddot{y}_c \\
F_z - mg &= m\ddot{z}_c
\end{align*}
$$

(38)-(40)
\[ M_x = I_x \dot{\phi}_x - (I_y - I_z) \omega_x \omega_z \]
\[ M_y = I_y \dot{\phi}_y - (I_z - I_x) \omega_x \omega_z \]
\[ M_z + M_w = I_z \dot{\phi}_z - (I_x - I_y) \omega_x \omega_y \]

(41)-(43)

\[ \dot{\phi}_c = \omega_x + (\omega_y \phi_c + \omega_z) \theta_c \]
\[ \dot{\theta}_c = \omega_y + \omega_z \phi_c \]
\[ \dot{\psi}_c = \omega_y \phi_c + \omega_z \]

(44)-(46)

where: \( m \) is the mass of vehicle; \( g \) is the acceleration due to gravity; \( I_x, I_y, \) and \( I_z \) are the roll, pitch, and yaw moments of inertia of the carbody, respectively; \( \omega_x, \omega_y, \) and \( \omega_z \) are the roll, pitch, and yaw angular velocities, respectively, in the carbody frame; and \( F_w \) and \( M_w \) are the aerodynamic force and moment, respectively, acting at the CG due to the crosswind gust. Equations (38)-(40) are the translational equations of motion given by Newton’s second law. Equations (41)-(43) are the rotational equations of motion from Euler’s equation. Equations (44)-(46) are the angle-angular velocity relations, which relate the vehicle angular velocities to the roll, pitch, and yaw angles, assuming small angles. Equations (38)-(46), which are equivalent to ten first-order differential equations, characterize the vehicle dynamics.

In summary, the input/output relation of the realistic vehicle model can be represented by the block diagram of Figure 10. The disturbances applied to the model are the lateral, \( y_g \), and vertical, \( z_g \), guideway deviations seen by each module \( (j = 1, \ldots, 2N_m) \):

\[ w = \left[ y_{g1} \cdots y_{g(2N_m)} \ z_{g1} \cdots z_{g(2N_m)} \right]^T \]

(47)

and the aerodynamic force, \( F_w \), and moment, \( M_w \), due to the crosswind gust, if present. The controllable inputs are the magnet forces at each module

\[ f = \left[ f_{1} \cdots f_{2N_m} \right]^T \]

(48)

The outputs of the model are the ten vehicle state variables:

\[ x_v = \left[ y_c \ z_c \ \phi_c \ \theta_c \ \psi_c \ \dot{y}_c \ \dot{z}_c \ \omega_x \ \omega_y \ \omega_z \right]^T \]

(49)
and the air gaps at each module

\[ h = \left[ h_1 \ldots h_{2N_m} \right]^T \]  

(50)

\[ w \]

\[ f \]

\[ F_w, M_w \]

5 DOF Vehicle

\[ x_v \]

\[ h, \dot{h} \]

**Figure 10. Block Diagram of Realistic Vehicle Model**

and associated rates. The guideway deviations are due to the guideway flexibility (described in Subsection 2.3) and guideway irregularities (described in Subsection 3.2). Note that the realistic maglev vehicle model, as presented here, does not include the SC magnet modules. (This is in contrast to the development in Subsection 2.2.1.) The magnet forces are generated by the SC magnet system and adjusted by the controller (described in Subsection 2.4).

### 2.3 GUIDEWAY MODEL

The maglev guideway consists of sequences of spans that are elevated, being supported at the ends. The spans can be modeled as distributed flexible beams, and described mathematically using the Bernoulli-Euler beam equation. This equation, which assumes that shear effects are negligible, can be used to determine the translational displacement history of points along the beam. (If shear effects are important, then a Timoshenko beam model can be adopted.) Angular displacements can be determined using the torsional St. Venant's equations. Boundary conditions are specified depending on the physical configuration. For example, if the spans are constrained in translation but free in rotation, a pinned-pinned model is appropriate. If the spans are resting on elastomeric pads at the ends, a free-free beam model with vertical viscoelastic components may be a more realistic representation.
The mathematical model developed below assumes homogeneous spans with prismatic geometry. This may be an acceptable approximation assuming that the inertia and cross-sectional geometry do not change along the length of the span. If the geometry and/or make-up do change (such as spans with thinner flared sections at the center and/or spans with embedded devices at discrete locations along their lengths), then it may be necessary to incorporate a more complicated beam description. If the guideway is not elevated but "at-grade" as in a tunnel, then the beam model may be replaced with a distributed stiffness and damping model, such as a beam on a viscoelastic foundation model that has been developed for railroad track. Furthermore, the mathematical model assumes straight (tangent) guideway spans. Since tangent as well as curved, banked spans may be encountered by a maglev vehicle, future models may need to account for both situations.

As mentioned in Subsection 2.2.2, guideway deviations seen by each magnet module can be attributed to vehicle dynamic loading on the flexible guideway spans. The guideway model, developed here, relates the magnet force and the corresponding guideway deflection for each magnet module. Subsection 2.3.1 models the dynamics of a guideway span subject to general loading. Subsection 2.3.2 applies the maglev vehicle loading to two sequential spans and solves for the guideway deflection observed by each magnet module. A multiple-span elevated guideway is considered and the assumption is that each span is straight, prismatic, vertically flexible only, and simply supported.

### 2.3.1 Beam Dynamics

A Bernoulli-Euler beam model with pinned-pinned ends is assumed for each guideway span. The equation of motion for a span can be derived as

\[
EI \frac{\partial^4 \tilde{w}_s(x,t)}{\partial x_s^4} + c \frac{\partial \tilde{w}_s(x,t)}{\partial t} + \gamma \frac{\partial^2 \tilde{w}_s(x,t)}{\partial t^2} = \tilde{f}(x,s,t) \tag{51}
\]

where \(x_s\) is the axial coordinate of the beam, \(t\) is time, \(EI\) is the bending rigidity, \(c\) is the viscous damping coefficient, \(\gamma\) is the mass per unit length of the beam, \(\tilde{w}_s(x,t)\) is the vertical deflection of the beam, and \(\tilde{f}(x,t)\) is the loading force per unit length due to the moving vehicle acting
on the beam. (The use of the superscript "−" denotes functional dependence on both space and time.)

To develop the solution of equation (51), a modal analysis method is used in which the deflection of the beam is expressed as

$$w_j(x_s,t) = \sum_{j=1}^{n_s} a_j(t) \sin\left(\frac{j\pi x_s}{L}\right)$$  \hspace{1cm} (52)

where \(a_j(t)\) is the time-varying modal amplitude, \(L\) is the span length, \(\sin(\frac{j\pi x_s}{L})\) is the mode shape of the pinned-pinned beam, and \(n_s\) is the number of mode shapes included in the solution. Substituting equation (52) into equation (51) and then multiplying by \(\sin(\frac{k\pi x_s}{L})\) and integrating from \(x_s = 0\) to \(x_s = L\) gives the resulting differential equation for the modal amplitude \(a_j(t)\) as

$$\ddot{a}_j(t) + \frac{c}{\gamma} \dot{a}_j(t) + \frac{EI}{\gamma} \left(\frac{j\pi}{L}\right)^4 a_j(t) = \frac{2}{\gamma L} \int_0^L f(x_s,t) \sin\left(\frac{j\pi x_s}{L}\right)dx_s, \quad a_j(0) = a_{j0} (53)$$

for \(j = 1, \ldots, n_s\). From equation (53), the circular frequencies and the modal damping ratio can be identified as

$$\omega_j = \sqrt{\frac{EI}{\gamma} \left(\frac{j\pi}{L}\right)^4}$$ \hspace{1cm} (54)

and

$$\zeta_j = \frac{c}{2\gamma \omega_j}$$ \hspace{1cm} (55)

for \(j = 1, \ldots, n_s\). The beam deflection can be determined by equation (52) after solving the set of differential equations (53).

### 2.3.2 Vehicle/Guideway Interaction

The vehicle is assumed to negotiate a multiple-span guideway as depicted in Figure 11. The vehicle/guideway interaction is considered in the time interval \([t_0, t_f]\). At \(t = t_0\) the vehicle is completely located on Span I and just about to enter Span II. As time increases, the vehicle
load excites both Span I and Span II simultaneously. In this study, it is assumed that the vehicle length is less than the guideway span length. As a result, the vehicle is completely located on Span II at $t = t_f$. For multi-span configurations, additional spans can be "daisy-chained" (i.e., at $t = t_f$, the clock is reset to $t = t_0$ and the same algorithm for the following span is applied).

Figure 11. Vehicle Initial/Final Positions

The dynamic interaction between a moving vehicle and a flexible guideway has been studied intensively (e.g., Kortum and Wormley, 1981; Smith and Wormley, 1974). The process for deriving the guideway dynamics involves three steps. The first step is to convert the magnet forces on the modules into the distributed loading forces on Spans I and II. It should be noted that the excitation on the spans is modeled as a distributed, time-varying moving force for the vehicle/guideway system in this study. The second step is to solve for the distributed span deflections. The final step is to obtain the guideway deflection at each module.

Let $f_I (x'_s, t)$ and $f_{II} (x_s, t)$ be the loading forces for Span I and Span II with axial coordinates $x'_s$ and $x_s$, respectively. Since the magnet forces are assumed to be uniformly distributed over each magnet module, the loading forces on the guideway can be derived as

$$f_I (x'_s, t) = \sum_{j=1}^{N_m} \left[ u_s (Vt - j l_m, x'_s) - u_s (Vt - (j+1) l_m, x'_s) \right] \left( f_j + f_{j+N_m} \right) \cos \beta / l_m \quad (56)$$

$$f_{II} (x_s, t) = \sum_{j=1}^{N_m} \left[ u_s (Vt - j l_m, x_s) - u_s (Vt - (j+1) l_m, x_s) \right] \left( f_j + f_{j+N_m} \right) \cos \beta / l_m \quad (57)$$
where \( V \) is the vehicle traveling speed, \( l_m \) is the magnet module length, \( f_j \) is the magnet force applied to module \( j \), \( \beta \) is the magnet cant angle, and \( u_s \) is the unit step function defined by

\[
u_s(x_s, x_z) = \begin{cases} 
1, & x_s \geq x_{s0} \\
0, & x_s < x_{s0} 
\end{cases}
\] (58)

Using equation (53), the equations of motion for Span I and II can be represented by

\[
\ddot{a}_j(t) + \frac{c}{\gamma} \dot{a}_j(t) + \frac{EI}{\gamma} \left( \frac{j\pi}{L} \right)^4 a_j(t) = \left( \frac{2}{\gamma L} \right) \int_0^L \tilde{f}_I(x'_z, t) \sin \left( \frac{j\pi{x'_z}}{L} \right) dx'_z 
\] (59)

\[
\ddot{b}_j(t) + \frac{c}{\gamma} \dot{b}_j(t) + \frac{EI}{\gamma} \left( \frac{j\pi}{L} \right)^4 b_j(t) = \left( \frac{2}{\gamma L} \right) \int_0^L \tilde{f}_II(x_z, t) \sin \left( \frac{j\pi{x_z}}{L} \right) dx_z
\] (60)

for \( j=1,\ldots,n_m \), where \( a_j(t) \) and \( b_j(t) \) are the modal amplitude for Span I and Span II, respectively. Since the loading forces in equations (56) and (57) are both staircase functions of time and axial coordinates, the integrals on the right-hand sides of equations (59) and (60) can be solved analytically as

\[
\int_0^L \tilde{f}_I(x'_z, t) \sin \left( \frac{j\pi{x'_z}}{L} \right) dx'_z = \frac{L \cos \beta}{jn_{1m}} \left( f_j + f_{j+N_m} \right) \left[ \cos \left( \max(jl_m - Vt, 0) \right) - \cos \left( \max(jl_m - l_m - Vt, 0) \right) \right]
\] (61)

\[
\int_0^L \tilde{f}_II(x_z, t) \sin \left( \frac{j\pi{x_z}}{L} \right) dx_z = \frac{L \cos \beta}{jn_{2m}} \left( f_j + f_{j+N_m} \right) \left[ \cos \left( \max(Vt - jl_m + l_m, 0) \right) - \cos \left( \max(Vt - jl_m, 0) \right) \right]
\] (62)

After solving the modal amplitudes \( a_j \) and \( b_j \) from equations (59) and (60), the guideway deflection at module \( j \) can be determined as

\[
w_{x,j}(t) = w_{x,j+N_m}(t) = \begin{cases} 
\sum_{k=1}^{n_z} a_k(t) \sin \left[ k\pi(Vt - jl_m + l_m/2) / L \right], & t < (jl_m - l_m/2) / V \\
\sum_{k=1}^{n_z} b_k(t) \sin \left[ k\pi(Vt - jl_m + l_m/2) / L \right], & t \geq (jl_m - l_m/2) / V
\end{cases}
\] (63)

for \( j=1,\ldots,N_m \). Thus, the guideway dynamics are completely described by equations (56)-(63). Equations (56)-(57) convert the module magnet forces into the distributed loading forces on
Span I and Span II. The distributed span deflections are then solved via equations (59)-(60). Finally, the guideway deflection observed by each module is obtained from equation (63).

\[ \begin{align*}
  f & \longrightarrow \text{Guideway} \longrightarrow w_s \\
\end{align*} \]

Figure 12. Block Diagram of Guideway Model

In summary, the input to the guideway model is the magnet force at each module and the output is the corresponding guideway deflection. The guideway model accounts for two sequential spans and then concatenates them for guideways involving multiple (i.e., >2) spans. Figure 12 presents a block diagram of the guideway model. The input, \( f \), is the vector of the magnet forces which is the same as the input to the five DOF vehicle model. The output is the vertical guideway deflection observed at each module:

\[ w_s = [w_{s,1} \cdots w_{s,2N_w}]^T \] (64)

Table 4 lists the guideway parameters chosen to be representative of a typical elevated span. The first mode and second mode natural frequencies are 6.19 Hz and 24.7 Hz, respectively. These correspond to (constant load) vehicle critical speeds of 154.8 m/s and 618.7 m/s, respectively, representing resonant conditions.

2.4 CONTROLLER MODEL

The dynamic models for the SC magnet modules, vehicle, and guideway have been derived in previous subsections. The block diagram shown in Figure 13 depicts the interaction among these individual components, and represents the realistic maglev system without a controller. (Figure 13 may also represent the simplified maglev system if the guideway model is absent and there are no aerodynamic loads.) The symbol, \( w_d \), denotes a vector of guideway disturbances which will be described in Section 3.2. The next step is to design a control strategy that shapes the applied voltage \( u \), subject to guideway disturbance, \( w_d \), and wind loads, \( F_w \) and \( M_w \), to achieve a nominal air gap, \( h \), and acceptable ride quality.
Table 4. Guideway Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bending Rigidity</td>
<td>$EI$</td>
<td>$3 \times 10^{16}$</td>
<td>N-m$^2$</td>
</tr>
<tr>
<td>Span Mass Per Unit Length</td>
<td>$\gamma$</td>
<td>5,000</td>
<td>kg-m$^2$</td>
</tr>
<tr>
<td>Span Length</td>
<td>$L$</td>
<td>25</td>
<td>m</td>
</tr>
<tr>
<td>Span Damping Ratio (for each mode)</td>
<td>$\zeta$</td>
<td>0.03</td>
<td>None</td>
</tr>
</tbody>
</table>

Figure 13. Block Diagram of Realistic Maglev System Model

The SC magnet is an EMS-type magnet which is inherently unstable. A key goal of the controller is to stabilize the EMS system. Further, the controller must successfully reject the disturbances while regulating the air gap. In this work, a Linear Quadratic (LQ) optimal control strategy with integral action and preview is applied to the maglev vehicle. The objective is to minimize the passenger accelerations without exceeding limits on air gap variations.

Conventional LQ optimal control can be used to stabilize the EMS system and regulate the air gap (Kortum and Utzt, 1984). However, a non-zero steady-state gap error can occur due to constant disturbances, such as a step guideway error or a constant aerodynamic load. To circumvent this problem, an optimal integral control technique (Anderson and Moore, 1990) is applied in which integrators are inserted in the controller to eliminate steady-state errors due to
constant disturbances. Furthermore, if information about the disturbance input is known \textit{a priori}, it can be used by an extended LQ optimal controller called an optimal preview controller (Hac, 1992). Optimal preview control has been proposed for automotive applications (Hac, 1992; Huisman et al., 1993) where promising results have been shown. Here, optimal preview control is tested on a simple maglev model to enhance performance such as reducing the air gap error, improving ride comfort, and minimizing energy demand.

The proposed control method requires a linearized maglev system. A linearized maglev system representing the single DOF vehicle with the SC magnet model has been derived in Subsection 2.2.1. Subsection 2.4.1 develops a linear model for the realistic maglev model (i.e., five DOF vehicle model plus magnet model). The linearized model is then augmented to incorporate the integral action. Based on the augmented system, an optimal control law is determined based on optimal preview control. In the design process, the weighting matrices in the performance index are adjusted to give the desired performance. In the simulation studies, presented in Section 4, both the simple linear and more realistic nonlinear vehicle models are controlled with the designed controllers.

2.4.1 Linearization of Realistic Maglev System Model

A linearized maglev model for the five DOF nonlinear vehicle model incorporating the SC magnets can be determined analytically or numerically. The analytical derivation yields an accurate linear model, yet the derivation is tedious and the resulting equations are lengthy. The numerical linearization bypasses a detailed derivation, however, the model may not be as accurate as the analytical model, especially for high order systems.

In this work, the linearized maglev model is determined numerically. Numerical linearization functions require that the inputs, outputs, and state variables of the nonlinear maglev model be identified first. The inputs include the controlled voltages of the magnet modules and the disturbances due to guideway flexibility, guideway irregularities, and crosswind gust. The outputs are the air gaps at the centroids of the magnet modules. The state variables consist of the vehicle CG displacements and velocities, and the trim currents in the magnet modules. The nonlinear maglev vehicle model is linearized in the neighborhood of the nominal point where all
state variables are set to zero (zero trim current and a nominal gap of \( h_0 \) at each module). The linearized maglev vehicle can be represented by the linear state equation and the output equation as

\[
\dot{x} = Ax + Bu + Ev, \quad x(0) = x_0 \tag{65}
\]
\[
y = Cx + Dv \tag{66}
\]

where \( x \) is the system state vector,

\[
x = [y_c \ z_c \ \phi_c \ \theta_c \ \varphi_c \ \dot{y}_c \ \dot{z}_c \ \omega_x \ \omega_y \ \omega_z \ i_1 \ \ldots \ i_{2N_m}]^T \tag{67}
\]

\( u \) is the controlled voltages vector,

\[
u = [u_1 \ \ldots \ u_{2N_m}]^T \tag{68}
\]

\( v \) is the disturbance vector,

\[
v = \begin{bmatrix} w \\ \dot{w} \\ F_w \\ M_w \end{bmatrix} \tag{69}
\]

where \( F_w \) and \( M_w \) are the aerodynamic force and moment due to crosswind gust and \( w \) is the vector containing the lateral and vertical guideway displacements,

\[
w = [y_{g1} \ \ldots \ y_{g(2N_m)} \ z_{g1} \ \ldots \ z_{g(2N_m)}]^T \tag{70}
\]

and \( y \) is the vector of air gap errors at the centroids of the magnet modules,

\[
y = [(h_1 - h_0) \ \ldots \ (h_{2N_m} - h_0)]^T \tag{71}
\]

In equations (65) and (66), \( A, B, C, D, \) and \( E \) are constant matrices which can be derived analytically or determined from MATLAB's linearization function \texttt{linmod}. The MATLAB code used to determine the linear model is listed in Appendix A.
2.4.2 **LQ Optimal Preview Control**

In this subsection, the LQ optimal preview control problem is formulated and the solution is derived. The steady-state gap error due to a constant disturbance is also determined. The result motivates the use of integrators, described in Subsection 2.4.3, to eliminate the steady-state error which is shown to exist.

2.4.2.1 **Problem Formulation and Solution** - The formulation of the LQ optimal preview control problem is as follows. Consider the system described by equations (65) and (66). The objective is to find an optimal control, $u$, that minimizes the quadratic performance index

$$ J = \int_0^\infty \left( x^T Q x + u^T R u \right) dt $$

(72)

where $Q$ and $R$ are symmetric weighting matrices that are semi-positive and positive definite, respectively. Assuming system $(A,J)$ is stabilizable$^1$ and system $(A,H)$, where $H^T H = Q$, is detectable$^2$, the optimal control law can be expressed as

$$ u = -Kx - R^{-1}B^T r $$

(73)

where

$$ K = R^{-1}B^T P $$

(74)

in which $P$ is the solution of the algebraic Riccati equation

$$ PA + A^T P - PBR^{-1}B^T + Q = 0 $$

(75)

and the reference signal $r$ is defined by

$$ r(t) = \int_0^{t_p} e^{A_c \sigma} B_c y(t + \sigma) d\sigma $$

(76)

where $t_p$ is known as the preview time and

$$ A_c = A - BR^{-1}B^T P $$

(77)

$$ B_c = PE $$

(78)

$^1$A system is defined as stabilizable if the uncontrollable subsystem is stable (Friedland, 1986).

$^2$A system is defined as detectable if the unobservable subsystem is stable (Friedland, 1986).
The assumptions of stabilizability of \((A,B)\) and detectability of \((A,H)\) assure a unique solution, \(P\), of the algebraic Riccati equation (75) and a stable closed-loop system. In equation (76), it is assumed that preview information, \(v(t+\sigma)\) for \(\sigma \in [0,\tau]\), is available. Notice that without preview information, \(r(t)\) in equation (76) is zero and hence equation (73) becomes the control law of a conventional LQ optimal control problem.

2.4.2.2 Steady-State Analysis - The steady-state gap error is studied for the worst case, i.e., optimal control without preview. In this case, the control law can be simplified as

\[
u = -Kx
\]  

(79)

By taking the Laplace transform of the state equation (65), the output equation (66), and the control law (79), the Laplace transform of the system output can be derived as

\[
Y(s) = \left[ C(sI_n - A + BK)^{-1}E + D \right]V(s)
\]  

(80)

where \(I_n\) is an identity matrix and \(Y(s)\) and \(V(s)\) are the Laplace transforms of the output vector, \(y\), and disturbance vector, \(v\), respectively. Using the final value theorem, the steady-state gap error can be determined by

\[
\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} s \left[ C(sI_n - A + BK)^{-1}E + D \right]V(s)
\]  

(81)

Considering a step disturbance vector, \(v\), with Laplace transform

\[
V(s) = \frac{1}{s}[1 \cdots 1]^T
\]  

(82)

where \(V(s)\) has length \(2N_m\), the corresponding steady-state gap error can be obtained by substituting equation (82) in equation (81) to give

\[
\lim_{t \to \infty} y(t) = C(-A + BK)^{-1}E + D
\]  

(83)

which is generally non-zero. Equation (83) implies that a finite steady-state gap error exists. If constant disturbances, such as step guideway offsets and crosswind gusts, are encountered by the maglev vehicle, LQ optimal control alone will not be suitable since it will incur constant offsets. In the following subsection, integral action is introduced to overcome this problem.
2.4.3 **LQ Optimal Preview Control With Integral Action**

To eliminate non-zero steady-state gap errors due to constant disturbances, the maglev system is augmented by adding integrators at its outputs. The LQ optimal preview control can then be applied to the augmented system. Subsection 2.4.3.1 gives the control law of the LQ optimal preview control with integral action. The resulting zero steady-state gap error is then proven in Subsection 2.4.3.2.

### 2.4.3.1 Problem Formulation and Solution

- By adding integrators at the system output, the output equation can be expressed as

\[
\dot{y}_f = y = Cx + Dv
\]

where \( y_f \) is called the integrator state vector. The augmented system dynamics are the combination of the plant dynamics (65) and the integrator dynamics (84), and can be represented in matrix form as

\[
\dot{x}_a = A_a x_a + B_a u + E_a v, \quad x_a(0) = x_{a0}
\]

where

\[
A_a = \begin{bmatrix} A & O \\ C & O \end{bmatrix}, \quad B_a = \begin{bmatrix} B \\ O \end{bmatrix}, \quad E_a = \begin{bmatrix} E \\ D \end{bmatrix}, \quad \begin{bmatrix} x \\ y_f \end{bmatrix}
\]

(Note that in equation (89) the augmented state vector \( x_a \) includes the integrator state vector \( y_f \).) Since equation (85) has the same form as the original state equation (65), the LQ optimal preview control can now be applied to the augmented system as follows.

The objective is to find the optimal control, \( u \), that minimizes the performance index

\[
J = \int_0^\infty (x_a^T Q x_a + u^T R u) dt
\]

subject to the state-space equation (85) of the augmented system. In equation (90), \( Q \) and \( R \) are weighting matrices, as defined before with appropriate dimensions. Assuming \( (A_a, B_a) \) is stabilizable and \( (A_a, H) \), where \( H^T H = Q \), is detectable, the optimal control law can be expressed as
\[ u = -K_c x_a - R^{-1} B_a^T r \]  

(91)

where

\[ K_c = R^{-1} B_a^T P \]  

(92)

in which \( P \) is the solution of the algebraic Riccati equation

\[ PA_a + A_a^T P - PB_a R^{-1} B_a^T + Q = 0 \]  

(93)

and \( r \) is defined by equation (76), and \( A_c \) and \( B_c \) [in equation (76)] are redefined as

\[ A_c = A_a - B_a R^{-1} B_a^T P \]  

(94)

\[ B_c = PE_a \]  

(95)

The assumptions of stabilizability of \( (A_a, B_a) \) and detectability of \( (A_a, H) \) assure a unique solution, \( P \), of the algebraic Riccati equation (93) and a stable closed-loop system.

By adding integrators at the system outputs, the integrals of the gap errors are introduced as new state variables of the augmented system. As the LQ optimal control is applied to the augmented system, the control law regulates the integrals of the gap errors to constant values for step disturbances. Hence, their derivatives, the gap errors, tend to zero (giving zero steady-state gap error). The following steady-state analysis verifies this result.

2.4.3.2 Steady-State Analysis - As in the analysis presented in Subsection 2.4.2.2, the control law (91) is considered without preview and can be written as

\[ u = -K_x x - K_I y_I \]  

(96)

where \( K_x \) and \( K_I \) are sub-matrices of the optimal feedback gain matrix \( K_c \) such that

\[ K_c = [K_x \\ K_I] \]  

(97)

Taking the Laplace transforms of the integrator dynamics (84), the augmented system dynamics (85), and the decoupled optimal control law (96), the transfer function from the system disturbance to the system output can be derived as

\[ Y(s) = s \left[ sl_n + C(sl_n - A + BK_x)^{-1} BK_I \right]^{-1} \left[ C(sl_n - A + BK_x)^{-1} E + D \right] V(s) \]  

(98)
Following the same procedure as in Subsection 2.4.2.2, the steady-state gap error due to a step disturbance vector, as described by equation (82), can be shown to be

\[
\lim_{t \to \infty} y(t) = \lim_{t \to 0} \left[ C(-A + BK_x)^{-1} BK_I \right]^{-1} \left[ C(-A + BK_x)^{-1} E + D \right] = 0 \tag{99}
\]

As indicated above, the LQ optimal control with integral action leads to zero steady-state gap errors for constant disturbances.

### 2.4.4 Preview Controller Structure

The control law (91) of the LQ optimal preview control with integral action consists of a feedback term and a feedforward term. As depicted in Figure 14, the first term on the right-hand side of equation (91) is represented by the feedback controller which includes integrators and requires the trim current, \(i\), the vehicle state, \(x_v\), and the air gap, \(h\), as inputs. The second term on the right-hand side of equation (91) is represented by the feedforward controller. The input of the feedforward controller is the preview information, \(v(t+\sigma)\) for \(\sigma \in [0,t_p]\), where \(t_p\) is the preview time. The feedforward controller accepts the preview information to shape the applied voltage, using equation (76). If no preview information is available, the controller consists solely of the feedback controller.

![Figure 14. Block Diagram of Controller Structure](image-url)

The feedback controller requires the trim current, \(i\), the vehicle state, \(x_v\), and the air gap, \(h\), as inputs.
2.4.5 Quadratic Weight Selection

The weighting matrices $Q$ and $R$ are chosen to meet the required performance specifications, such as limiting the magnitude of key state and control variables. In this subsection, a method for selecting $Q$ and $R$ is introduced.

To determine the weighting matrices, the limits for the state and control variables (such as known physical constraints and/or estimated maximum values) are first specified. Here, the realistic (five DOF) maglev vehicle model is considered. The limits of the augmented state vector and the control vector can be denoted as

$$x_{a, \text{max}} = [x_{a1, \text{max}}, \ldots, x_{a(10+4N_{m}), \text{max}}]^T$$

$$u_{\text{max}} = [u_{1, \text{max}}, \ldots, u_{2N_{m}, \text{max}}]^T$$

For simplicity, the weighting matrices are assumed to be diagonal and can be represented as

$$Q = \text{diag}(q_1, \ldots, q_{10+4N_{m}})$$

$$R = \text{diag}(r_1, \ldots, r_{2N_{m}})$$

The diagonal elements in $Q$ and $R$ are determined in such a way that each term in the criteria, $x_{a, \text{max}}^TQx_{a, \text{max}} + u_{\text{max}}^TRu_{\text{max}}$, has equal contribution. This aim can be achieved by setting

$$q_j = x_{a_j, \text{max}}^{-2}, \quad j = 1, \ldots, 10 + 4N_{m}$$

$$r_j = u_{j, \text{max}}^{-2}, \quad j = 1, \ldots, 2N_{m}$$

It should be noted that the translation of specifications into the selection of $Q$ and $R$ matrices is not straightforward nor unique, and often further adjustments are needed. A general principle of weighting adjustment is that when a variable of interest takes too high a value, the weighing of that variable is increased in the performance index. More details on weighting selection can be found in (Anderson and Moore, 1990).

Based on the approach described above, the estimated limits used in the weighting matrices $Q$ and $R$ were determined and are summarized in Table 5. The limit on the lateral and vertical CG displacements, $y_{\text{max}}$ and $z_{\text{max}}$, respectively, is selected as 0.01 m which is of the same order as
the allowed gap variation. The limit on the roll, pitch, and yaw displacements is small, i.e., 0.01 rad (0.57°), to reflect the kinematic constraints on these displacements. The weighting on the CG velocities affects the speed of vehicle response. They can be adjusted to achieve desired damping ratios and natural frequencies. These weightings are initially chosen as 1 m/s for the translational velocities and 1 rad/s for the rotational velocities. Maximum values of 1000 A and 100 V are used for the limits of trim current and input voltage, respectively. The speed of integral action is dominated by the weightings on the integrator state variables where an estimated limit of 0.001 m-s is initially selected.

2.5 COMPLETE MAGLEV SYSTEM

The complete maglev system, consisting of the realistic vehicle model, guideway model, magnet system, and controller, can be represented by the block diagram of Figure 15. The inputs to the overall system are the aerodynamic force and moment, $F_v$ and $M_{wv}$, due to the crosswind gust and the guideway disturbance, $w_d$, which is discussed in Section 3. If preview control is implemented, then there is an additional input, $v$, which is the preview information

<table>
<thead>
<tr>
<th>Parameter Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{max}, z_{max}$</td>
<td>0.01</td>
<td>m</td>
</tr>
<tr>
<td>$p_{max}, q_{max}, r_{max}$</td>
<td>0.01</td>
<td>rad</td>
</tr>
<tr>
<td>$\dot{y}<em>{max}, z</em>{max}$</td>
<td>1.0</td>
<td>m/s</td>
</tr>
<tr>
<td>$\omega_{x_{max}}, \omega_{y_{max}}, \omega_{z_{max}}$</td>
<td>1.0</td>
<td>rad/s</td>
</tr>
<tr>
<td>$i_{i_{max}} (j = 1, ..., 2N_m)$</td>
<td>1000</td>
<td>A</td>
</tr>
<tr>
<td>$u_{i_{max}} (j = 1, ..., 2N_m)$</td>
<td>100</td>
<td>V</td>
</tr>
<tr>
<td>$\psi_{j_{max}} (j = 1, ..., 2N_m)$</td>
<td>0.001</td>
<td>m-s</td>
</tr>
</tbody>
</table>
Figure 15. Block Diagram of Complete Maglev System

(i.e., the measured guideway displacements in front of the vehicle). The outputs of the system are the air gap, \( h \), and the vehicle state, \( x_v \). Figure 15 may also represent a simplified maglev system in which the realistic vehicle model and the magnet system are replaced by the simplified maglev vehicle model (Subsection 2.2.1) and the guideway model is removed.

In the following pages, the names "Model I" and "Model II" are used to denote the simplified maglev system and the realistic (complete) maglev system, respectively. The dynamics of Model I are represented by the augmented system [equation (85)] with control law [equation (91)] where the augmented system state variables are the vehicle vertical displacement and velocity, magnet trim current, and the integral of the air gap. Thus, four state variables (two for the vehicle, one for the magnet, and one for the controller) are needed to characterize Model I. The dynamics of Model II are described by the combination of the dynamic equations of each subsystem: ten first order differential equations [equations (38)-(46)] for the vehicle model; one first order differential equation [equation (5)] for each of the \( 2N_m \) modules; \( 2n_s \) second order differential equations [equations (59)-(60)] for the guideway model; and \( 2N_m \) first order differential equations [equations (84)] representing the integrator dynamics.

The number of state variables for the different subsystems is summarized in Table 6. The total number of state variables is \( 10 + 4(N_m + n_s) \). For a vehicle with four magnet modules (two per
side) and a guideway modeled with two modes per span, 26 state variables are needed to characterize the total dynamic system.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>State Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle</td>
<td>10</td>
<td>lateral, vertical, roll, pitch, yaw displacements &amp; rates</td>
</tr>
<tr>
<td>SC Magnet</td>
<td>$2N_m$</td>
<td>current at each module; $2N_m$ modules</td>
</tr>
<tr>
<td>Guideway</td>
<td>$4n_s$</td>
<td>2 spans; $n_s$ modes per span; 2 state variables per mode</td>
</tr>
<tr>
<td>Controller</td>
<td>$2N_m$</td>
<td>integrator at each module; $2N_m$ modules</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$10+4(N_m+n_s)$</td>
<td></td>
</tr>
</tbody>
</table>
3. DISTURBANCE INPUTS

Two main types of external disturbances are applied to the maglev vehicle. These are crosswind gusts and guideway irregularities. The crosswind gust is modeled as a lateral aerodynamic force which is equivalent to a force and a moment acting at the carbody CG. The velocity of the crosswind gust is assumed to be uniformly distributed; more complicated wind gust models may be considered in future studies. The guideway irregularities may be caused by surface roughness of the guideway iron rails, misalignment of the guideway spans, column height variations, and intentionally imposed camber of the guideway spans to compensate for vehicle loading.

3.1 CROSSWIND GUST

The crosswind gust is modeled as a lateral air flow across the carbody. The drag force due to the crosswind gust can be expressed as

$$ F_d = \frac{1}{2} C_d \rho_a A_s V_a^2 $$

(106)

where $C_d$ is the drag coefficient, $\rho_a$ is the density of air, $A_s$ is the carbody side area, and $V_a$ is the average air flow velocity. The value of the drag coefficient depends on the vehicle geometry.

In the simulation model it is assumed that the crosswind gust velocity may change from span to span and is uniformly distributed over each guideway span. Considering Figure 16 which shows a plan view with crosswind gust $V_{a1}$ and $V_{aII}$ for Spans I and II, respectively, the corresponding drag forces per unit length on the carbody can be expressed as

$$ f_{aI} = \frac{1}{2L_v} C_d \rho_a A_s V_{aI}^2 $$

(107)

$$ f_{aII} = \frac{1}{2L_v} C_d \rho_a A_s V_{aII}^2 $$

(108)
where $L_v$ is the vehicle length. As shown in Figure 17, both $f_{dl}$ and $f_{all}$ may produce time-dependent forces and moment on the carbody CG. The total aerodynamic force and moment due to the crosswind gusts from Spans I and II can be derived as

\[
F_w = \begin{cases} 
(f_{all} - f_{dl})Vt + f_{dl}L_v, & t \leq L_v/V \\
 f_{all}L_v, & t > L_v/V 
\end{cases} 
\]  \hspace{1cm} (109)

\[
M_w = \begin{cases} 
(f_{all} - f_{dl})(L_v - Vt)Vt/2, & t \leq L_v/V \\
0, & t > L_v/V 
\end{cases} 
\]  \hspace{1cm} (110)

acting at the vehicle CG.
Table 7. Crosswind Gust Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of Air</td>
<td>( \rho_a )</td>
<td>1.23</td>
<td>kg/m(^2)</td>
</tr>
<tr>
<td>Vehicle Side Area</td>
<td>( A_s )</td>
<td>70.2</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Drag Coefficient</td>
<td>( C_d )</td>
<td>1.2</td>
<td>None</td>
</tr>
</tbody>
</table>

In this study, the parameters used for the crosswind gust model are listed in Table 7. The drag coefficient \( C_d = 1.2 \), obtained from (Fox and McDonald, 1985, p. 460), assumes a rectangular cylinder for the carbody.

3.2 GUIDEWAY IRREGULARITIES

Guideway irregularities include random roughness, step, ramp, camber, and versine displacement deviations. The sum of these irregularities is the combined guideway irregularity. In the following subsections, the guideway irregularities are described by the displacements of the right and left rails as functions of distance along the guideway span. All irregularities are assumed to be deviations in the vertical direction except for the random roughness irregularity, which occurs in the normal direction of the rail face (i.e., 35° from vertical).

3.2.1 Random Roughness Irregularity

The random guideway roughness is a consequence of the manufacturing process for the rails. A commonly applied model (Fries and Coeffey, 1990) is adopted which describes the guideway roughness as a Power Spectral Density (PSD) function of the form:

\[
\Phi(\omega) = \frac{A_r}{\omega^2}
\]

where \( A_r \) is a roughness parameter. The appropriate roughness parameter depends on the manufacturing tolerances of the guideway. Here, the value \( A_r = 1.2 \times 10^{-6} \) m, corresponding to a welded steel rail (Barrows, et al., 1991), is selected.

The guideway roughness described by the PSD in equation (111) can be generated by passing a white noise process through the linear system.
\[ \frac{dy_r(x_s)}{dx_s} = \sqrt{4} w_h(x_s) \]  

(112)

where \( x_s \) is the distance along the guideway span, \( y_r \) is the deviation of the rail face in the normal direction due to guideway roughness, and \( w_h \) is a white noise signal with unit intensity. Different white noise signals are used to generate the guideway roughness on the right and left rails.

### 3.2.2 Step Irregularity

The misalignment of the guideway spans is modeled as a step discontinuity from span to span. For each span, the step irregularity is described simply by

\[ y_s(x_s) = d_s \]  

(113)

where \( d_s \) is the step deviation of the rail in the vertical direction. The step deviation can be positive or negative and it is assumed the step irregularities on the right and left rails are the same.

### 3.2.3 Ramp Irregularity

The ramp guideway irregularity is attributed to column height differences which may occur. This effect is modeled as

\[ y_p(x_s) = d_{p1} + (d_{p2} - d_{p1}) x_s / L \]  

(114)

where \( d_{p1} \) and \( d_{p2} \) are the column heights for the beginning and ending columns of the span, respectively. This irregularity is in the vertical direction and is assumed to be the same for the right and left rails.

### 3.2.4 Camber Irregularity

The camber irregularity is a result of pre-stressing the guideway to account for the vehicle loading. After long-term loading, the nominal pre-stress camber may change and the camber error can be modeled as

\[ y_b(x_s) = d_s \sin(\pi x_s / L) \]  

(115)
where $d_b$ is the camber amplitude which can be positive and negative (due to sagging). This irregularity is in the vertical direction and is assumed to be the same for the right and left rails.

### 3.2.5 Versine Irregularity

An alternate model of span pre-displacement to compensate for the guideway static loading is the versine guideway geometry described by

$$y_v(x_s) = d_v \left[ 1 - \cos \left( \frac{2\pi x_s}{L} \right) \right] / 2$$

where $d_v$ is the versine amplitude. The versine guideway geometry has zero slope at both ends of the span.

### 3.2.6 Guideway Displacement at Each Magnet Module

The guideway irregularities described in Subsections 3.2.1-3.2.5 are functions of distance. To find the guideway irregularity at each magnet module, the location of each module at any time, $t$, is identified first. The vehicle is considered to travel along a two span guideway with initial and final positions as shown in Figure 11 (Subsection 2.3.2.). The location of magnet module $j$ ($j = 1, \ldots, N_m$) can be expressed as

$$x_{s,j} = x_{s,j+N_m} = Vt + l_m / 2 - j l_m$$

(117)

If $x_{s,j}$ is positive, the module is located on Span II, and if negative, on Span I. The guideway irregularity at module $j$ can be determined by

$$w_{d,j}(t) = \begin{cases} 
  y_{I,v}(x_{s,j} + L), & t < (j - 1/2) l_m / V \\
  y_{II,v}(x_{s,j}), & t \geq (j - 1/2) l_m / V 
\end{cases}$$

(118)

$$w_{d,j+N_m}(t) = \begin{cases} 
  y_{I,v}(x_{s,j} + L), & t < (j - 1/2) l_m / V \\
  y_{II,v}(x_{s,j}), & t \geq (j - 1/2) l_m / V 
\end{cases}$$

(119)

for $j = 1, \ldots, N_m$, where the combined guideway irregularities on the right and left rails are $y_{I,v}(x_s)$ and $y_{II,v}(x_s)$ for Span I and $y_{II,v}(x_s)$ and $y_{II,v}(x_s)$ for Span II, respectively. Note that the guideway irregularity in equations (118) and (119) may have components, $w_{dy,j}$ and $w_{dz,j}$, in the lateral and vertical directions, respectively. Then, the guideway disturbance vector, shown in the block...
The total guideway displacement is the combination of guideway flexibility (acting in the vertical direction only) and guideway irregularity and can be represented in its lateral and vertical components as

\[ y_{gf} = w_{dy,j} \]
\[ z_{gf} = w_{dz,j} + w_{s,j} \]

for \( j = 1, \ldots, 2N_m \), where \( w_{s,j} \) is the vertical guideway deflection in equation (63) (from Subsection 2.3.2). Equations (121) and (122) are the elements of the guideway displacement vector, \( w \), i.e., the disturbance applied to the five DOF vehicle model, as shown in equation (47) (Subsection 2.2.2).
4. SIMULATION STUDIES

This section presents results of simulation studies of two maglev system models, i.e., a simplified model, Model I, and a more realistic maglev system, Model II. Model I, which includes the single DOF vehicle model and the SC magnet model (operating on a rigid guideway), was developed to evaluate the effectiveness of the LQ optimal controller, with and without preview. Model II was constructed to simulate more realistic situations. It represents a five DOF vehicle, housing multiple magnet modules, traversing a multi-span vertically flexible guideway. The vehicle is exposed to crosswind gusts and guideway irregularities. Overall maglev system performance is measured in terms of specifications on the allowed air gap, the maximum control voltage, and the International Standardization Organization (ISO) ride quality criteria.

There are three subsections organized as follows. Subsection 4.1 compares the performance of Model I to that of passive mechanical vehicle models with conventional suspensions. It also studies the effectiveness of preview control for Model I. Subsection 4.2 describes the performance measures used for Model II. Simulation results for Model II with different sets of inputs are presented in Subsection 4.3.

4.1 MODEL I SIMULATION

This subsection presents a preliminary simulation study of the preview control strategy applied to Model I. The model includes two state variables to describe the vehicle and a single state variable to represent the magnet module dynamics. Also, all state variables in the augmented maglev system are assumed measurable.

The open-loop system dynamics of Model I are represented by equations (15) and (21), with parameter values listed in Table 2. The transfer function from the input voltage, \( u \), to the air gap error, \( y \), [(defined in equation (22))] can be determined as

\[
\frac{Y(s)}{U(s)} = \frac{-6.01}{(s - 20.9)(s + 23.8)(s + 176.1)}
\]  

(123)
where $U(s)$ and $Y(s)$ represent the Laplace transforms of $u$ and $y$, respectively. The resulting pole-zero map is plotted in Figure 18. It clearly indicates an unstable pole located at $+20.9$. This pole is associated with the inherent instability of the EMS magnet system. There are also two real poles in the left-half plane. The pole at $-23.8$ represents a stable vehicle mode with a time constant of 0.042 s ($=1/23.8$). As noted in Subsection 2.1.3, this time constant is much slower than the time constant of the magnet, i.e., 0.0056 s.

To stabilize the system, the control law (96) is applied to the augmented maglev model (85) to form a stable closed-loop system. The weighting matrices, used to determine the control law, are selected by the rule described in Subsection 2.4.5, i.e.,

$$Q = \text{diag}\left[ z_{\text{max}}^{-2} \quad \dot{z}_{\text{max}}^{-2} \quad i_{\text{max}}^{-2} \quad y_{i,\text{max}}^{-2} \right]$$

(124)

$$R = u_{\text{max}}^{-2}$$

(125)
where the estimated limits are adopted from Table 5, i.e., \( \ell_{\text{max}} = 0.01 \text{m} \), \( \dot{\ell}_{\text{max}} = 1 \text{m/s} \), \( i_{\text{max}} = 1000 \text{A} \), \( y_{f,\text{max}} = 0.001 \text{m/s} \), and \( u_{\text{max}} = 100 \text{V} \). These values are the estimated limits of the augmented state variables \((z, \dot{z}, \dot{i}, y_f)\) and the control variable \((u)\). Applying Laplace transforms to the closed-loop system equations, the closed-loop transfer function from the guideway displacement, \(w\), to the air gap error, \(y\), can be derived as

\[
\frac{Y(s)}{W(s)} = \frac{-s(s + 181.7)(s^2 + 47.3s + 1460)}{(s + 176.9)(s + 5.93)(s^2 + 46.1s + 573.2)}
\]  

(126)

where \(W(s)\) is the Laplace transform of \(w\). Figure 19 shows that all closed-loop poles have negative real parts implying a stable system. The controller with integral action adds a closed-loop zero at the origin to achieve a zero steady-state gap error due to step guideway displacement. With the selection of \(Q\) and \(R\), the pair of complex conjugate eigenvalues has a natural frequency of 23.9 rad/s (3.80 Hz) and a damping ratio of 0.962.
The simulation studies assume that the vehicle is traveling horizontally at a constant speed of 100 m/s (360 kph, 224 mph) over a step guideway disturbance of magnitude 10 mm. Figure 20 shows the guideway displacement with a step disturbance of 10 mm occurring at 0.1 s (10 m from the initial position). (For the maglev system, the guideway displacement, w, is positive in the down direction, as indicated in Figure 5.) As shown in Figure 20, the step increases from 0 to 10 mm in 0.01 s. As such, it is not a theoretical step and its (numerical) derivative has a finite value.

4.1.1 Comparison to Conventional Suspension Design

To gain an understanding of the simplified maglev system (without preview), its performance is compared to that of vehicles with conventional (i.e., mechanical mass-spring-damper-type) suspensions. In particular, two different suspension designs are considered, i.e., a single DOF mechanical model, as shown in Figure 21a, and a two DOF model, as represented in Figure 21b. In both cases, the conventional vehicle model is assumed to consist of linear passive mechanical components. The assumed convention for positive displacements is vertically up.
Here $w$ is assumed to be the guideway displacement, and the gap error is $z - w$ for the one DOF model and $z_1 - w$ for the two DOF model.

One difficulty in the comparison to the maglev vehicle performance is the selection of model parameters for the conventional suspension. Here, for the conventional one DOF model two sets of values are examined, as presented in Table 8. Specific parameter values are reported in Table 9 for the two DOF model. For the two DOF model, a mass ratio of 5:1 is assumed for the ratio of secondary to primary masses, with the total mass being the same as that of the two cases of the one DOF model. The primary suspension is assumed to have a 5 Hz natural frequency and 5 percent damping and the secondary suspension is selected to have a 1 Hz natural frequency and 30 percent damping. Note that for the one DOF model, Case A is based on a primary suspension design, and Case B is representative of a secondary suspension design. Traditionally, ground-based vehicles have used a primary suspension with a relatively high natural frequency (5 to 10 Hz) and low damping (0 to 5 percent of critical damping) to closely follow the guideway, and a secondary suspension with a relatively low natural frequency (~1 Hz) and relatively high damping (30 to 50 percent) to isolate the passengers.

\[ m\ddot{z} + b\dot{z} + kz = bw + kw \]

**Figure 21.** (a) One DOF and (b) Two DOF Conventional Vehicle Models
Table 8. Parameter Values of One DOF Conventional Vehicle Model

<table>
<thead>
<tr>
<th>Case</th>
<th>Mass $m$ (kg)</th>
<th>Damping $b$ (N·s/m)</th>
<th>Stiffness $k$ (N/m)</th>
<th>Natural Freq $\omega_n$ (Hz)</th>
<th>Damping Ratio $\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2340</td>
<td>7350</td>
<td>2,309,000</td>
<td>5</td>
<td>0.05</td>
</tr>
<tr>
<td>B</td>
<td>2340</td>
<td>8820</td>
<td>92,400</td>
<td>1</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 9. Parameter Values of Two DOF Conventional Vehicle Model

<table>
<thead>
<tr>
<th>System</th>
<th>Mass $m$ (kg)</th>
<th>Damping $b$ (N·s/m)</th>
<th>Stiffness $k$ (N/m)</th>
<th>Natural Freq $\omega_n$ (Hz)</th>
<th>Damping Ratio $\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary (subscript 1)</td>
<td>390</td>
<td>1225</td>
<td>384,900</td>
<td>5</td>
<td>0.05</td>
</tr>
<tr>
<td>Secondary (subscript 2)</td>
<td>1950</td>
<td>7350</td>
<td>77,000</td>
<td>1</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Figure 22 shows the gap error history for the simplified EMS model and the two cases of the one DOF model with conventional suspension. In all cases there is an instantaneous drop in gap error due to the step change in guideway displacement, indicating that the vehicle is approaching the guideway. This result is expected for the maglev model based on the form of the transfer function in equation (126). (Since the numerator and denominator have the same order polynomials, it indicates a direct transmission from input to output.) For the maglev model, the gap error continues to drop (meaning the vehicle moves closer to the guideway) and then returns toward zero, ultimately achieving zero steady-state error. This behavior is explained as follows. As a result of the step, the gap is decreased creating a large magnet force. This magnet force drives the vehicle closer toward the guideway (i.e., the gap continues its descent). The controller responds by commanding the current, and the associated magnet force, to drop. The result is that the air gap increases and then is regulated to its nominal value.

The gap error histories for the conventional suspension shown in Figure 22 highlight several distinct behaviors. Upon encountering the step, the restoring forces act immediately to
increase the gap. (This is in contrast to the EMS maglev model for which the force is non-restoring, and hence unstable without the controller.) For both Cases A and B, the responses exhibit underdamped behavior (in contrast to the maglev response which behaves like a critically damped system) with relatively long settling times. The slow speed of convergence is especially evident for Case B with minimal damping.

The acceleration time histories for the simplified maglev model and for the two cases of the one DOF model with conventional suspension are plotted in Figure 23. For the maglev model there is an instantaneous decrease in acceleration due to the step change in guideway displacement. This result can be anticipated from equation (13), which shows that the acceleration is linearly proportional to the guideway displacement at the instant of the offset (when the current and vehicle displacement are zero). Upon encountering the step, the initial value of the acceleration (here -0.5 g) is independent of the controller (since \( i \) is initially zero). Subsequently, \( i \) is adjusted by the controller. Equation (13) governs the acceleration response which exhibits overshoot and then regulation to zero.
For the acceleration histories of the conventional suspension models, the initial responses are shown in Figure 23 as finite positive values. (Theoretically they are impulses with infinite magnitude, but appear as finite values due to the assumed shape of Figure 20. The sign of the initial acceleration can be determined from the equation of motion presented in Figure 21. Initially, the displacement and velocity of the mass are zero, and the guideway step \( w \) is positive.) The acceleration history for Case A is lightly damped and shows large peak accelerations. For Case B the accelerations are small except at the step (where there is a theoretically infinite acceleration). Whereas the initial acceleration of the maglev system is fixed [determined by equation (13)], the magnitudes of the initial acceleration of the conventional suspension models depend on the time interval of the step rise.

In addition to the comparison to the single DOF passive mechanical system, the maglev system has been studied relative to a two DOF conventional model, as shown in Figure 21b, consisting of primary and secondary suspension subsystems. The gap error history for the two DOF...
Figure 24. Gap Error for Two DOF Conventional & EMS

Figure 25. Acceleration for Two DOF Conventional & EMS
system shows improved performance in comparison to the one DOF conventional model in terms of smaller overshoot and faster settling time. Relative to the maglev system, the two DOF conventional model exhibits improved performance in terms of gap deviation (i.e., primary suspension stroke, $z_1 - w$) and acceleration (i.e., sprung mass acceleration, $\dot{z}_2$) and slightly worse performance in terms of settling time. The acceleration of the sprung mass exhibits no discontinuity when the vehicle encounters the guideway step, due to the filtering of the secondary suspension. The performance of the maglev system relative to that of the two DOF mechanical model is challenged in that the maglev system has only a primary suspension, albeit active, and no secondary suspension system.

In summary, this simulation study tests the simplified maglev model, Model I, using LQ optimal control (augmented with integration) without preview, and compares its performance to one and two DOF passive mechanical models. The results for the maglev model (without preview) show that:

- LQ optimal control leads to stable system behavior;
- the integral action leads to a zero steady-state gap error due to a step disturbance;
- its performance has advantages relative to a single DOF conventional model; and
- its performance is compromised relative to a two DOF conventional model, essentially due to the lack of a secondary suspension.

These results for the maglev system are dependent on the design of the controller. The controller tested here has been selected to satisfy multiple performance specifications (as reflected by the weighting matrices.) It is possible to tailor the controller to achieve improved performance for any given specification.

4.1.2 Preview Control Performance

To investigate the influence of preview time, simulations with three different preview times (0.01 s, 0.05 s, and 0.1 s) and without preview (equivalent to LQ control with integral action, as tested in the previous subsection) are considered. (The preview times of 0.01 s, 0.05 s, and 0.1 s correspond to preview distances of 1 m, 5 m, and 10 m, respectively.) In testing the
Figure 26. Magnet Input Voltage for Step Disturbance

Figure 27. Magnet Force for Step Disturbance
Figure 28. Air Gap Error for Step Disturbance

Figure 29. Vehicle Acceleration for Step Disturbance
effectiveness of preview control, it is assumed that the guideway displacement can be measured perfectly.

As noted earlier, Figure 20 shows the guideway displacement with a step disturbance of 10 mm occurring at 0.1 s (10 m from the initial position). The associated magnet input voltages, magnet forces, air gap errors, and vehicle accelerations are shown in Figures 26-29, respectively. Figure 26 indicates that the input voltage may be reduced substantially with increasing preview time. With preview control, the feedforward controller acts before encountering the step disturbance to reduce the input voltage. The system without preview reacts only after the vehicle reaches the step. Note that the voltage history for this no-preview case is the step response of equation (126). The nominal magnet force, as indicated in Figure 27, is 22.95 kN and serves to balance the vehicle weight. In general, the magnet force variations decrease in magnitude as the preview time increases. Figure 28 shows that the air gap error can be less than the step magnitude of 10 mm when preview control is applied. (A positive air gap error corresponds to an air gap larger than the nominal gap of 40 mm.) Without preview, the air gap error is always greater than the step magnitude. Further, the steady-state gap error approaches zero as a result of the incorporated integrator. Figures 28 and 29 show that the peak gap error and acceleration decrease as the preview time increases. However, the improvement in acceleration achieved by using a large preview time (e.g., 0.05 s and 0.1 s) is limited. It implies a tradeoff between the gap error and the acceleration, in which both error and acceleration may not be minimized simultaneously. The maximum and minimum values in Figures 26 - 29 are summarized in Table 10.

In summary, this simulation study tests the simplified maglev model, Model I, using LQ preview control (augmented with integration). The results lead to several conclusions, including:
- LQ optimal control stabilizes the maglev system;
- the integral action leads to a zero steady-state gap error due to step disturbances; and
- preview control enhances the system performance by reducing the input voltage, magnet force, gap error and vehicle acceleration.

<table>
<thead>
<tr>
<th>Preview Time (s)</th>
<th>Input Voltage (V) Max Min</th>
<th>Magnet Force (kN) Max Min</th>
<th>Gap Error (mm) Max Min</th>
<th>Acceleration (g) Max Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0</td>
<td>34.4 19.4</td>
<td>0 -14.4</td>
<td>0.15 -0.50</td>
</tr>
<tr>
<td>0.01</td>
<td>26.8 -204</td>
<td>32.7 20.6</td>
<td>0 -12.0</td>
<td>0.16 -0.43</td>
</tr>
<tr>
<td>0.05</td>
<td>4.3 -114</td>
<td>26.5 16.5</td>
<td>0.8 -9.0</td>
<td>0.28 -0.15</td>
</tr>
<tr>
<td>0.1</td>
<td>-0.2 -67</td>
<td>27.2 17.6</td>
<td>2.5 -6.6</td>
<td>0.23 -0.18</td>
</tr>
</tbody>
</table>

### 4.2 PERFORMANCE MEASURES FOR MODEL II

A number of safety- and comfort-related performance measures can be identified for Model II, the more sophisticated maglev system model. It is required that this model:

- maintain each magnet module/iron rail air gap within a safety margin to prevent vehicle/guideway contact;
- keep the control voltages within feasible limits; and
- maximize ride comfort.

In this study, the allowed air gap is chosen to be between 30 mm and 50 mm (40 mm nominal gap with ±10 mm maximum gap error). The smallest acceptable air gap of 30 mm precludes vehicle/guideway contact while assuring a reasonable safety margin. The required control voltages are constrained within ±600 V (Kortum and Utzt, 1984) to prevent saturation of the magnet forces. The ride comfort is measured by comparing carbody accelerations at the car front and rear to the ISO ride quality criteria (ISO, 1978).
The ISO ride quality criteria specifies limits on Root Mean Square (RMS) lateral and vertical accelerations in one-third octave bands over a specified range of frequencies. The RMS value at any center frequency $\omega_c$ in one-third octave bands can be calculated numerically by

$$p_{rms} = \left[ \frac{1}{\omega_u - \omega_l} \int_{\omega_l}^{\omega_u} S_p(\omega) d\omega \right]^{1/2}$$

(127)

where $p_{rms}$ is the RMS value of the time-varying function, $p(t)$, which may represent the lateral or vertical accelerations of the carbody, $S_p(\omega)$ is the corresponding power spectral density, and $\omega_l$ and $\omega_u$ are the lower and upper bounds for the one-third octave band, respectively, where

$$\omega_l = \omega_c \exp\left(\frac{1}{6} \ln 2\right) = 1.122 \omega_c$$

(128)

$$\omega_u = \omega_c \exp\left(-\frac{1}{6} \ln 2\right) = 0.891 \omega_c$$

(129)

<table>
<thead>
<tr>
<th>Center Frequency of Third-Octave Band (Hz)</th>
<th>Lateral RMS Acceleration (g)</th>
<th>Vertical RMS Acceleration (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.0275</td>
<td>0.0764</td>
</tr>
<tr>
<td>1.25</td>
<td>0.0275</td>
<td>0.0686</td>
</tr>
<tr>
<td>1.60</td>
<td>0.0275</td>
<td>0.0615</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0275</td>
<td>0.0550</td>
</tr>
<tr>
<td>2.50</td>
<td>0.0343</td>
<td>0.0485</td>
</tr>
<tr>
<td>3.15</td>
<td>0.0427</td>
<td>0.0427</td>
</tr>
<tr>
<td>4.00</td>
<td>0.0550</td>
<td>0.0382</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0686</td>
<td>0.0382</td>
</tr>
<tr>
<td>6.30</td>
<td>0.0858</td>
<td>0.0382</td>
</tr>
<tr>
<td>8.00</td>
<td>0.1084</td>
<td>0.0382</td>
</tr>
<tr>
<td>10.0</td>
<td>0.1375</td>
<td>0.0485</td>
</tr>
<tr>
<td>12.5</td>
<td>0.1715</td>
<td>0.0615</td>
</tr>
<tr>
<td>16.0</td>
<td>0.2168</td>
<td>0.0764</td>
</tr>
<tr>
<td>20.0</td>
<td>0.2751</td>
<td>0.0971</td>
</tr>
</tbody>
</table>
The power spectral density, $S_p(\omega)$, can be determined numerically using MATLAB's power spectral density function `psd`. The MATLAB code used to determine the RMS value is listed in Appendix B.

The ISO reduced comfort boundaries for one-hour exposure are summarized in Table 11 (ISO, 1978). In the simulations, it is required that the lateral and vertical RMS accelerations for points at the car front and rear are below the ISO one-hour reduced comfort boundaries.

### 4.3 MODEL II SIMULATION

The nonlinear maglev system Model II, described in Subsection 2.5, includes the effects of guideway flexibility, crosswind gust, and guideway irregularities such as random roughness, step, ramp, and camber errors. This model is the basis for the simulation study presented here. In the block diagram of Figure 15, it is assumed that preview information, $v$, is not available and hence only feedback control is employed. Also, all the inputs of the feedback controller are assumed measurable such that full state feedback can be applied.

The weighting matrices $Q$ and $R$, used to determine the controller parameters, are selected as

\[
Q = \text{diag}(y_{\text{max}}^{-2}, z_{\text{max}}^{-2}, \phi_{\text{max}}^{-2}, \theta_{\text{max}}^{-2}, \phi_{\text{max}}^{-2}, \dot{z}_{\text{max}}^{-2}, \omega_{x,\text{max}}^{-2}, \omega_{y,\text{max}}^{-2}, \omega_{z,\text{max}}^{-2})
\]

\[
R = \text{diag}(\mu_{1,\text{max}}^{-2}, \cdots, \mu_{2N_m,\text{max}}^{-2})
\]

where the diagonal elements are the maximum value estimates as reported in Table 5.

The parameter values used in the simulation study are listed in Table 1 for the SC magnets, Table 3 for the vehicle model, and Table 4 for the guideway model. In addition, it is assumed that (i) there are two magnet modules on each side ($N_m = 2$), with each module housing twelve magnets, and (ii) only the first two modes ($n_s = 2$) are significant for the guideway span dynamics. Figure 30 shows the guideway mid-point deflections using one, two, and three.
modes as the vehicle travels over a flexible guideway span at a constant speed of 100 m/s. The results suggest that a two-mode guideway model is sufficient.

The vehicle model was simulated on a four-span guideway at a speed of $V = 100$ m/s, and subjected to vertical guideway flexibility and disturbance inputs such as guideway irregularities and crosswind gusts. Several cases are presented below. The first set of cases examines the individual effects of the guideway flexibility and the disturbance inputs. Then, a sample case with combined disturbance inputs as well as guideway flexibility is presented.

4.3.1 Case 1: Guideway Flexibility

Figures 31, 32, and 33 show the magnet input voltages, magnet forces, and the air gap errors at each module, respectively, for the case with guideway flexibility. The peak values of the magnet input voltages are well below the required 600 V and the maximum gap error is within the safety margin of 10 mm. The nominal magnet force at each module required to balance the vehicle weight is 179.6 kN. The maximum force of 200.3 kN occurs at modules 2 and 4 (i.e.,
the rear modules on the right and left), representing a 11.5 percent increase over the nominal value.

The accelerations at the front and rear of the carbody are depicted in Figure 34. The lateral accelerations are essentially zero, since the vehicle encounters only vertical guideway flexibility. The peak acceleration occurs as a 0.107g deceleration at the rear of the car. The RMS accelerations in both lateral and vertical directions, shown in Figures 35 and 36, respectively, comply with the ISO one-hour ride quality criterion. The vertical guideway flexibility has little influence on the lateral RMS acceleration. The vertical RMS acceleration has a maximum value of 0.0257 g at 8.32 Hz.

![Magnet Input Voltages for Right-Side Modules](image)
![Magnet Input Voltages for Left-Side Modules](image)

*Figure 31. Magnet Input Voltages for the Case of Guideway Flexibility*
Figure 32. Magnet Forces for the Case of Guideway Flexibility

Figure 33. Air Gap Errors for the Case of Guideway Flexibility
Figure 34. Accelerations at Front/Rear for the Case of Guideway Flexibility

Figure 35. Lateral RMS Accelerations for the Case of Guideway Flexibility
4.3.2 Case 2: Crosswind Gust

Figure 37 shows the distribution of the crosswind gust with a lateral air flow velocity of 30 m/s (108 kph, 67 mph) that starts at the beginning of the second span (at 25 m). The direction of the air flow is in the $Y_f$ direction of the inertial frame (from the right of the vehicle). Figure 38 indicates that the voltages for the right-side modules decrease while the voltages for the left-side modules increase. This is a result of the crosswind gust from the right of the vehicle. The controller adjusts the voltages to produce the extra magnet force to counteract the aerodynamic force and moment generated by the crosswind gust. The maximum voltage for a 30 m/s crosswind gust is 43.9 V at module 3 which is much smaller than the limit of 600 V. Figure 39 shows that the maximum air gap error is only 1.1 mm at module 3 and the steady-state gap error approaches zero as a result of the integral action of the controller. Figures 40 and 41 show that the ISO ride quality criterion for the one-hour limit is satisfied. The crosswind gust contributes a maximum RMS acceleration of 0.0061 g at 4.17 Hz in the lateral direction.
Figure 37. Distribution of Crosswind Gust Along Guideway

Figure 38. Magnet Input Voltages for the Case of Crosswind Gust
Figure 39. Air Gap Errors for the Case of Crosswind Gust

Figure 40. Lateral RMS Accelerations for the Case of Crosswind Gust
4.3.3 Case 3: Step Irregularity

Figure 42 shows a guideway step irregularity with a magnitude of 5 mm at the junction of the first and second spans. The step irregularity is assumed to be in the vertical direction. Figure 43 shows the resulting voltages for the right-side and left-side modules. The maximum voltage for the step irregularity is 64.7 V and occurs at modules 1 and 3. Figure 44 shows the air gap errors. The maximum error is 5.3 mm at modules 1 and 3. The steady-state gap error approaches zero as a result of the integral action of the controller. Figures 45 and 46, for the lateral and vertical RMS accelerations, respectively, show that the ISO ride quality criterion with the one-hour limit is satisfied. The step irregularity contributes a maximum RMS acceleration of 0.0109 g at 6.31 Hz in the vertical direction.
Figure 42. Guideway Displacements for the Case of Step Irregularity

Figure 43. Magnet Input Voltages for the Case of Step Irregularity
Figure 44. Air Gap Errors for the Case of Step Irregularity

Figure 45. Lateral RMS Accelerations for the Case of Step Irregularity
4.3.4 Case 4: Combined Disturbance Inputs with Guideway Flexibility

The combined effects of guideway flexibility, crosswind gust, and guideway irregularity are considered here. The velocity distribution of the crosswind gust was shown in Figure 37. The guideway irregularity consists of guideway roughness, step, ramp, and camber irregularities. Individual components of the combined guideway irregularity are shown in Figures 47 and 48, where the guideway roughness is in the normal direction of the rail face while the guideway step, ramp, and camber irregularities are in the vertical direction. The step deviation, the column height, and the camber amplitude for each span are generated randomly using MATLAB's normal distribution function, `randn`, with a 2 mm standard deviation (with probability 99.7% the value will be less than 6 mm). Figure 49 shows the combined guideway irregularity in the lateral and vertical directions.

Figure 46. Vertical RMS Accelerations for the Case of Step Irregularity
Figure 47. Guideway Roughness and Step Irregularities

Figure 48. Guideway Ramp and Camber Irregularities
The magnet input voltages at each module are shown in Figure 50. The maximum voltage is 95.9 V at module 3 which is substantially less than the limit of 600 V. Figure 51 shows the magnet force generated by each module. Initially, the magnet force for all the modules is 180 kN which is supplied by the SC coil to balance the vehicle weight. The force variation is controlled by the voltages and has a maximum magnitude of 51 kN (129 kN at module 1) which is less than 30% of the steady-state value. Figure 52 shows that all the gap errors are below the allowable 10 mm safety margin (the maximum of 7.7 mm occurs at module 4). Figure 53 depicts the vehicle accelerations for car front and car rear in the lateral and vertical directions. The lateral accelerations are mainly attributed to the crosswind gust and the guideway roughness while the vertical accelerations are due to the guideway flexibility and the guideway irregularity. To evaluate ride comfort, the one-third octave RMS accelerations in the lateral and vertical directions are computed and compared with the ISO ride quality criteria. Figures 54 and 55 show the lateral and vertical RMS accelerations for car front and car rear, respectively. The results indicate that both the lateral and vertical acceleration levels are below
the ISO 1-hour limit. Table 12 summarizes the maximum and minimum values of the simulation results.

Figure 50. Magnet Input Voltages for the Combined Disturbance Case

Figure 51. Magnet Forces for the Combined Disturbance Case
Figure 52. Air Gap Errors for the Combined Disturbance Case

Figure 53. Lateral and Vertical Accelerations for the Combined Disturbance Case
Figure 54. Lateral RMS Accelerations for the Combined Disturbance Case

Figure 55. Vertical RMS Accelerations for the Combined Disturbance Case
Table 12. Maximum and Minimum Values of Model II Simulation

<table>
<thead>
<tr>
<th></th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet Input Voltage (V)</td>
<td>95.9</td>
<td>−93.5</td>
</tr>
<tr>
<td>Trim Current (A)</td>
<td>91.5</td>
<td>−85.1</td>
</tr>
<tr>
<td>Magnet Force (kN)</td>
<td>225</td>
<td>129</td>
</tr>
<tr>
<td>Air Gap Error (mm)</td>
<td>5.40</td>
<td>−7.7</td>
</tr>
<tr>
<td>Lateral Peak Acceleration (g)</td>
<td>0.048</td>
<td>−0.056</td>
</tr>
<tr>
<td>Vertical Peak Acceleration (g)</td>
<td>0.130</td>
<td>−0.172</td>
</tr>
<tr>
<td>Lateral RMS Acceleration (g)</td>
<td>0.008 @ 8.3 Hz</td>
<td>−</td>
</tr>
<tr>
<td>Vertical RMS Acceleration (g)</td>
<td>0.028 @ 11.7 Hz</td>
<td>−</td>
</tr>
</tbody>
</table>

In summary, this simulation study tests the system performance of Model II under disturbances including guideway flexibility, crosswind gusts and guideway irregularities. The behavior of the EMS-type maglev system when exposed to individual or combined disturbances can be determined. The simulation results show that the requirements on the air gap safety margin, the control voltage limit, and the ride quality can be satisfied for the simulation cases.
5. CLOSURE

5.1 MODELING CAPABILITIES

In this study two simulation models were developed for EMS-type maglev vehicles: a simplified linear model, Model I, and a more realistic nonlinear model, Model II. The modeling capabilities and assumptions of both models are summarized below.

5.1.1 Features of Model I

Model I is a linear maglev model, which is useful for preliminary investigations of EMS-type maglev control systems. It consists of a single mass representing the vehicle, a SC magnet mounted on the vehicle, and a control system to regulate the air gap under the influence of guideway irregularities. It is assumed that the vehicle is characterized by a single DOF in the vertical direction (i.e., a bounce model) and that the guideway is rigid. The SC magnet supplies levitation force and is a linear version of the nonlinear SC magnet used in Model II. The control system consists of both feedback and feedforward controllers. The feedback controller stabilizes the vehicle in its equilibrium position and regulates the air gap as the vehicle encounters guideway deviations. The feedforward controller takes in guideway information in front of the vehicle to reduce acceleration and gap error further. Model I is represented by four state variables: vehicle displacement; vehicle velocity; magnet current; and the integral of gap error (for the controller).

5.1.2 Features of Model II

Model II simulates an EMS-type maglev vehicle on a multiple-span vertically flexible guideway subject to disturbance inputs, including crosswind gust and guideway irregularities. The components of Model II contain a nonlinear SC magnet system, a five DOF vehicle model, a multiple-DOF guideway model, and a (dynamic) feedback control system. The modeling capabilities and assumptions of the subsystems and disturbance inputs are summarized as follows.
• The SC magnet model accepts input voltage and produces magnet force. The assumptions are (i) negligible magnetic flux loss and (ii) constant current in the SC coil. To satisfy the first assumption, the maximum air gap should be small. In this study, it is set to 50 mm (10 mm gap error with nominal gap 40 mm). To relax the second assumption, a control system for regulating the SC current could be included in future models.

• The vehicle is assumed to be rigid and has DOFs in the lateral (sway), vertical (bounce), roll, pitch, and yaw directions. The magnet modules are mounted along the vehicle with a cant angle to provide simultaneous guidance and levitation. The magnet force of each magnet module is assumed to be uniformly distributed. In the analysis, the small angle assumption has been made for the roll, pitch, and yaw displacements.

• The guideway model determines the vertical deflection of the spans seen by each magnet module. It is assumed that each guideway span is simply supported. Further, the vehicle length is assumed to be less than the guideway span length such that only the dynamics of two sequential spans need to be considered.

• The feedforward controller is not available in Model II. The control system uses feedback control alone and the feedback signals are all assumed measurable.

• Crosswind gust is modeled as a lateral air flow with constant speed over each guideway span. As the vehicle enters a particular span, the air flow produces an equivalent lateral force and a yaw moment acting at the vehicle CG.

• Five different types of guideway irregularities are considered. These are: surface roughness, step, ramp, camber and versine. It is assumed that the surface roughness affects the right and left rails independently while the variations for the step, ramp, camber, and versine irregularities are the same for the left and the right rails.
5.2 MODEL EXTENSIONS

Several modeling improvements are envisioned to extend the capabilities of Model II.

5.2.1 Extended SC Magnet Model

Future SC magnet models may relax the assumptions on (i) negligible magnetic flux and (ii) constant SC current. To account for the magnetic flux loss, a 3-D finite element electromagnetic analysis can be performed to find the magnet force as a function of the SC current, trim current, and the air gap. This more detailed functional relationship will replace equation (3) as the SC magnet force characteristic. To account for the more realistic situation in which the vehicle loading varies, the SC current is no longer a constant due to the change of the total vehicle weight. In this case, the voltage equation (5) needs to be modified to account for the *emf* induced by the SC coil.

5.2.2 Extended Vehicle Model

The five DOF vehicle model is assumed to be rigid and travels along a straight, horizontal guideway at a constant speed. Future models may include: (i) a secondary suspension system for better ride comfort; (ii) a carbody tilt mechanism; (iii) curving dynamics with and without banked guideway; (iv) vehicle flexibility to account for carbody modes; (v) longitudinal dynamics to form a complete six DOF vehicle model; and (iv) a multiple-car vehicle consist for more passenger capacity.

5.2.3 Extended Guideway Model

Future guideway models may include guideway span torsional dynamics and boundary conditions other than simply-supported for the guideway spans. A possible boundary condition may involve a linear spring and damper to support the span in the vertical direction and a torsional spring in the roll direction.
5.2.4 **Extended Control System**

Control system extensions may include: (i) preview control (feedforward controller) for the five DOF vehicle maglev system; (ii) a vehicle state estimator in the feedback controller to estimate the vehicle state variables which are not measurable; and (iii) a regulator for adjusting the SC current to account for changing vehicle weight.

5.3 **SUMMARY**

This study reports upon two simulation models that were developed for representing the governing behavior of EMS-type maglev systems. The simplified model offers insights into the nature of the SC magnet system and the effectiveness of the LQ optimal controller. The practical model predicts vehicle behaviors under the influence of guideway flexibility, guideway irregularities, and crosswind gusts. A simulation program is developed which allows users to select various types and magnitudes of disturbance inputs. Program outputs include the applied voltages, the magnet forces, the air gaps, and the RMS accelerations (compared with the ISO ride quality criterion). An example simulation case with a vehicle speed of 100 m/s and guideway flexibility, crosswind gust, and a combination of guideway irregularities demonstrates the effectiveness of the LQ optimal controller to satisfy the desired performance specifications. The simulation models are analytical tools providing the capability for further exploration of vehicle, guideway, and controller design issues.
APPENDIX A: NUMERICAL LINEARIZATION OF THE FIVE DOF MAGLEV VEHICLE

The state equation (65) and the output equation (66) can be written in the standard form:

\[
\dot{x} = A_p x + B_p u_p \tag{A.1}
\]

\[
y = C_p x + D_p u_p \tag{A.2}
\]

where

\[
A_p = A \tag{A.3}
\]

\[
B_p = \begin{bmatrix} B \\ E \end{bmatrix} \tag{A.4}
\]

\[
C_p = C \tag{A.5}
\]

\[
D_p = \begin{bmatrix} O \\ D \end{bmatrix} \tag{A.6}
\]

\[
u_p = \begin{bmatrix} u \\ v \end{bmatrix} \tag{A.7}
\]

In equation (A.6) \( O \) is a zero matrix with appropriate dimensions. To obtain the linear model, the coefficient matrices \( A_p, B_p, C_p, \) and \( D_p \) are first determined by SIMULINK's linearization function \texttt{linmod}. The coefficient matrices of the original state and output equations [i.e., \( A, B, C, D, \) and \( E \) in equations (65) and (66)] are then recovered from equations (A.3)-(A.6). The MATLAB code for linearization is shown below:

```matlab
% Linearization of the 5 DOF maglev system

[Ap, Bp, Cp, Dp] = linmod('plant');
Nm=2; % number of modules on each side
A=Ap;
B=Bp(:,1:2*Nm);
C=Cp;
D=Dp(:,2*Nm+1:6*Nm+2);
E=Bp(:,2*Nm+1:6*Nm+2);
```

where \texttt{plant} is an S-function defining the nonlinear maglev system. (See the User's Guide for more information on SIMULINK's S-functions.) The function \texttt{plant} is listed as follows:

```matlab
function [sys,x0]=plant(t,x,u,flag)
% PLANT S-function of the 5 DOF maglev system
```

83
% System parameters
Nm=2; % number of modules on each side
nm=12; % number of magnets per module
mu0=pi*4e-7; % permeability of air (H/m)
Am=.04; % face area of each magnetic pole (m)
N=1020; % number of turns in SC coil
I=42.8; % SC current (A)
n=96; % number of turns in normal coil
m=64; % vehicle mass (kg)
g=9.81; % acceleration due to gravity (m/s^2)
Ix=0.148e6; % moment of inertia about carbody X axis (kg-m^2)
Iy=1.73e6; % moment of inertia about carbody Y axis (kg-m^2)
Iz=1.91e6; % moment of inertia about carbody Z axis (kg-m^2)
Rc=1.04; % resistance of the normal coil (ohm)
lm=9; % module length (m)
w0=0.763; % width, magnet centroid to car C.G. (m)
ho=.04; % nominal air gap (m)
beta=35*pi/180; % angle of magnet inclination from vertical (rad)

for j=1:Nm,
    ra(1,j)=lm*(Nm/2+.5-j);
    ra(2,j)=-wc;
    ra(3,j)=-hc;
    ra(1,j+Nm)=ra(1,j);
    ra(2,j+Nm)=wc;
    ra(3,j+Nm)=-hc;
end

for j=1:Nm,
    rb(1,j)=lm*(Nm/2+.5-j);
    rb(2,j)=-wc+h0*sin(beta);
    rb(3,j)=-hc+h0*cos(beta);
    rb(1,j+Nm)=rb(1,j);
    rb(2,j+Nm)=wc-h0*sin(beta);
    rb(3,j+Nm)=-hc+h0*cos(beta);
end

% S-function system description

if flag == 0
    sys=[10+2*Nm 0 2*Nm 6*Nm+2 0 1];
    x0=zeros(10+2*Nm,1);
elseif abs(flag) == 1 | flag == 3
    % State variables
    % x(1) : yc
    % x(2) : zc
    % x(3) : phi_c
    % x(4) : theta_c
    % x(5) : psi_c
    % x(6) : ycd
    % x(7) : zcd
    % x(8) : omega_x
    % x(9) : omega_y
    % x(10) : omega_z
    x(j=11...10+2Nm): trim current (i)
    % Input/Disturbance variables
    u(j=1...2Nm) : input voltage (u)
    u(j=2Nm+1...3Nm) : lateral guideway displacement (yg)
    u(j=3Nm+1...4Nm) : vertical guideway displacement (zg)
    u(j=4Nm+1...5Nm) : lateral guideway velocity (ygd)
    u(j=5Nm+1...6Nm) : vertical guideway velocity (zgd)

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% u (j=6Nm+1...6Nm+2): crosswind gust (Fw,Mw)
i=x(11:10+2*Nm);
yg=u(2*Nm+1:3*Nm);
yg(3*Nm+1:4*Nm)=yg;
zyg=u(4*Nm+1:5*Nm);
ygd(u(5*Nm+1:6*Nm);
ygd(6*Nm+1:7*Nm)=zgd;
Fw=u(6*Nm+1);
Mw=u(6*Nm+2);
\% Carbody/Inertial coordinate transformation
\% Tc : carbody roll-pitch-yaw transformation matrix
\% Ted: time derivative of Tc
Tc=[1-x(5) x(4);x(5) 1-x(3);x(4) x(3) 1];
Tcd=Tc*[0 -x(10) x(9);x(10) 0 -x(8);x(9) x(8) 0];
\% Magnet forces and resulting moments
\% SF: sum of the magnet forces in initial frame
\% SM: sum of the resulting moments in carbody frame
rc=[0;x(1);x(2)];
rcd=[0;x(6);x(7)];
rm=Tc*ra;
rmd=Tcd*ra;
for j=1:2*Nm,
    rg(:,j)=[0;yg(j);zg(j)];
    rgd(:,j)=[0;ygd(j);zgd(j)];
    rf(:,j)=rg(:,j)+rb(:,j)-rm(:,j)-rc;
    rfd(:,j)=rgd(:,j)-rm(:,j)-rc;
    h(j)=norm(rh(:,j));
    hhd(j)=rh(:,j)'*rhd(:,j)/h(j);
    f(j)=nm*muO*Am*(N*I+n*i(j))^2/4/h(j)^2;
    M(:,j)=M(2:j) rm(2:j);M(3:j) 0 -rm(1,j); ... 
    end
SF=sum(F')';
SM=Tc'*sum(M')';
\% Dynamic equations
xd(1)=x(6);
xd(2)=x(7);
xd(3)=x(8)+(x(9)*x(3)+x(10))*x(4);
xd(4)=x(9)-x(10)*x(3);
xd(5)=x(9)*x(3)+x(10);
xd(6)=(SF(2)+Fw)/m;
xd(7)=SF(3)/m-g;
xd(8)=(1x(3)+x(10)+SM(1))/Iy;
xd(9)=(1x(4)+x(8)+SM(2))/Iy;
xd(10)=(1x(5)+x(9)+SM(3)+Mw)/Iy;
for j=1:2*Nm,
    xd(10+j)=[(u(j)-Rc*i(j)+muO*Am*n*(N*I*n*i(j))*h(j))/2/ 
    end
if abs(flag) == 1
    sys=xd';
else
    sys=h';
end
else
    sys=[];
end
APPENDIX B: CALCULATION OF RMS VALUES IN ONE-THIRD OCTAVE BANDS

The RMS values of a time-varying function \( p(t) \) in one-third octave bands can be calculated by the MATLAB function \( \text{rmsv} \) listed below. The inputs to \( \text{rmsv} \) are vectors \( t \) and \( p \) where \( t \) is the time vector and the vector \( p \) represents the time-varying function. The outputs of \( \text{rmsv} \) are the vector of center frequencies \( f \) and the vector of RMS values \( \text{prms} \). The function \( \text{rmsv} \) requires the power spectral density function \( \text{psd} \) in the Signal Processing Toolbox.

```matlab
function [f,prms]=rmsv(t,p);
%KMSV [f,prms]=rmsv(t,p) returns the RMS value of a time-varying function p(t) in one-third octave bands. prms is the vector of RMS values and f is the vector of center frequencies.
%
% time-varying function p
ptab=[t p];
te=t(length(t));
Na=500;
ta=linspace(0,te,Na+1);
for j=1:Na+1,
    p(j)=table1(ptab,ta(j));
end
%
% Scaled power spectral density
na=100;
freq=logspace(-1,2,na+1);
X=p(j);
[Pxx,fp]=psd(X,[],Na/te); % Power spectral density
msv=trapz(ta,X.^2)/te; % Mean square value
fac=msv/trapz(fp,Pxx)/2/pi; % Factor for scaling
S=fac*Pxx; % Scaled PSD
%
% RMS value in one-third octave bands
psdtab=[fp S];
for k=1:na+1,
    fl=.891*freq(k);
    fu=1.122*freq(k);
    fb=linspace(fl,fu,11);
    for l=1:11,
        Sb(l)=table1(psdtab,fb(l));
    end
    prms(k,j)=sqrt(2*pi*trapz(fb,Sb));
end
```

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REFERENCES


